QC
851
.G73
V. 2

Air Ministry
Meteorological Research Committee
MRP 284-451 (non-inclusive)

MRP No.	Title	Author
284	The Audibility of the Burton-on-Trent Bomb Dump Explosion.	R.J. Murgatroyd
293	An Investigation of the Stability of Radio-altimeter S.C.R.-718.	C.J.M. Aanensen
297	A Preliminary Note on Summer Atmospherics.	C.K.M. Douglas
298	The Theory of Tropical Cyclones.	J.S. Sawyer
309	The Size Distribution of Cloud Particles.	A.C. Best
318	The Present Position of Weather Forecasting in Tropical Regions.	A.G. Forsdyke
319	An Appreciation of Various Methods of Measuring the Light Scattered by the Upper Atmosphere from a Search-light Beam.	F.E. Jones
320	The Diurnal Variation of Fog at Sea.	H. Jameson
328	A Review of Progress in the Development of Some Meteorological Instruments.	
329	Report of the Tropical Meteorology Panel.	
330	The Shape of Raindrops and Mode of Disintegration of Large Drops.	A.C. Best
331	An Approximate Method of Estimating the Pressure Pattern Flight Path for Minimum Flight Time in a Stationary System of Geostrophic Winds.	J.S. Sawyer
334	Seasonal Charts of the Mean Height of the Tropopause.	D. Dewar J.S. Sawyer
338	The Fine Structure of Wind in the Free Air.	C.S. Durst
341	High Altitude Sounding Balloons.	O.M. Ashford
344	A Note on the Adiabatic Lapse Rate for Saturated Air at Temperatures Below 0 Degrees C.	J.K. Bannon
346	Anomalous Sound Reception Experiments.	R.J. Murgatroyd
350	Index of M.R.P.'s Approved for General Distribution.	
352	The Size Distribution of Rain Drops.	A.C. Best

MRP No.	Title	Author
353	The Estimation of Large Scale Vertical Currents from the Rate of Rainfall.	J.K. Bannon
360	The Upper Atmosphere - Estimated Distribution of Temperature, Pressure and Wind up to the 45 Km Level.	A.H.R. Goldie
362	A Preliminary Study of the Vertical Exchange of Moisture Near the Ground.	F. Pasquill
363	A Tentative Evaluation of Evaporation From Grassland in Terms of Temperature Measurements Near the Surface.	F. Pasquill
364	Comparison of G.L.IIIB and S.C.R. 658 Wind Finding Equipment.	R.F. Apps
372	Solar Radiation at Kew Observatory.	J.M. Stagg
376	The Magnitude of the Casual Variations of Temperature in the Free Air.	R. Frith
377	A Contribution to the Study of Convection in the Lowest Layers of the Atmosphere.	G.D. Robinson
383	Note on the Insufficiency of Gradient Wind Approximation in Disturbed Conditions.	J.K. Bannon
387 AD 044465	The Errors of G.L.III Observations.	D.N. Harrison
392	The Angular Deviation of the Wind From the Isobars at Liverpool.	J.K. Bannon
398	The Development of the M.O. Radar Reflector Mk.IIB.	O.M. Ashford H.J. Ferrer
401	Index of M.R.P.'s Approved for General Distribution.	
402	Small Scale Temperature and Humidity Patterns in the Free Air.	R. Frith
405	The Speed Correction Coefficient of the M.O. Knife-edge Electrical Resistance Thermometer and its Variation with Air Density.	R. Frith H.C. Shellard
406	Errors in Winds Measured with G.L.III Radar Equipment.	J.K. Bannon
409	Upper Winds over the World.	C.S. Durst C.E.P. Brooks
422	The Accuracy of Mk.II R/S Observations.	D.N. Harrison

MRP No.	Title	Author
424	Eddy diffusivity and Conductivity Near the Ground and the Indirect Evaluation of Natural Evaporation.	F. Pasquill
428	Measures of Atmospheric Circulation.	A.H.R. Goldie
436	Meteorlogical Factors in the Occurrence of Turbulence at High Altitudes - Interim Report.	J.K. Bannon
437	A Note on a More General Criterion of Turbulence (after Richardson) and its Application to the Atmosphere.	J.K. Bannon
451	Index of M.R.P.'s Approved for General Distribution.	

8 5544

AIR MINISTRY

20th May, 1948.

METEOROLOGICAL RESEARCH COMMITTEE

INDEX OF M.R.P.'s APPROVED FOR GENERAL DISTRIBUTION.

(Appendix No.4. of M.R.P. 200)

M.R.P.	TITLE	AUTHOR
346	Anomalous Sound Reception Experiments	R.J. Murgatroyd
350	Index of M.R.P.'s approved for General Distribution.	
352	The Size Distribution of Rain Drops.	A.C. Best
353	The Estimation of Large Scale Vertical Currents from the Rate of Rainfall.	J.K. Bannon
360	The Upper Atmosphere - Estimated Distribution of Temperature, Pressure and Wind up to the 45 Km level.	A.H.R. Goldie
362	A Preliminary Study of the Vertical Exchange of Moisture near the Ground	F. Pasquill
363	A Tentative Evaluation of Evaporation from Grassland in terms of Temperature Measurements near the Surface.	F. Pasquill
364	Comparison of G.L. IIIB and S.C.R. 658 Wind Finding Equipment.	R.F. Apps
372	Solar Radiation at Kew Observatory	J.M. Stagg
376	The Magnitude of the Casual Variations of Temperature in the Free Air.	R. Frith
377	A Contribution to the study of Convection in the lowest layers of the Atmosphere	G.D. Robinson
383	Note on the Insufficiency of Gradient Wind approximation in Disturbed Conditions.	J.K. Bannon
385	Research Programme	
387	The Errors of G.L. III observations.	D.N. Harrison

G.198612/D.S./5/48.

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

NOV 17 1952

MRP's 284, 293, 297, 298, 309
MRP's 376, 379, 383, 387, 392

CONFIDENTIALLY Its contents must not be quoted.

M. R. P. 451.
20th November, 1948.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

INDEX OF M. R. Ps. APPROVED FOR GENERAL DISTRIBUTION
(Appendix No. 5 of M. R. P. 200)

M. R. P.	TITLE	AUTHOR
392	The Angular Deviation of the Wind from the Isobars at Liverpool.	J.K. Bannon
398	The Development of the M.O. Radar Reflector Mk. IIB.	O.M. Ashford & H.J. Ferrer
401	Index of M. R. Ps. approved for General Distribution	
402	Small Scale Temperature and Humidity Patterns in the Free Air.	R. Frith
405	The Speed Correction Coefficient of the M.O. Knife-edge Electrical Resistance Thermometer and its variation with Air Density.	R. Frith & H.C. Shellard
406	Errors in Winds measured with G.L. III Radar Equipment.	J.K. Bannon
409	Upper Winds over the World.	C.S. Durst & C.E.P. Brooks
422	The Accuracy of Mk. II R/S Observations.	D.N. Harrison
424	Eddy Diffusivity and Conductivity near the Ground and the Indirect Evaluation of Natural Evaporation.	F. Pasquill
428	Measures of Atmospheric Circulation.	A.H.R. Goldie
436	Met. Factors in the occurrence of Turbulence at High Altitudes - Interim Report.	J.K. Bannon
437 (Revised)	A Note on a more General Criterion of Turbulence (after Richardson) and its application to the Atmosphere.	J.K. Bannon

224033/PH/3/49/250.

Encl 118

MRP's 284, 293, 292, 298, 309
[illegible] 326, 329, 383, 387, 392

CONFIDENTIAL

Classification changed to unclassified by M.R.P. 305 of 30 Oct. 1946 JP

MRP 284

Air Ministry

27th November 1944

Meteorological Research Committee

THE AUDIBILITY OF THE BURTON-ON-TRENT BOMB DUMP EXPLOSION

by

F/Lt. R. J. Murgatroyd

I. INTRODUCTION

In the immediate region of an explosion sound travels at much higher speeds than normal, but in a very short distance starts to propagate at usual speeds determined by the temperature and wind structure in the first few thousand feet above the ground. Sound which travels in this layer of the atmosphere accounts for the "inner" zone of audibility of an explosion and it should be possible to relate the dimensions and shape of this zone to the wind and temperature structure of the lower troposphere. The intensity of the sound will generally decrease with distance from the source until it becomes inaudible. It has been found that at still greater ranges from the explosion it may become audible again, so that the general picture comprises an "inner" zone of audibility followed by a "silent" zone with an "outer" zone of audibility usually at ranges of 100-200 miles from the explosion itself. The sound reaching the outer zone is considered by many investigators to travel mainly in the stratosphere and attain a maximum height of about 40 - 50 Km., from the earth's surface. A knowledge of the location and shape of the outer zone will give a certain amount of indirect information on the wind and temperature structure at this height in the stratosphere. Although the main characteristics of the sound propagation are determined by the refraction in the atmosphere caused by wind and temperature variations with height, it would not be expected that the various zones of audibility will be well marked - with the large energy of a great explosion the effects of scattering, diffraction and multiple reflections are likely to be considerable. A description is given below of the results of an investigation of the audibility phenomena of the Burton-on-Trent explosion.

2. THE EXPLOSION AT BURTON-ON-TRENT

The disaster occurred at approximately 11h. 10 m. 39 s B.S.T. on 27th November, 1944 at an R.A.F. bomb dump near Hanbury, Burton-on-Trent. There were two explosions with 10 - 15 seconds interval between. The second explosion was much the greater of the two and about 3,000 tons of explosive were lost in all. A main crater about 900 feet long by 700 feet wide and 80 feet deep was formed with many small craters, and there was a considerable loss of life and damage to property. The explosion was described on the B.B.C. and reported in the Press at the time.

3. METHOD OF OBTAINING INFORMATION OF THE AUDIBILITY OF THE EXPLOSION

The following statement was issued to the Press on the 14th December, 1944. The request was also broadcast by the B.B.C.

"The Meteorological Research Committee of the Air Ministry wishes to know at what places in the British Isles the recent explosion at Burton-on-Trent was heard. Reliable information upon this point would be of considerable scientific interest. Members of the public who heard the explosion are invited to assist in this investigation by sending a postcard stating where they were at the time. Postcards should be addressed to ..... etc".

In addition, Royal Observer Corps posts and Meteorological Office Observing Stations were asked to give reports on the audibility of this explosion.

The response to this invitation was good. 1,395 useful replies were received, with about another 20 which could not be used. Although the request to the public only asked for reports of whether it was heard, in the majority of cases other useful information was added in the replies, such as descriptions of the sound, the time it was heard, effects due to inaudible air waves etc.

OCT 30 8 48 E 1946
WASHINGTON, D.C.

For various reasons the request for this information was made more than a fortnight after the explosion which was perhaps advantageous from the point of view that only people who had definitely noticed it would have a clear recollection of it. Many people stated that they had noted it in their diaries at the time. But it also had the disadvantage that most of the observers had had the opportunity to read the Press reports in which the time of the explosion and its nature was described. This tended to make some include details of the type which can only have been imagined. In many replies for instance it was stated that the explosion was heard at 11.10 exactly in spite of the fact that the observer may have been up to 100 miles from the explosion and sound only travels at about 750 m.p.h.

One of the greatest difficlties in this investigation was, therefore, to separate the reliable and unreliable reports and the attempts to do this are described below.

ANALYSIS OF REPORTS

Of the 1,395 replies 1,045 observers heard the explosion and 850 observers felt its effect e.g. by rattling of windows, doors, etc. Some observers both heard and felt it. In Fig. 1 all these observations are plotted separately. The replies were separated into areas - small Ordanance Map Squares being used as convenient units and the reception over each area was then studied in detail. For some squares there were many corraborating replies with agreement as to times of reception, nature of sound, etc, and these were considered as areas of "certainly heard" and are shown in Fig. 1. Other areas where, according to the analysis there was a high probability of sound reception but not certainty, are delineated differently in Fig. 1. In the remaining areas where there were reports all the observation points are plotted and some idea can be gained of the possibility of reception in other areas.

The zones given are therefore tentative and it is instructive to compare them with apopulation map (Fig. 2). The existence of a silent zone between an inner and outer zone to the east of the explosion is probable although it is not well confirmed. Certainly there are no well marked zones as have been described in connection with previous investigations of this type.

Considering first the inner zone there appears to be little doubt that the explosion was heard practically everywhere to the South of Burton-on-Trent between azimuths of about 120° - 210° up to ranges of 100 - 150 miles. The reports at still greater ranges to the South, are not improbable.

To the West there are very few reports from Wales and at a first glance that would be explained by the low population of central Wales. On the other hand if the propagation had been symmetrical more reports would have been expected from the heavily populated South Wales and Lancashire areas.

To the North sound was certainly heard in the Sheffield area, although it is not so certain it was heard in North Derbyshire.

To the North-East and East it was certainly heard in the Grimsby, Hull and Wash areas. It is thought that this zone constitutes the outer zone, and what silent zone there was existed mainly in Lincolnshire and South Yorkshire.

As well as the plot of observations and boundaries of zones, Fig. 1 contains:

(i) Lines which would define the theoretical azimuth boundaries of the inner zone. This is described in paragraph 9(a).

(ii) A line which shows the inner boundary of the area considered to be the outer zone. This seems to be fairly well defined to the North-East. In the North it includes the observations in

the Sheffield area, as it is difficult to ascribe these to any other sound paths. In the East it is also fairly well defined although the increase of observations in the Wash area tends to agree with a relative increase in population there.

The suggestion that the inner zone and outer zone tend to amalgamate to the South-East is based on the fact that two sets of sounds with about 90 secs. interval between them were recorded on the Kew seismograph, in addition to the seismic wave from the explosion. The sound which arrived first i.e. by the lower path had by far the greater intensity of the two.

(iii) The "audibility bowl" contours upwind for the lower atmosphere; these are described in paragraph 9(b).

In drawing the zones as shown it can be seen that many of the observations have been neglected. Many of these may be due to diffraction effects and the other possibility is that of confusion with other sources of sound at the same time. The results of the various attempts to find what other large sounds were possible at the time, are listed below:-

(i) There were no V weapons falling on this country at that time.

(ii) No aircraft crashed about that time.

(iii) At the various Army Firing Ranges it was ascertained that 25 pounders were being fired at Larkhill, Salisbury Plain and 6 pounders on the Yorkshire moors. The sound of these guns is not likely to be confused with that of a heavy explosion.

(iv) There were no reports of any naval firing of heavy guns.

(v) The R.A.F. were carrying out bombing practice at several ranges and were using 10 lb. smoke filled bombs.

It was not possible to check on blasting at quarries etc. and there were several other possibilities e.g. bomb disposal, but as far as could be ascertained there were no other large sources of sound at about the same time as the explosion. Thus many of the reports are unaccounted for in the zones drawn on Fig. 1, and hence the zones shown can only be regarded as the most probable areas of audibility.

5. REPORTS OF THE TIMES AT WHICH THE SOUND WAS HEARD

In Fig. 3 two circles are drawn about the point of explosion showing where the sound would have been heard at 11.15 a.m. and 11.20 a.m. respectively on the assumption that it travelled along the surface at 337.6 m/sec. This was simply calculated once the true time of the explosion was known. The latter was calculated to be 11 h. 10 m. 38 s. B.S.T. by Mr. JJ Shaw of West Bromwich from his seismological observations and 11 h. 10 m. 39 s. B.S.T. by Dr. A. Beer from the seismological observations at Kew Observatory, the difference being negligible as regards the sound propagation. The only instrumental recordings of the sound also come from Kew; the travel time of the sound in the path in the lower atmosphere being 542 secs. and that in the path in the upper atmosphere 633 secs. over a distance of about 180 Km. from Burton-on-Trent to Kew. Fig. 3 gives a plot of the times recorded by observers who glanced at their clocks when they heard the explosion. Within the usual errors of ordinary household clocks and watches these results agree very well with those expected and supply a certain amount of confirmation of the zones shown.

6. REPORTS OF MULTIPLE ARRIVALS

In previous investigations of this type it has often been remarked that although there may have been only one explosion many observers frequently heard two or more reports. Unfortunately in this case there were two explosions although one was much larger than the other, and so it is not possible to

distinguish between the possibilities of (a) the direct reception of the two sounds and (b) the complete attenuation of the smaller sound accompanied by the splitting into multiple reports of the larger sound. The observations of multiplearrivals are plotted in Fig. 4 and the theoretical aspect of multiple arrivals is discussed in paragraph 9 (c).

7. NATURE OF THE SOUND AND OTHER EFFECTS:

The various observers described the sound as a heavy long rumble, boom or thud, a muffled report, a dull explosion, or simply stated that it was heard quite distinctly. Estimates of its intensity varied considerably in the same locality. It was generally recognised as being different from local blasting and was often compared to thunder or to bombs and V weapons bursting at various distances. When several explosions were heard it was notalways agreed which was the loudest. In some cases accurate estimates of the direction from which the sound arrived were given, and occasionally the sound was thought to come downwards from the sky.

In addition to the reports of audibility, there were many reports of other effects particularly the rattling of windows and doors or the shuddering of the whole building. In some cases multiple arrivals were noticed when a door or window rattled, stopped, and then rattled again. Reports of earth tremors were common - sometimes the ground was felt to move and other times objects fell over, or a bed or chair was felt to vibrate even at a distances of 100 miles or so from the explosions. At shorter distances the effects were even more pronounced and reports of curtains blowing in, broken windows, cracked ceilings and chimney pots were common. Several meteorological stations reported that a barograph had time-marked itself. It is not known how many of these various effects can be attributed to seismic waves and how many to infrasonic air waves. It was noted in the case of large cities comparatively near the explosion, e.g. Birmingham and Leicester, that the sound was observed with great intensity on the outskirts and was hardly heard at all in the centre of the city; this was due either to the noise of the traffic or the effect of being in a sound shadow.

It was usually found that the earth tremor was observed first, then effects of inaudible waves such as windows rattling, and finally the sound itself. Earth waves, of course, travel much faster than sound waves and therefore a tremor naturally would be observed first. But as regards the infrasonic and sound waves, it is not thought that the speed of sound in air varies at all appreciably with its frequency, but rather that the waveform of the sound wave would modify as it progressed, with high frequencies suffering greater attenuation than low frequencies. The observation of relative arrival times of infrasonic and audible waves wouldthen depend on the effect of this on the orginal wave form of the explosion wave. This would also explain the preponderance of "felt" reports at the longer ranges in the "inner" zone of audibility.

Various effects were noticed on birds and animals. Cock pheasants screeched in Yorkshire, Kent, Norfolk and Warwickshire. In the latter county also dogs howled, cows ran, chickens and rooks flew, and ducks paddled to the side of a pond. The Norfolk report stated that the pheasants screeched before the sound was heard. It was noticed that a large tarpaulin first sucked in and then blew out in a wood in Bedfordshire. No reliable reports of the duration of the sound were available although there were various estimates ranging from 1 second to 3 minutes.

In North Wales, a pilot who was flying at 2,500 feet felt violent turbulence lasting for 2 minutes, then eventually dying at 11.19 hours. It is difficult to connect this observation directly with the explosion at Burton-on-Trent.

8. WIND AND TEMPERATURE VALUES AT THE TIME OF THE EXPLOSION

Tabulated below are the wind and temperature values up to 60,000 feet given by the midday radio-sonde ascents at Liverpool, Downham Market and Larkhill on 27th November, 1944.

Height in feet	Liverpool Temp. °F	Liverpool Wind Direction	Liverpool Wind Speed f.p.s.	Bowman Market Temp. °F	Bowman Market Wind Direction	Bowman Market Wind Speed f.p.s.	Larkhill Temp. °F	Larkhill Wind Direction	Larkhill Wind Speed f.p.s.
0	35	130	9	41	Calm		40	360	9
5,000	29	180	13	26	040	17	29	340	18
10,000	17	300	24	10	360	19	14	320	47
15,000	5	310	48	- 1	350	45	3	340	69
20,000	- 7	320	90	-13	350	99	-7	340	114
25,000	-28	320	111	-31	340	162	-26	340	138
30,000	-46	330	133	-52	340	180	-49	340	165
35,000	-66	320	159	-71	340	183	-71	350	165
40,000	-82	330	171	-78	340	195	-82	350	117
45,000	-78	340	90	-73	330	81	-72	340	93
50,000	-71	330	54	-73	330	61	-67	320	67
55,000	-69	310	443	-73	330	41			
60,000	-70	300	51	-70	330	37			

These values have been used for the theoretical discussion below. Further data on the weather at the time can be obtained from the Daily Weather Report for 27th November 1944.

9. THEORETICAL ASPECTS

(a) The a + w curves - good audibility considerations

Through the atmosphere the speed of sound a is independent of pressure and varies little with humidity. It is proportional to the square root of the absolute temperature T so that $a = \text{constant} \times \sqrt{T}$. The velocity relative to the ground when the sound "ray" is horizontal is given by the sum of a and w where w is the wind component blowing along the azimuth considered. The condition for good audibility is that sound propagated into the air shall be refracted down to the ground again and this occurs when the function a + w increases with height. a increases with height under inversion conditions and decreases with height under lapse conditions. w increases with height in a manner depending on the variation of both wind speed and direction with height. The effects of a and w will tend either to oppose or reinforce each other and the final a + w values along any azimuth will determine the audibility pattern along that azimuth. If the surface value of a + w is more than that at any greather altitude, sound rays will all be refracted upwards and theoretically there will be no audibility on the surface except that due to scattering or diffraction. In the case where the a + w values increase with height, the height of the maximum value of a + w will determine the greatest height attained by any sound waves returning to the ground on that particular azimuth.

The a + w curves for the wind and temperature structure of the Burton-on-Trent explosion have been drawn for azimuths at intervals of 30° on Figure 5. It will be seen that the surface value of a + w is only exceeded at greater heights on azimuths between above 120° and 210° below heights of 60,000 feet. Hence, as regards sound waves travelling in the troposphere and being refracted down to produce the "inner" zone of audibility, we should only expect good audibility between azimuths of 120° and 210°. This is in fair agreement with the reports plotted and zones shown in Figure 1, and the boundary azimuths of 120° and 210° as defined by the a + w curves are shown on that Figure.

It can also be seen from Figure 5 that the maximum value of a + w tends to occur at heights up to 25,000 or 30,000 feet between 120° and 210°, and the sound rays travelling in the "inner" zone must therefore have been able to attain maximum heights of this magnitude. It should be noted that since these curves only extend up to the limits of measurement of wind and temperature available, (60,000 ft.) they give no hint of whether sound rays penetrating to greater altitudes could be refracted down to the ground. The condition for

this is that the surface value of a + w along the azimuth considered shall be exceeded at these great heights. If we dismiss the possibility of any rays being reflected down at a sudden discontinuity in the atmosphere it will be seen that a sound ray travelling to a great height will necessarily have attained a considerable range from the explosion point before it reaches a height at which the surface value of a + w is exceeded. Any ray which reaches the higher stratosphere and returns to the surface will then have a minimum range at which it returns to the earth's surface. In this case it will be the distance to the Sheffield area "outer" zone to the north, the Hull-Grimsby "outer" zone to the north-east and the Wash "outer" zone to the east. The surface a + w values along these azimuths are all approximately 1100 ft/sec. The values of a + w at 60,000 feet are about 900 ft/sec. It appears from the rates at which a + w vary with height in the troposphere that values of 1100 ft/sec. are not likely to be attained below above 90,000 or 100,000 feet at the least. Hence at heights of 100,000 feet or more, the a + w value in the north-easterly direction is at least 1100 ft/sec. There is no means of separating the effects of a and w except to note that since the audibility appears to be towards the east and north-east and not the west and north-west, some at least of the effect must be due to a following wind.

Some various possibilities with a south-westerly or west-south-westerly wind at 100,000 feet or more are;-

a + w	a	w (ft/sec)	Temperature °F
1100	950	150	-100
	1000	100	- 50
	1050	50	0

The stratosphere temperature at about 60,000 feet was approximately -70°F. A series of wind measurements carried out recently at 100,000 feet over the whole year has shown that speeds of 100 m.p.h. from the west at this level are quite usual in winter. It is therefore not unreasonable to suppose that the "outer" zone was due in part to sound waves attaining heights of about 100,000 feet, with the stratosphere remaining isothermal up to this level and a west-south-westerly wind of abut 100 m.p.h. blowing at this height. This is, of course, only one possibility and is not necessarily in conflict with the hypothesis of "outer" zones of audibility being mainly a result of high temperatures at great heights. The whole question depends on the variation of a + w with height, and the only fact that can be deduced with any certainty is the likelihood of a wind mainly from the west at these heights at the time of the explosion.

(b) The a + w curves - poor audibility considerations

Between azimuths of 210° and 120° we should expect theoretically no audibility due to sound travelling in the troposphere only. Even a sound ray starting horizontally (the "grazing ray") will be bent concove upwards and leave the earth's surface. If the path of this ray is worked out along the various azimuths, the envelope within which sound can be heard, or the so called "audibility bowl", can be plotted. If the ranges, at which this "grazing ray" attains a height of, say, 20,000 feet, are joined up on a surface map, it can be seen at a glance how high up in the air an observer would theoretically need to be hear the explosion. (This problem is simply the reverse of the problem of determining at what range an approaching aircraft can be heard. Theoretically the aircraft can only be heard when it enters the "bowl of audibility" - the source of sound the, of course, being up in the air). These contours then indicate that from pure considerations of refraction the sound from the "grazing ray" had attained a height of about 50,000 feet by the time it was over, say, Sheffield (Figure 1). It seems that while the grazing ray" was below 20,000 feet approximately between the azimuths of 210° and 120° , the sound was audible on the surface due to diffraction and scattering effects from the edge of the

sound envelope, and the shape of the "audibility bowl" and the "inner" zone in the "upwind" direction tend to coincide. The region of good audibility around Sheffield is thought to be due to refractions down from the high stratosphere. The envelope of this sound as it descends will also produce an extension of the "outer" zone towards the explosion point due to diffraction effects.

(c) The a + w curves - considerations of multiple arrivals and timings

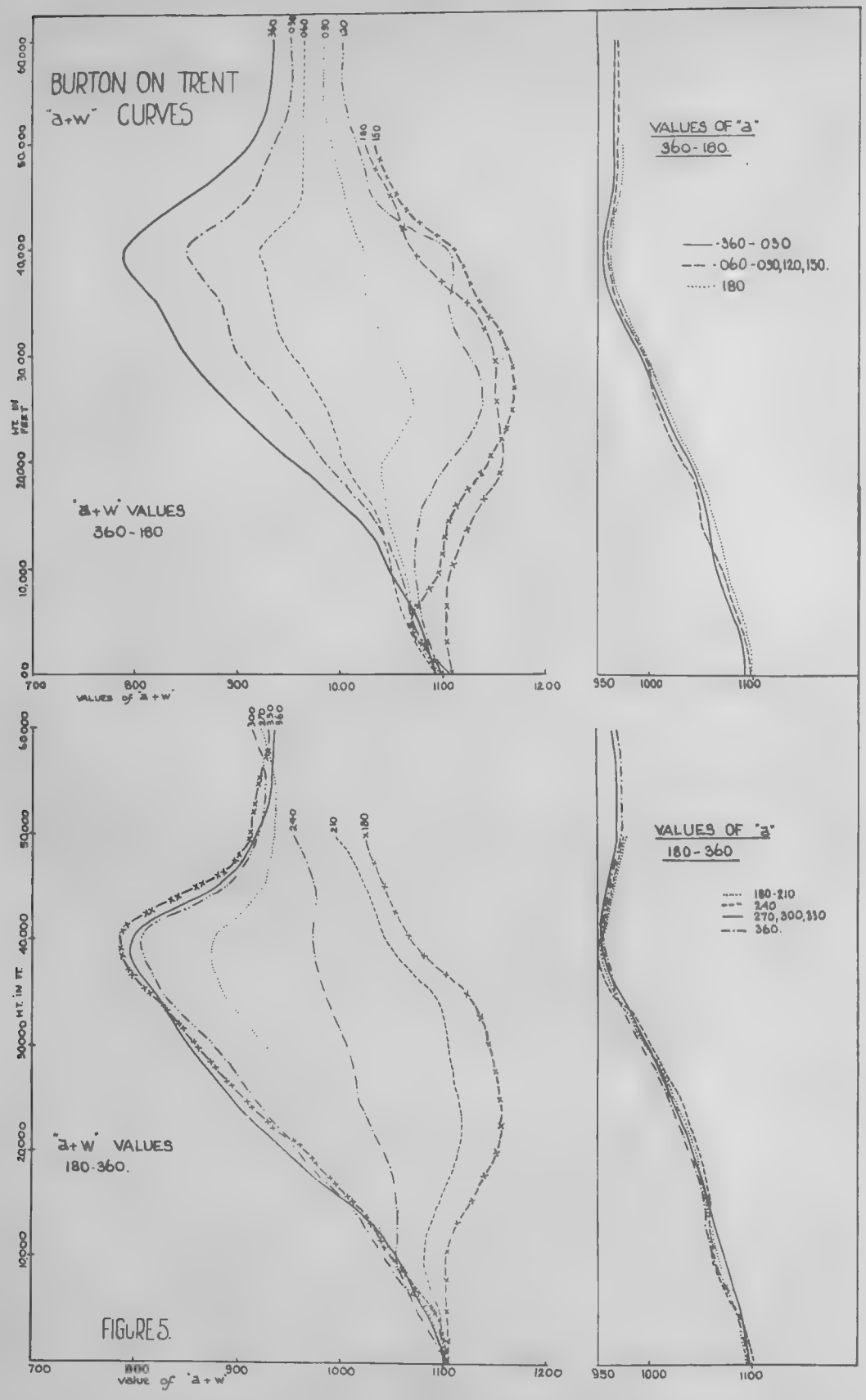
In previous investigations of this type no explanation has been given of multiple arrivals. Also the work here on multiple arrivals is inconclusive. Types of a + w curves from which multiple arrivals are likely are shown in Figures 6A and 6B, which show two a + w curves and some of the resulting sound paths.

In case B an a + w curve is shown with a maximum above the surface and a greater maximum at a higher altitude. When an a + w curve becomes vertical the sound can attain any range theoretically up to infinity. If A 1 is the maximum value at the lower height, there will be a continous variation up to infinity of range with height of rays attaining maximum heights below that corresponding to A1. Between A1 and A2 all rays will be refracted upwards and then when A1 is exceeded they will be refracted down again. Hence there will be regions where both rays not quite reaching the height corresponding to A1 and those reaching heights slightly greater than that corresponding to A2, will descend. The time each of these rays takes over its path will in general not be equal, and hence there will be ranges where double reports from a single explosion will be heard. The question as to which of two rays reaching the same range arrives first, depends on their respective path lengths and the average speed at which they travel in these paths, and this may differ for different shapes of a + w curves.

In case A is shown the simplest form of a + w curve which would account for the phenomena of an "outer" zone with a minimum range to its "inner" boundary and double arrivals within its zone. It can be shown that as a first approximation the range X of any sound ray is given by

$$X = \text{constant} \int_0^H \frac{dh}{\sqrt{A - (a + w)}}$$

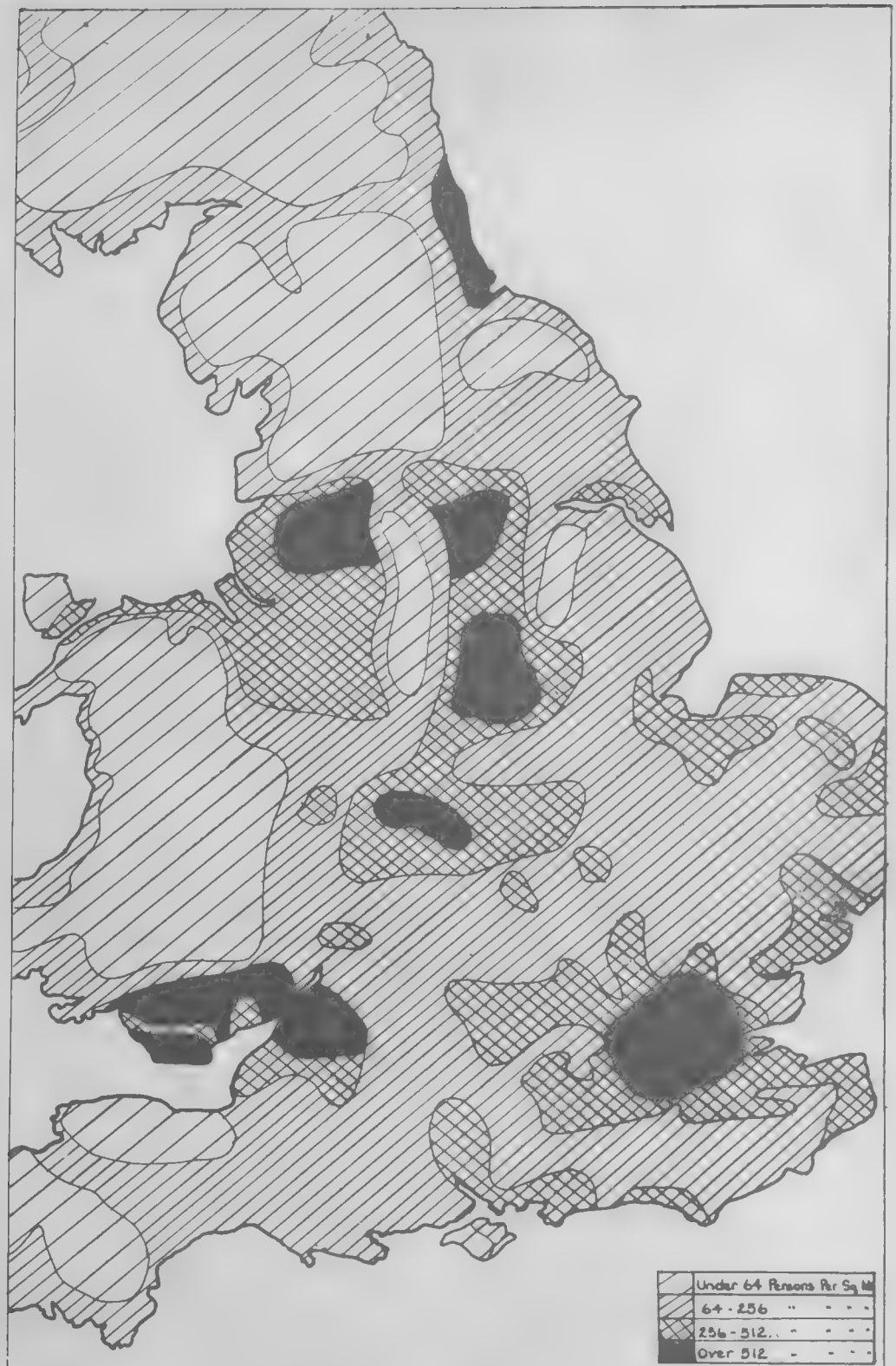
where H is the maximum height attained by the ray and A the value of (a + w) which occurs at that height. If we consider a ray with a maximum (a + w) value of A3 this will have a very large range because H is very large. If we consider a ray having a maximum (a + w) value of A1 it will also have a very large range because $\frac{1}{\sqrt{A - (a + w)}}$ is very large both near the surface and near the height corresponding to A1. Somewhere between A1 and A3 there will be a ray with maximum (a + w) value of A2 which will have less range than either those going above or below it. This will account for the minimum range of the "outer" zone which is often experienced. At greater ranges than this two reports will generally be received from one explosion. It is possible to get combinations of both 6A and 6B in order to obtain three or four arrivals, In addition there is no reason why on certain azimuths both rays in the troposphere and those travelling up to the stratosphere whould not be received, providing t at the lager value of A, when the "inner" and "outer" zones will amalgamate. It is thought that this may have happened in the London area in this case.



Printed by Photographic Reproduction Branch, Air Ministry. London. S.W.1. 1/1183.

FIGURE 1.

Printed by Photographic Reproduction Branch Air Ministry. London. S.W.1. 1/1183



POPULATION MAP

FIGURE 2.

Printed by Photographic Reproduction Branch, Air Ministry, London, S.W.1. 1/1183

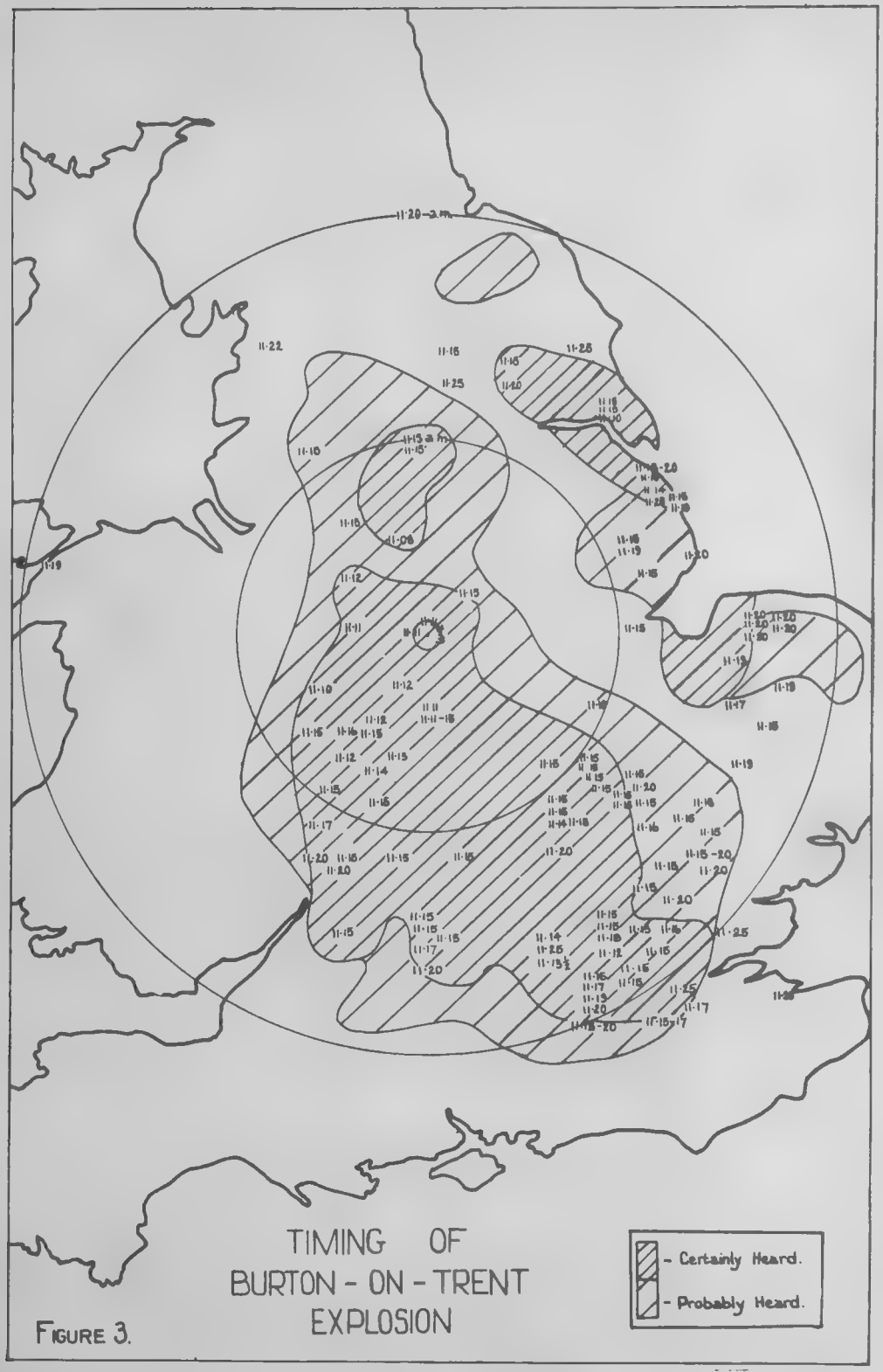
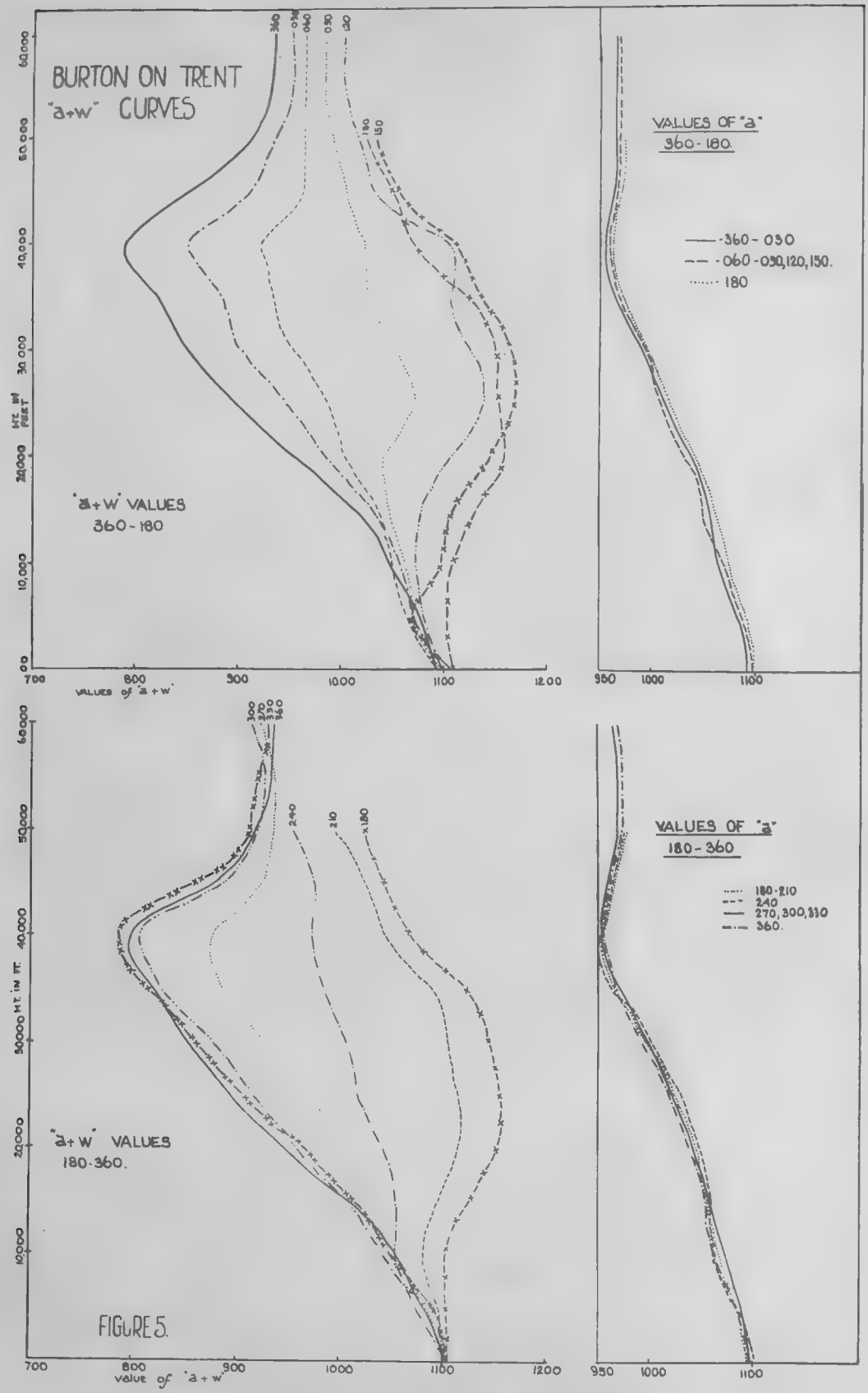


FIGURE 3.

Printed by Photographic Reproduction Branch, Air Ministry, London, S.W.1. 1/1183

FIGURE 4.

Printed by Photographic Reproduction Branch, Air Ministry, London, S.W.1 1/1183



Printed by Photographic Reproduction Branch, Air Ministry, London, S.W.1. 1/1183

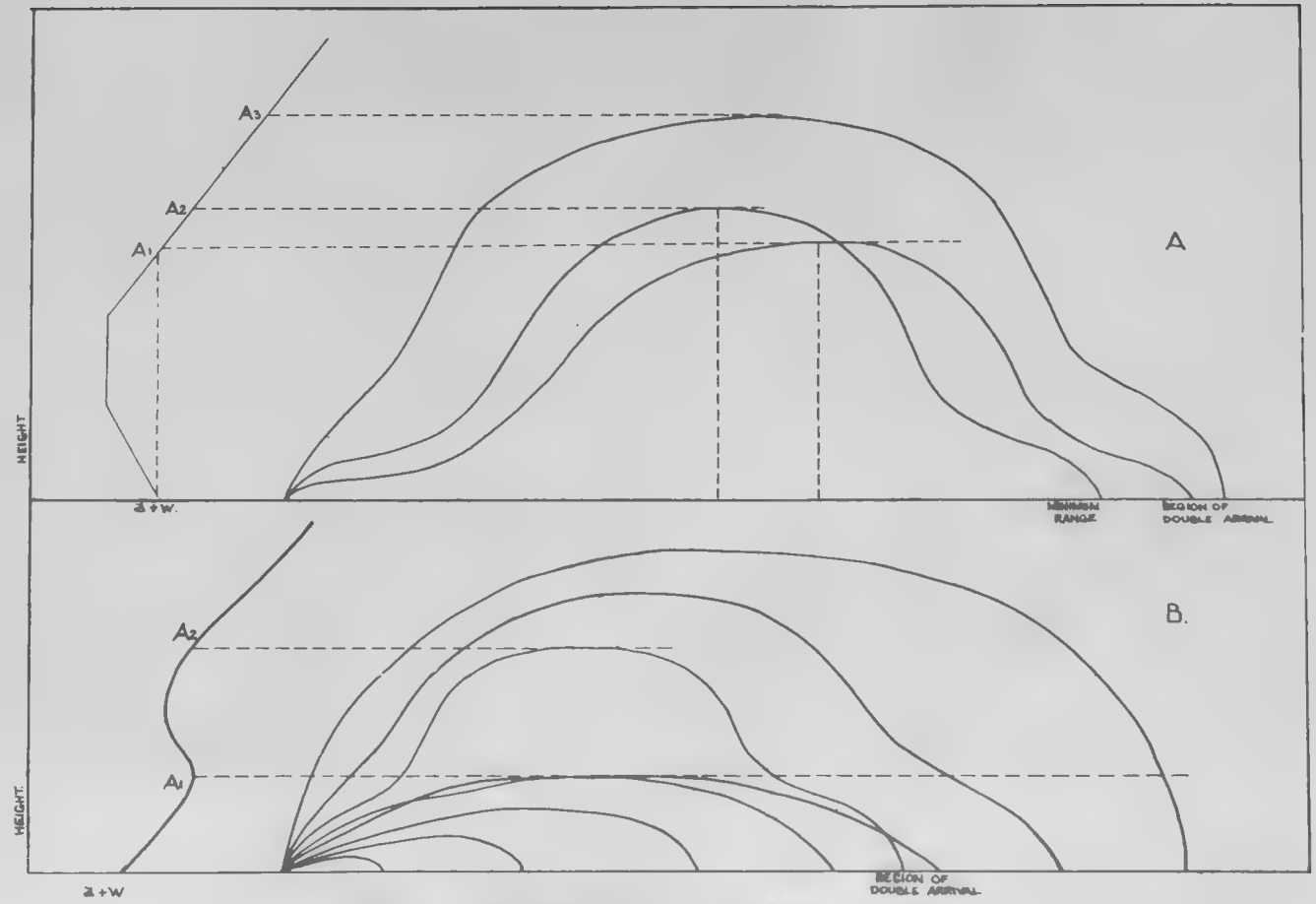


DIAGRAM ILLUSTRATING CASES OF DOUBLE ARRIVAL

FIGURE 6.

Printed by Photographic Reproduction Branch, Air Ministry, London, S.W.1. 1/1183

IJPE

Acc. #7.1980

M.08.133.01

Air Ministry
Meteorological Research Committee

MRP 293
13th June 1946

An Investigation of the Stability of Radio-altimeter S.C.R.-718

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

by

C.J.M. Aanensen.

This report has not yet been published and is to be treated CONFIDENTIALLY. Its contents must not be quoted.

Summary. Meteorological reconnaissance flights including a high level leg offer a possibility of testing the stability of the radio-altimeter. Statistical analysis of observations shows that there is no evidence for instability of the radio-altimeter, though there is some evidence for a small systematic difference between changes of height as observed with the radio-altimeter and as calculated from pressure, temperature and humidity observations.

1. Meteorological reconnaissance flights include a climb from near sea level to 500 mb. (about 18,000 feet), a flight of a certain distance (varying between 250 and 400 nautical miles according to the track being flown) at 500 mb., followed by a descent to near sea level. The ascent and descent take place many miles from land and each is made in the form of a spiral or "box" not extending more than 10 miles from a specified position. Temperature, humidity and pressure are observed throughout the flight; sea level pressure measurements are made at the foot of the ascent and descent. The information thus obtained allows the height of the 500 mb. pressure level to be calculated at the positions of ascent and descent. These heights can be compared with heights read from the radio-altimeter SCR-718 which is carried in the aircraft.

2. Let the calculated height of the 500 mb. level at the ascent be h_o.

Let the calculated height of the 500 mb. level at the descent be h_1.

Let the height of the 500 mb. level at the ascent recorded by the radio altimeter be H_o.

Let the height of 500 mb. level at the descent recorded by the radio altimeter be H_1.

Then the change in height along the 500 mb. leg according to calculations $= h_1 - h_o$

and the change in height according to radio altimeter readings $= H_1 - H_o$

The difference, $\Delta = (H_1 - H_o) - (h_1 - h_o)$, will give information regarding the stability of the radio-altimeter during the run on the high level leg, within the accuracy available from the method of calculation using pressure, temperature and humidity readings.

3. The errors of the calculated height of the 500 mb. level arise from (1) errors of temperature and humidity determinations, (2) error of the sea level pressure determinations, and (3) error arising from aircraft position whereby the sounding is not strictly vertical. In addition, in comparing calculated heights with radio-altimeter heights there is a fourth possibility of error, one in navigation whereby at the time of the radio-altimeter reading the aircraft is not vertically above the position at which the sea level pressure has been or will be found. Moreover underlying the derivation of Δ also is the assumption of a static atmosphere, that is that the hydrostatic equation $\partial p/\partial z = -g\rho$ is valid; this is not always strictly true and in bumpy conditions may lead to finite errors in Δ, but in general the assumption of a static atmosphere will hold good. It is difficult to assess the magnitude of these various errors but the following is given as a considered opinion. The probable error arising from

(1) is about ± 15 feet (see Appendix I); that from (2) ± 30 feet (this corresponds to ± 1.1 mb. reported in S.D.T.M. No. 106); that from (3) is very difficult to judge, and in consideration of the magnitude of the difference, Δ , the errors arising from (3) will be inextricably mixed up with the fourth error. The combined effect of these two can be roughly treated as the difference in the height of the 500 mb. level between the positions of the bottom and top of the sounding. A sounding takes about half-an-hour and if the navigator uses winds which are in error by, say, 50 knots on the average the aircraft will drift 25 n. miles during the sounding. At the worst this drift could be across the 500 mb. contours and, with a mean wind of say 40 knots the difference in height of the 500 mb. level at the two ends of a sounding would be about 30 feet. Since in general the error of navigating winds will not be 50 knots and neither will the aircraft drift across the contour it is likely that the probable error will not be more than ± 20 feet.

It follows that for purposes of comparing heights the probable error of a simple calculated height

$$= \pm \sqrt{30^2 + 15^2 + 20^2} = \pm 39 \text{ feet}$$

Hence the probable error of the change of calculated heights along the high level leg

$$= \pm 39\sqrt{2} = \pm 55 \text{ feet.}$$

4. The makers of the radio-altimeter S.C.R.-718 state that the measurements of height are subject to an error of 50 feet + 0.25% of height, which for 18,000 feet is 50 + 45 = 95 feet. (The stated error is split into two parts because it arises from two distinct causes. The first term, 50 feet arises from the counting mechanism of the radio-altimeter; the minimum unit of length is 100 feet, so that height cannot be measured to a greater accuracy than ± 50 feet. The second term arises from the usual type of instrumental errors). If we assume that this is a 95% error the corresponding probable error is ± 32 feet.

The instructions for Meteorlogical Air Observers state that any radio-altimeter height reported shall be the mean of 4 readings. Hence the probable error of the mean reading should be ± 16 feet.

The probable error of the change of error during the high level leg as obtained from the radio-altimeter should therefore be

$$\pm 16\sqrt{2} = \pm 23 \text{ feet.}$$

Hence if the manufacturer's figures and our assumptions are correct we can expect a probable error in Δ

$$= \pm\sqrt{55^2 + 23^2} = \pm 60 \text{ feet}$$

We shall now see how the probable error of the actual observations compares with this figure.

5. Equipment of meteorological reconnaissance aircraft with radio-altimeters commenced in the summer of 1945. By the end of September enough observations had accumulated from four flights to warrant preliminary analysis. The flights * concerned were Epicure, Bismuth, Allah and Sharon. The lengths of the high

/level

* The approximate positions of the ascent and descent, between which the flights at 500 mbs. take place, on the Meteorological Reconnaissance Flights are as follows

Allah	48°N.	20°W.	and	52°N.	18°W.
Sharon	47°N.	19°W.	"	44°N.	23°W.
Epicure	45°N.	17°W.	"	47°N.	10°W.
Bismuth	55°N.	23°W.	"	61°N.	17°W.
Nocturnal	38°N.	20°W.	"	37°N.	13°W.

level leg were 350, 400, 250 and 250 nautical miles respectively. The observations of Allah and Sharon were combined since the lengths of the high level leg were identical. For the three lengths of high level leg for which information was available the mean values of Δ and the probable errors were calculated; the results are given in Table 1.

Table 1. Analysis of values of Δ obtained to end of September.

Flight.	Allah and Sharon	Epicure	Bismuth	All Flights
Length of high level leg (n. miles)	250	350	400	-
No. of Differences Δ	87	56	63	206
Mean value of Δ (ft)	-20	-27	+0.3	-12.4
Probable error of mean Δ (ft)	±7.1	±4.9	± 14	± 5.3
P.E. of a single Δ (ft)	± 67	± 37	±110	± 76

6. In considering these results it must be remembered that the aircraft of all the flights were equipped with the same type of radio-altimeter and that operating instructions were identical. Any differences in the results must be due to errors, avoidable and unavoidable, on the part of the aircraft crew and differences in maintenance of the radio-altimeter. From an inspection of Table 1 the following appears:-

(1) There is considerable evidence that the mean value of Δ is negative. The Bismuth observations do not support this conclusion but they are less reliable than the others (as may seem by comparing the probable errors). This negative mean difference can only be interpreted as (a) the S.C.R.-718 indicates heights which are too low after a run of about two hours duration or (b) the P-T-U method of calculating heights has the reverse error. There is no evidence to show which of these suggestions is correct.

(2) There is no clear evidence that the value of Δ increases with length of high level leg, i.e. with duration of flight.

(3) There is a very considerable difference in the size of the probable errors of one squadron compared with another. It is possible that this is due to better instrumental maintenance and more careful observation. Another suggestion, which seems to have some truth behind it, is that the bumpier conditions associated with the more northerly track of Bismuth may contribute towards the increase in the probable error of Δ, since the calculation of height as pointed out earlier, depends on the hydrostatic equation.

(4) The size of the mean value of Δ for all flights is of the order expected from the considerations of paragraphs 2 - 4 above. The small probable error of Δ for Epicure flights is however noteworthy. It is considerably less than that expected and this can only mean that we have considerably overestimated the size of the larger component probable errors. Returning to the considerations of paragraph 3, the largest component probable errors were associated with the sea level pressure measurement and must have been less than as assessed in paragraph 3.

7. Before giving further attention to the question of errors it is of interest to consider what happened after the first two months. Arrangements were made to ensure that servicing instructions were uniform throughout the Meteorological Reconnaissance Squadrons and as a result certain maintenance personnel were interchanged between squadrons. It was however unfortunate that the Allah and Sharon flights were discontinued and the information from the other flights became less regular because of maintenance problems consequent upon demobilisation. The following is an analysis of the observations received from Nocturnal flight for July to December and for Epicure flight for the period November to February.

Table 2. Analysis of further values of Δ .

Flight.	Epicure	Nocturnal
Length of high level leg (n. miles)	350	350
No of Differences (Δ)	59	92
Mean value of Δ (ft)	-1.7	- 10
P.E. of a single difference (ft).	$\pm$ 57	$\pm$ 35

8. It is noticeable that for Epicure the mean value of the difference Δ has decreased arithmetically, whilst the probable error has increased almost up to the originally expected value. On the other hand the figures supplied by Nocturnal flight which gives a negative value for the mean Δ , show a probable error almost identical with that of the original Epicure value (in Table 1). One seems bound to draw the following conclusions:-

(i) that the mean value of Δ is about - 10 feet.

(ii) that with careful maintenance and experienced crews working under good conditions the probable error of a single Δ is about $\pm$ 35 feet.

It may be noticed in passing that part of the deterioration of probable error on Epicure flights may be ascribed to the increased winds, bumpiness and poor weather of the winter compared with mid-summer. Apart from the invalidity of the hydrostatic equation in bumpy conditions it is the general experience of Meteorological Air Observers that good measurements of the various quantities concerned in the evaluation of Δ can only be obtained in good flying conditions.

9. Finally we may consider what alterations to the assumptions of paragraphs 2 - 4 can be made in order to reduce the expected p.e. of Δ from $\pm$ 60 ft. to $\pm$ 35 feet. It is obvious that the assumed value of $\pm$ 1.1 mb. for the p.e. of a sea-level pressure observation must be reduced. A lower limit of this p.e. must be about $\pm$ 0.5 mb. (equal to $\pm$ 15 feet for height calculation). This alteration will bring the expected p.e. of Δ to $\pm$ 47 feet which is still larger than the lowest actual value. To reduce the value sufficiently a reduction must be made in the assumed values of the other errors; however we have no evidence to show by how much each one must be diminished. It is hardly likely that any one kind of error considered above should be negligible but it is of interest to note that the following values of indivudal errors would give a probable error of Δ of $\pm$ 35 feet.

/P.E.

Appendix I.

Estimation of the probable error of height calculated from aircraft measurements of temperature, pressure and humidity.

There are no appreciable systematic errors in temperature caused by radiation effects or by thermometer errors. The casual errors arise from:-

(i) error of reading of thermometers (about $\pm$ 0.1°F on reconnaissance flights by Meteorological Air Observers).

(ii) errors of correction for airspeed (about $\pm$ 0.2°F)

(iii) temperature equivalent of pressure error

(iv) reporting error

(v) humidity correction error

(vi) error in reduction of pressure, temperature data to heights arising in method of computing.

Of these (i) and (ii), combined give probable error = $\pm$ 0.22°.
The temperature equivalent of the pressure error (iii) = $\frac{\partial T}{\partial p} \cdot \Delta p$ where Δp is the pressure error. Mk.II aneroids may not be used with more than $\pm$ 6 mb. error in one direction at any point of the scale (the method of describing this tolerance is actually in terms of means of rising and falling pressures and deviation from the mean). The distribution of errors is not linear since the aim is for zero error, hence the probable error will not be as much as $\pm$ 3 mb. Suppose it is $\pm$ 2 mb. Then for I.C.A.N. conditions the p.e. at various pressures is given by

	1000 mb.	750 mb.	500 mb.
$\partial T/\partial p$ (°F/mb.)	0.10	0.13	0.20
p.e.	$\pm$ 0.20	$\pm$0.26	$\pm$0.40

Since in obtaining height the logarithmic mean temperature is involved we use the logarithmic mean error over the interval 1000 to 500 mb. This is 0.29°F.

The humidity error (v) is quite small (of the order of 0.1°F. for 10% humidity error in terms of virtual temperature) and can be neglected.

Since temperatures are reported to the nearest whole degree Fahrenheit the distribution is uniform and the probable error in reporting is $\pm$ 0.25°.

Hence the total probable error in the mean virtual temperature error

$$= \pm \sqrt{.22^2 + .29^2 + .25^2} = \pm 0.45°\text{F. approx.}$$

The equation connecting equivalent errors in height (Δh) and mean temperature ($\overline{\Delta T}$) is Δh (feet) = $122.8\ \overline{\Delta T} \log_{10} \frac{(1000)}{p}$ ..... for $\overline{\Delta T}$ in °F.

Hence for a sounding to 500 mb.

$$\Delta h = 122.8 \times 0.45 \log_{10} \frac{(1000)}{(500)} \text{ feet.}$$

$$= 16.5 \text{ feet}$$

/With

With regard to error (vi), computors do not read the mean temperatures themselves but corresponding layer thicknesses. Test of the technique indicate that the probable error over the pressure range 1000 to 500 mb. is ± 7 feet.

Hence the total combined p.e. in height = $\pm\sqrt{16.5^2 + 7^2}$ = ± 17.5 feet. This value has been rounded to ± 15 feet in the paper. This is permissible in view of the approximations used in its derivation.

P.E. of temperature observations = ± 15

P.E. of sea -level pressure = ± 15 feet.

P.E. of position of aircraft = ± 10 feet.

P.E. of S.C.R.-718 readings = ± 8 feet.

That is the estimated p.e. of all sources with the exception of temperature errors have been reduced to about half. For the radio-altimeter this is equivalent to omitting the 50 feet uncertainty, which might be possible in that the sea surface is not smooth relatively to the aircraft and a mean indication of the height is given by the instrument. This is however on the borders of speculation.

10. That the mean value of Δ is about - 10 feet seems to be well founded. But it cannot be stated whether this systematic change occurs in the radio-altimeter or, for instance, in the aneroid. As a general conclusion it appears that a statistical investigation of observations on routine meteorological reconnaissance flights cannot test the stability of the radio-altimeter. The most that can be said is that there is no evidence of any systematic or unsystematic instability of the radio-altimeter readings over periods of the order of two hours greater than could arise within the errors quoted by the manufacturers of the instrument.

/

the
ind
± 7

Thi
in

12th August, 1946.

A preliminary note on summer atmospherics

by
C.K.M. Douglas

This report has not yet been published and is to be treated CONFIDENTIALLY. Its contents must not be quoted.

1. Introduction

The following note is based on a brief survey of atmospherics recorded during the summer half-year in the period 1943 to 1945. Atmospherics during the winter 1942-1943 were the subject of a detailed research by F.A. Berson and S. Petterssen (M.R.P. 130), and it is considered that most of the conclusions there stated are typical of winter atmospherics. There is now evidence of a somewhat more widely diffused area of maximum intensity in the Atlantic in winter with a higher frequency between latitudes 40° and 45° North than that shown in figure 1. of M.R.P. 130. A statistical investigation into the geographical distribution over sea and land at different times of day and seasons is being put in hand.

The observational technique during the period 1943-1945 was substantially the same as described in that memorandum. On June 17th, 1945, a new station was opened at Irvinestown, Ireland, and this has improved the accuracy of the plots. The observations are made for 10 minutes at intervals of 2 or 3 hours during the daylight hours. It is considered that the majority of flashes within from 500 to 1000 miles of the British Isles during the "runs" are recorded, though in periods of exceptional frequency it is impossible to record them all. Some locations have been obtained at distances up to 1500 miles, but the possibility of this is affected by the number of flashes in the nearer areas.

It is probable that lightning discharges can occur so high that thunder is not heard on the ground. A few atmospherics have been located in areas where the only lightning flashes believed to be possible in the existing meteorological conditions are at a high level, possibly above 15,000 feet. Thunderstorms have occurred in the British Isles with their base at 10,000 feet or somewhat higher, and in the Mediterranean area this height may be appreciably exceeded. Aircraft have sometimes reported "static" (i.e. electric sparks in the aircraft) when there are only medium cloud forms with no reports of thunder or rain on the ground. Rain from a high level may evaporate before reaching the ground, though the penetrating power of thundery rain is great, partly because much of it is soft hail at high levels. Actually when there are a few atmospherics without thunder there is usually at least slight rain at ground level.

The value of the observations to short-period weather forecasting is very great. During the war they sometimes decided whether bomber operations could be carried our or not, since major thunderstorm outbreaks commonly last into the night, and sometimes all through the night, drifting in the upper current. In peace time they are usually the sole source of information of thundery activity over sea areas, and they are of value both in frontal analysis and in the direct forecasting of thunder. Even over land areas with many ground stations the atmospherics frequently give the earliest intimation of thunder. Successive sets of observations show the movement of storms or groups of storms and the variation of intensity. The ultimate ideal is frequent (or automatic) recording throughout the 24 hours.

2. Atmospherics over sea areas.

2.1 Annual variation of frequency over sea areas.

During the summer months there is a decrease of activity over the North Atlantic with a minimum in June, but there is still an appreciable number of outbreaks, mainly of a minor character. The number of days with atmospherics between 15° and 40° West, North of 30° North, in the three-year period is given below, the columns under (a) being the number of days with three or more locations (generally considerably more) and those under (b) being the number of days with only one or two.

	J	F	M	A	M	J	J	A	S	O	N	D
(a)	61	47	54	43	23	11	16	29	35	44	70	60
(b)	11	10	14	7	5	3	12	7	13	5	6	14

/2.2

2.2 Geographical Distribution and Causes of Summer Atmospherics Over Sea Areas.

From April till October the frequency is highest round the Azores and decreases to north of latitude 45°. To north of 55° there are only a few scattered atmospherics. In producing conditions favourable for atmospherics it is clear that the effect of the higher sea temperature in the southern area outweighs that of the greater cyclonic activity further north. The water content of the air is probably the decisive feature. Though the Azores area is on the average in the high pressure belt, it is visited by fronts and depressions at times, including some shallow depressions which spread up from south, and it is in these conditions that thunderstorms are likely to break out. The majority are at cold fronts, but there are a few outbreaks ahead of warm fronts, and in warm sectors. Those in cold air masses are largely limited in summer to the central areas of old depressions and the troughs immediately to south of their centres.

There is increased activity near the Iberian Peninsula and still more in partly enclosed seas like the Bay of Biscay, the English Channel, and the North Sea. Many of the thunderstorms in these areas have drifted off the land, and others are due to advection of warm air from the land. Over the North Sea about half the storms from June to August are independent of the surface layers, which are stable owing to the sea being colder than the overlying air or to an inversion or marked stability above shallow cold air. (In June to August 1943-1945, 26 out of 55 appreciable outbreaks in the North Sea were of this type; in September only 2 out of 15). The frequency of atmospherics over the North Sea has a peak at about 0700-0800 G.M.T. with a marked fall by noon, and the same is true in the Bay of Biscay and the Channel area. It is probably due largely to divergence and subsidence associated with the difference between sea and land of the 24 hour term in the diurnal pressure variation, related to the sea-breeze. Patches of atmospherics occasionally move N. or N.W. over the seas between Norway and Iceland. Their source of warmth is over the land.

3. Atmospherics over Land Areas and Associated Meteorological Conditions

3.1. Annual Variations & Geographical Distribution over Land

There is a marked rise in the frequency over land from April to June, and it remains high till late August, dropping rapidly in September. In accordance with the known distribution of thunderstorms, the number increases towards S.E., across the British Isles and most of Europe. Very many locations have been made of atmospherics in S.E. Europe, but the possibility of detecting distant atmospherics is conditioned by the number in the more important nearer areas. The increase southward to the Pyrenees and the Alps and eastwards across Germany is shown up by the observations.

3.2 Diurnal Variation of Thunderstorm Activity

The time of the afternoon maximum frequency is determined by the time of occurrence of thunderstorms which occur in maritime polar air, the most frequent type of air over this country and West Europe. The high level hot-weather storms have a later maximum, especially the frontal storms, which sometimes break out after dusk and are often most intense at night.

3.3 Effect of Large-Scale Pressure Developments

The conditions of humidity and lapse-rate required for thunderstorms are well known in a general way. The distribution of both these variables, and also the chance of thunderstorm development in the marginal conditions often observed, are greatly affected by general development over areas large compared with that of an individual thunderstorm. Convergence may be localised along the lines where the storms develop, but it is conditioned by the large-scale developments. Thunderstorms have a far closer relation with cyclonic vorticity than with any particular kind of

/air

air mass. This is mainly due to the relation between convergence and the development of cyclonic vorticity, but even after development has ceased (e.g. in a dying depression) surface friction has an appreciable influence. On the other hand, the effect of divergence and subsidence is to dry and stabilise the air. After originally cold air has subsided there is complete freedom from thunder for a few days, over an area which may cover most of West Europe, though outlying regions like Spain have generally a different regime.

3.4. The Significance of High Surface Temperature

Very high temperatures tend to occur in anticyclonic air which has subsided and is often too dry for thunderstorm formation. In the British Isles there is not much relation between temperature and the incidence of thunder, and in fact the expectation of thunder is below average on days with a maximum temperature over 80°F. Nevertheless out-standingly severe storms are generally associated with very high temperatures within some 200 miles.

The increase in the frequency and severity of thunderstorms towards south and east on the Continent can be attributed to higher temperatures. A supply of moisture is also necessary, and there is no doubt that evaporation from vegetation is important. Maritime tropical air contains plenty of moisture, at least at low levels, but this cannot cross mountains, and penetration into most of Europe can only take place from west. It becomes unstable from the surface upwards at temperatures round 85° to 90° at the hottest season.

When thunderstorms break out in very warm air it is not necessarily of sub-tropical origin. A consideration of developments over several days brings out the curious point that subsidence, though it operates initially against thunder, may help to produce conditions which favour severe storms later on. With high temperatures at about 5000 feet, heat and moisture (the latter evaporated from vegetation) accumulate below, and if the stable layer just above 5000 feet can be broken through, the cumulo-nimbus clouds can generally grow to enormous heights. It is a matter of general experience that hot-weather storms are difficult to start, but tend to become violent when they do break out. They generally require a front or a line of convergence to act as a trigger, at least on low ground. For obvious reasons a shallow stable layer can be more readily penetrated over mountains than elsewhere.

A stable dry layer is often found also in tropical air above the damp lower layers, owing to previous subsidence in the sub-tropical anticyclone. It is the normal condition in the high pressure belt which includes Southern Europe in summer and frequently affects France, Germany and Southern England, with narrow damp belts along the fronts or zones of convergence between the anti-cyclonic cells. Complexities of structure in this climatic zone arise from the fact that the zones of convergence tend to travel with the upper currents, faster than the air masses low down, giving frontogenetic troughs ahead of the surface fronts.

3.5 Horizontal Temperature Gradients.

The effect of heating over the land is to set up a gradient of temperature near the coastal areas, especially at low levels, and this is super-posed on gradients due to other causes. The majority of air masses have temperature gradients in them and this influences the pattern of thunderstorm development. The consequence of temperature gradients is that many outbreaks of thunder in the air mass develop in belts and have quasi-frontal properties. Whether a particular group of storms should be classified as frontal or not often depends on whether the analysis is a microscopic one or is confined to the main frontal system.

/The

The relation between temperature gradient and upper wind structure is a well-known principle of meteorology, and it has an important effect on the motion of thunderstorms, which depends mainly on the wind somewhat above the base of the main cumulonimbus cloud mass. In hot weather the cloud base is often above 5000 feet and the motion agrees better with the wind at 10,000 feet or even higher than with that at lower levels. The observations of atmospherics supply a very powerful tool for following the motion of the storms.

The most important thunderstorms reaching the British Isles generally come from some southerly point with high temperatures to east. In the most conspicuous cases there is a cold air mass with an upper depression or trough to west or southwest. If it is to S.W. the upper wind is S.E., and though S.E. upper currents are not frequent the percentage of thunderstorms is very high when they do occur.

3.6. Convectional Storms Associated with Deep Cold Air

In those parts of Europe within a few hundred miles of the Atlantic or North Sea the nearest approach to pure convectional storms scattered over a substantial area occurs in deep cold air. This is found in the upper trough or upper depression behind the surface depression or right over the centre of an old depression, often with instability for some hundreds of miles south of it. The closed upper depression or 'cold pool' is much more frequent in our area than in most parts of the temperate zone, and is associated with the relatively weak latitudinal temperature gradient. They fairly frequently move slowly east or south over the British Isles and adjacent continental areas, and sometimes remain stationary, with widespread thunderstorms, for several successive days with only a slow warming of the air mass. As a rule there is an old dying depression at sea level, but there are sometimes complications in the pressure distribution. The air mass is of maritime polar or occasionally of arctic origin, but since the subsiding masses of anticyclonic air are of similar origin, the mere specification of air mass type without reference to the isobaric system has no significance. Since maritime polar air is much the most frequent type of air, the majority of thunderstorms naturally occur in it, though they are on the whole less violent than those in very warm air.

3.7. Warm Fronts and Thunder Activity

In the three summers considered no warm front from the Atlantic gave atmospherics over the British Isles, though there were a few out-breaks far out at sea.

Atlantic warm fronts in summer rarely penetrate far into Europe.

Cool air from Baltic and North Sea sometimes gives rise to fronts which are most intense low down, and may act as warm fronts to a depression moving NE or N., occasionally NW., even if the depression is of a distant origin.

3.8. Cold Fronts and Thunder Activity

Among cold fronts three quarters of those which crossed part of the British Isles gave no atmospherics in the period covered by the observations. Of these that did give atmospherics, there were some major outbreaks associated with really warm air ahead, generally contintental, and there were also a number of outbreaks at secondary cold fronts. It is only occasionally that a cold front with unheated maritime tropical air ahead of it gives any thunder, since one day of heating over land is insufficient to produce instability.

On the continent the longer travel of the air over heated land gives rise to a different situation and the number of atmospherics associated with cold fronts increases eastward and southward. The structure is often complex and one rarely sees a well-marked belt of atmospherics, such as is sometimes shown in the Atlantic, especially in winter. There is a large scatter and there are often many atmospherics in front of or behind the front, sometimes both. When the warm air is unstable, thunderstorms

/may

may break out anywhere in it, more especially in the region of convergence ahead of the front. A trough of low pressure generally forms in this region, and frequently a new frontal system is developed ahead of the main front coming in from the Atlantic. The most violent outbreak of thunderstorms is then almost always on the most easterly front at the boundary of the hottest air. It was a wave on one of these newly formed fronts east of the main front which gave the severe storm in S.E. and E.England on the night of July 3rd - 4th, 1946.

Behind a cold front there are two distinct classes of storms, those in the upper warm current and those in the cold air. The height of the base of the upper storms may be as much as 10,000 feet, or even more in extreme cases. Those in the cold air require a depth of 15,000 feet of non-subsided cold air, which is not always attained. As a rule there is a belt free of storms between the two types of storm, but in some complex sluggish situations when the frontal surface is not well marked, and when the cold air has been heated from below, there is no substantial area near the front free from atmospherics in the afternoon.

If the air near the ground has a low relative humidity, as is normal in hot weather, the condensation level is high, and it is natural that the main thunderstorm outbreak should be west of the front. The distance is generally less than 50 miles but it is occasionally as much as 200 miles, when the front is slow moving or stationary, and the upper current is roughly parallel to it. This large difference is most frequent on the Continent, but there was a case in England on August 24th, 1944, with the base of the storm at 10,000 feet. The more widespread outbreaks both west of and on the front are associated with small disturbances travelling along the front. Some of these have the nature of waves, but others are too small and too complex for clear identification. Only occasionally is the disturbance sufficiently pronounced to make the front move west. This happened on July 15th, 1945, but during most of the severe storm in England on the preceding night the front was stationary near the East Coast. The maximum rainfall was 85 miles from the front, and the west boundary of the storm about 120 miles from the front.

3.9. Direction of Movement and Duration of Cold Front Thunderstorms

A front associated with a large air-mass difference has a belt of strong upper winds over it, with a large component parallel to the front, southerly in the case of a cold front oriented north-south. The component of wind at right angles to the front, which in the ideal case is the same as the speed of the front, is generally much less. The thunderstorms drift with the wind between about 10,000 and 15,000 feet, and this generally exceeds 20 knots and occasionally exceeds 40 knots. In view of the patchy nature of thunderstorm distribution along fronts, the most important thing for the forecasters is the atmospherics activity up wind at the above levels, rather than the activity in the direction from which the front is approaching. The duration of the outbreaks varies from a few hours to twelve hours or more. A phase of extreme intensity is limited to a few hours, a fact which must be taken into account in forecasting. The severest storms in England have not followed the severest outbreaks in France.

Conditions are different at secondary cold fronts in polar air, when the duration of storms is less and the diurnal variation over land is larger than at primary fronts. Nevertheless the distribution is often patchy and considerable weight must be given to the wind between 5,000 and 10,000 feet.

3.10 Occlusions and Thunder Activity

Occlusions moving east over the British Isles occasionally give rise to thundery activity along or just behind them, generally with rather low upper air temperature. When they reach the Continent they generally act as a cold front.

Cold fronts swinging round depressions may come up from south and ultimately form north-moving occlusions, with atmospheric activity for anything up to 12 hours or even more after occlusion.

3.11 Predominance of Thundery Type Rain in Summer

The correlation of atmospherics with the rainfall in the years 1941-1945 has led to the conclusion that more than half of the May to August rainfall over most of England is of thundery type or thundery origin, sometimes formed locally but more often coming up from the Continent or the Bay of Biscay. The actual rain may not be accompanied by atmospherics but in such cases atmospherics are a feature of the early development of the rain area.

53 p8

METEOROLOGICAL OFFICE (M.O.15)

AIR MINISTRY,

LONDON.

9.10.46.

To Overseas Recipients of M.R.P.298

This paper was discussed at the last meeting of the Meteorological Research Committee and it was agreed that it would be highly desirable to obtain observational evidence bearing on the theories advanced in this paper.

2. It is appreciated that there may be considerable difficulty in obtaining such evidence but it was suggested that the forecaster, during his daily duties, might find evidence which either would not normally become incorporated in the weather records or alternatively would not be apparent to anyone examining the records some time after the event.

A.C. BEST
Secretary, M.R.C.

G. 137065

has l
over
local
The a
atmos

This report has not yet been

published ... to be treated
CONFIDENTIALLY. Its
contents must not be quoted.

M.R.P. 298

20th August 1946.

Air Ministry

Meteorological Research Committee.

The Theory of Tropical Cyclones.

by

J. S. Sawyer.

Summary.

The conditions under which continued outward motion is possible in a circular vortex are investigated theoretically, and the results applied to the development of tropical cyclones, in order to give a possible explanation of the removal of air from the storm centre and the production of low pressure.

The development of a storm by this mechanism is described and necessary conditions for such development are found to be:-

(a) A lapse rate which will give rise to vertical instability and development of cumulonimbus.

(b) A distribution of circulation at some level aloft such that anticyclonic circulation increases outwards at a rate exceeding a certain value.

(c) A distribution in the vertical of horizontal rotation such that cyclonic circulation decreases with height.

The importance of horizontal turbulence in producing the calm "eye" of the storm is considered, and the relation between tropical cyclones and fronts is briefly discussed.

Contents.

I. Introduction.
II. The stability of circular vortices.
III. Diagramatic representation of instability of a vortex.
IV. Representation of expanding and contracting vortices in the characteristic diagram.
V. Effect of dynamical instability on the formation of tropical storms.
VI. The effect of vertical motion on stability of vortices.
VII. Critical conditions for the development of a tropical storm.
VIII. The further development of a revolving storm.
IX. The fall of pressure in a revolving storm.
X. The formation of the "eye" of the storm.
XI. Downward flow in the lower layers of a cyclone.
XII. Fronts in the circulation of tropical cyclones.

I. Introduction.

Most meteorologists who have studied the mechanics of the tropical cyclone agree that its formation is connected with vertical instability in the atmosphere, and that the energy of the cyclone is derived from the latent heat of condensation released during the ascent of warm moist air from the surface.

It is explained that the formation of a warm column of air by convection causes the isobaric surfaces at the top of the air column to be raised, and an out-flow of air at this level results from the rise in pressure. The outflow causes a reduction in the total weight of the column and produces a fall of surface pressure; the surface winds then converge towards the newly formed "low" and produce a cyclonic circulation as a result of the Coriolis effect.

This briefly is the accepted explanation for the formation of cyclonic storms in the tropics as put forward by Shaw (1922), although the importance of converging air currents and of the inter-tropical front have also been stressed by various authors. It has however been realised (Brunt- Physical and Dynamical Meteorology p. 303) that there is an important omission in the theory of the tropical cyclone as outlined above:- it fails to explain what mechanism continues to remove air from the upper levels of the cyclone after the initial rise in pressure has been equalised and the circulation has become cyclonic at all levels.

Shaw attributes this eviction of air to the frictional drag of the rising air on its surroundings, but the mechanism is difficult fully to understand, or to treat mathematically. Durst and Sutcliffe (1938) have considered the "vertical motion" terms in the hydrodynamical equations, and have shown that, with certain assumed values for the velocities in a tropical storm, these terms are large enough to account for outward motion in the higher levels of the storm.

However, neither of these theories explains why convection should give rise to cyclonic development on some occasions, and in some latitudes, but not on other occasions and in other places. This paper puts forward another possible explanation of the method by which air is evicted from the upper part of cyclones, and deduces conditions under which development of tropical cyclones is and is not, possible. It also brings under critical review certain other facts and considerations regarding cyclone development in low latitudes, and endeavours to fit them into a composite picture of the development of a typical storm.

II. Stability of circular vortices.

In the study of meteorological dynamics in temperate latitudes, it is usually found that, if through any cause pressure at some level rises or falls, although initially winds blow out and from the region of rising pressure, or into the region of falling pressure, when the rise or fall of pressure ceases, a balance is set up and little further air enters or leaves the new pressure centre. This outflow from regions of rising pressure is known as the "isallobaric effect" and was first discussed by Brunt and Douglas (1928).

In order to explain the fall of pressure in a tropical cyclone we must seek a mechanism which will continue to remove air after the pressure aloft has ceased to rise, or even commenced to fall. We are therefore led to enquire under what circumstances no balanced state can be attained after a rise of pressure, and for this purpose we shall consider the motion which takes place in a steady circular vortex which is subjected to a small decrease in pressure gradient due to a small rise in the pressure at the centre.

The equations of motion of a fluid moving symmetrically about a vertical axis fixed relative to the earth may be written in cylindrical coordinates (Brunt - 1921) as:-

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + lv + K\frac{\partial^2 u}{\partial z^2} \quad \ldots\ldots\ldots(1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{uv}{r} + w\frac{\partial v}{\partial z} = -lu + K\frac{\partial^2 v}{\partial z^2} \quad \ldots\ldots\ldots(2)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \quad \ldots\ldots\ldots(3)$$

In these equations u, v and w are the velocities respectively along the directions of the radius r, horizontally at right angles to the radius vector in a cyclonic direction, and vertically in the direction of z increasing.

Other symbols are:-

ρ - the density of the air.
l - the Coriolis parameter: $-2\omega \sin \phi$
K - the coefficient of eddy diffusivity.
g - the acceleration due to gravity.

In the derivation of equations (1) and (2) certain small terms involving the vertical velocity (viz: $-lw \cot \phi \cos \theta$ and $-lw \cot \phi \sin \theta$ respectively) have been omitted from the right-hand side of the equations. (θ is the angle between the radius vector and the eastward direction). These terms are small so long as w is small compared with u and v, and they also act in opposite directions on opposite sides of the vortex, so that although they may introduce asymmetry into it they are not likely to be important as factors in its development. They have not been considered further.

We consider first the steady motion of a circular vortex without vertical or radial motion, and we shall write $v = v_o$ in this steady initial state.

For such conditions $u = 0$, $w = 0$ and $\partial v/\partial t = 0$

Then from equation (1)

$$lv_o + \frac{v_o^2}{r} = \frac{1}{\rho}\left(\frac{\partial p}{\partial r}\right)_o \quad \ldots\ldots\ldots(4)$$

We now consider a small disturbance of the steady motion caused by a decrease in the outward pressure gradient such that $\frac{\partial p}{\partial r} = \left(\frac{\partial p}{\partial r}\right)_o - \Delta$, and we shall write $v = v_o + v^1$ where v^1 is the small change in v resulting from this rise in the central pressure of the vortex.

Ignoring the frictional terms equation (1) may be rewritten in terms of v^1 and Δ as follows:-

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} - \frac{(v_o + v^1)^2}{r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial r}\right)_o + \frac{\Delta}{\rho} + l(v_o + v^1) \quad \ldots(5)$$

As we are considering a small disturbance from steady motion we may regard u, v^1 and w as small quantities and disregard terms which are higher than the first order in them and their derivatives. Thus utilising equation (4), equation (5) reduces to

$$\frac{\partial u}{\partial t} = \frac{2v_o v^1}{r} + lv^1 + \frac{\Delta}{\rho} \quad \ldots\ldots\ldots(6)$$

Similarly equation (2) gives

$$\frac{\partial v^1}{\partial t} + u\left\{\frac{\partial v_o}{\partial r} + \frac{\partial v^1}{\partial r}\right\} + \frac{u(v_o + v^1)}{r} + w\left\{\frac{\partial v_o}{\partial z} + \frac{\partial v^1}{\partial z}\right\} = -lu$$

which reduces, when second order terms are neglected, to

$$\frac{\partial v^1}{\partial t} = -u\left(l + \frac{\partial v_o}{\partial r} + \frac{v_o}{r}\right) - w\frac{\partial v_o}{\partial z} \quad \ldots\ldots\ldots(7)$$

but from (6)

$$\frac{\partial u}{\partial t} = \frac{\Delta}{\rho} + v^1 \left(\frac{2v_o}{r} + 1 \right)$$

Thus

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\Delta}{\rho}\right) + \frac{\partial v^1}{\partial t}\left(\frac{2v_o}{r} + 1 \right)$$

and substituting from (7)

$$\frac{\partial^2 u}{\partial t^2} = -u \left(\frac{2v_o}{r} + 1\right)\left(1 + \frac{\partial v_o}{\partial r} + \frac{v_o}{r}\right) + \frac{\partial}{\partial t}\left(\frac{\Delta}{\rho}\right) - w \frac{\partial v_o}{\partial z}\left(\frac{2v_o}{r} + 1\right) \quad \ldots\ldots(8)$$

writing

$$B \equiv \left(1 + \frac{2v_o}{r}\right)\left(1 + \frac{\partial v_o}{\partial r} + \frac{v_o}{r}\right)$$

equation (8) becomes

$$\frac{\partial^2 u}{\partial t^2} = -B\left(u - \frac{1}{B}\frac{\partial}{\partial t}\left(\frac{\Delta}{\rho}\right)\right) - w \frac{\partial v_o}{\partial z}\left(\frac{2v_o}{r} + 1\right) \quad \ldots\ldots(9)$$

We shall first consider the solution of this equation when there is no vertical motion ($w = 0$). As however the equation of continuity can be written for symmetrical motion in cylindrical coordinates $\frac{1}{r}\frac{\partial}{\partial r}(r\rho u) + \frac{\partial}{\partial z}(\rho w) = 0$ it follows that with an arbitrary distribution of u as a function of r, $\partial w/\partial z$ cannot in general be zero. Thus, in general, w will only be zero in a single layer at the top of the storm, and the results obtained by ignoring the second term on the right hand side of equation (9) will be valid only for a single thin layer. However it will be shown in paragraph VI how far the conclusions based on a thin layer may be extended to a "slice" of finite depth.

Putting $w = 0$ equation (9) becomes

$$\frac{\partial^2 u}{\partial t^2} = -B\left(u - \frac{1}{B}\frac{\partial}{\partial t}\left(\frac{\Delta}{\rho}\right)\right) \quad \ldots\ldots(10)$$

This may be regarded as determining u at a fixed distance r from the centre of the vortex and it may be solved easily in the case in which $\frac{\partial}{\partial t}\left(\frac{\Delta}{\rho}\right)$ is constant. We shall consider the motion under such a [illegible] pressure disturbance, which, if the variation in density is ignored, is equivalent to a steady increase in the outer pressure gradient.

It may be noted that it is [illegible] that it is possible for such a pressure disturbance to be imposed independent of the value of u. This can be done because the pressure distribution is determined by the distribution of mass in the higher layers of the storm [illegible] directly affected by the motion in a thin layer. In how far the analysis can be extended to a "deep" slice" of the atmosphere it will be necessary to consider the effect of the motion in the upper part of the "slice" on the pressure distribution in the lower part. For this reason, applications of deductions made on the basis of equations (9) and (10) will be confined to problems in which it is clear the nature of the pressure disturbance is imposed by causes outside the immediate influence of the layers concerned.

If B is positive equation (10) represents a simple harmonic oscillation of the radial velocity u, about a mean value $\frac{1}{B}\frac{\partial}{\partial t}\left(\frac{\Delta}{\rho}\right)$. This solution corresponds to the flow in a stable balanced vortex referred to earlier. If we ignore any variation in ρ, we see that the outflow continues only so long as the central pressure continues to rise, $(\partial\Delta/\partial t > 0)$. When the central pressure ceases to rise $\partial\Delta/\partial t = 0$ and u oscillates about a mean value of zero, i.e. the outflow ceases. This is an example of the well known isallobaric effect.

If, however, B is negative, $\partial^2 u/\partial t^2$ is always positive for positive values of $\left(u - \frac{1}{B}\frac{\partial}{\partial t}\frac{\Delta}{\rho}\right)$, and thus u increases without limit. The motion is thus unstable, and a small disturbance such as a rise in central pressure will cause continuous outflow, because balanced motion cannot be re-established.

until large changes in v etc. have been effected.

We therefore investigate under what conditions

$$B \equiv \left(1 + \frac{2v_o}{r}\right)\left(1 + \frac{\partial v_o}{\partial r} + \frac{v_o}{r}\right) < 0$$

There are two cases to consider

$$(a) \quad 1 + \frac{2v_o}{r} < 0 < 1 + \frac{\partial v_o}{\partial r} + \frac{v_o}{r}$$

$$(b) \quad 1 + \frac{2v_o}{r} > 0 > 1 + \frac{\partial v_o}{\partial r} + \frac{v_o}{r}$$

(It may be noted that $\frac{1}{2} + \frac{v_o}{r}$ is the angular velocity in space of the air about the axis of the vortex and $1 + \frac{\partial v_o}{\partial r} + \frac{v_o}{r}$ is its absolute vorticity. The condition $B < 0$ is thus that the absolute vorticity of the vortex should be of opposite sign to its rotation).

Case (a) occurs when

$$\left.\begin{aligned} \frac{v_o}{r} &< -\frac{1}{2} \\ \text{and} \quad \frac{1}{r}\frac{\partial}{\partial r}(rv_o) &> -1 \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

This represents a vortex with anticyclonic rotation in which the rotation does not increase too rapidly outward.

In temperate latitudes these conditions are not likely to be satisfied on a large scale because of the relatively large value of 1. However, in latitude 11 degrees (N or S) 1 is approximately 0.1 in units hrs^{-1} so that conditions (11) may be satisfied in an anticyclonic vortex in which the tangential velocity 200 miles from the centre exceeds 10 m.p.h.

The second case (b) occurs when

$$\left.\begin{aligned} \frac{v_o}{r} &> -\frac{1}{2} \\ \frac{1}{r}\frac{\partial}{\partial r}(rv_o) &< -1 \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

This represents a vortex with cyclonic rotation (or one with anticyclonic rotation for which $-v_o/r$ is less than 1/2) in which the cyclonic rotation decreases so rapidly outward that

$$\frac{\partial}{\partial r}(rv_o) < -1r$$

There is a third case (case (c)) in which the motion is not stable. This occurs when $B = 0$.

$$\text{i.e. either} \quad \frac{v_o}{r} + \frac{1}{2} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

$$\text{or} \quad \frac{1}{r}\frac{\partial}{\partial r}(rv_o) = -1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

This latter equation integrates giving

$$\frac{v_o}{r} + \frac{1}{2} = \frac{C}{r^2} \quad (C = \text{constant}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

It follows that if equation (13) is satisfied, equation (15) is necessarily so. Thus the latter equation may be regarded as a necessary and sufficient criterion for case (c).

Using equation (15) in conjunction with equations (6) and (7) we find (if $v = 0$)

$$\frac{\partial u}{\partial t} = \frac{2Cv^1}{r^2} + \frac{\Delta}{\rho} \quad .$$

and v^1 is constant.

As v^1 is initially zero it follows that $\partial u/\partial t = \Delta/\rho$ and the outflow takes place under the acceleration due to the imposed disturbance of the pressure gradient as it would if there were no circular motion. Case (c) is therefore also one in which outflow will continue so long as an increased central pressure is maintained.

III. Diagrammatic representation of instability of a vortex.

Case (a), (b) and (c) are conveniently represented in a diagram in which r^2 is taken as abscissa x and vr as ordinate y.

On such a diagram the lines of constant velocity v are represented by parabolas with axis along the axis of x and vertex at the origin. On this diagram any distribution of v with r can be represented by a characteristic curve typical of the vortex considered.

To draw the straight line $y = -1/2.x$ in this diagram. Figure 1 represents such a diagram on which the critical line ($y = -\frac{1}{2} x$) is drawn for (11 degrees latitude) and also for latitudes 5, 20, 30, 40, 50 and 70 degrees

For points below the critical line

$$y < -\frac{1}{2} x$$

or

$$\frac{v_o}{r} < -\frac{1}{2} .$$

and for points above the line

$$\frac{v_o}{r} > -\frac{1}{2}$$

Further, the slope of any characteristic curve drawn on the diagram may be written

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial r} / \frac{\partial x}{\partial r} = \frac{1}{2r} \frac{\partial (rv)}{\partial r}$$

But the slope of the line $y = -\frac{1}{2} x$ is 1/2 and thus for curves with a steeper slope $\frac{\partial y}{\partial x} < -\frac{1}{2}$ or $\frac{\partial (rv)}{\partial r} < -1r$ and for curves with a less steep slope $\frac{\partial (rv)}{\partial r} > -1r$.

It thus follows that the velocity distributions in the unstable vortices are represented by characteristic curves in the r^2. rv diagram as follows:-

Case (a) - Characteristic curves lie below the line $y = -\frac{1}{2} x$ and approach it with increasing r (i.e. slope greater than -1/2).

Case (b) - Characteristic curves lie above the line $y = -\frac{1}{2} x$ and approach it with increasing r (i.e. slope less than -1/2).

Case (c) - Characteristic curves are straight lines parallel to the line $y = -\frac{1}{2} x$.

To avoid confusing figure 1 which shows the position of the line $y = -\frac{1}{2} x$ for various latitudes, examples of curves characteristic of case (a) and (b) and (c) in relation to the line $y = -\frac{1}{2} x$ are shown in figure 2.

The results of the above analysis may therefore be summarised as follows.

The distribution of tangential velocity v as a function of r in a circular vortex may be represented by a characteristic curve in a diagram in which rv is plotted against r^2 . The vortex will be unstable for a change of pressure gradient if, and only if the characteristic curve is parallel to or steadily approaches the line $y = -\frac{1}{2}x$ for increasing values of r.

It will be noticed that the above analysis takes no account of the effect on the pressure distribution of the changes in the velocity distribution. This will be discussed later but such changes will clearly act in the direction of increasing stability.

It may be desirable to indicate briefly the physical interpretation of the results that have been obtained. In order to do this we must realise that if a ring of air rotating about a vertical axis fixed in the earth is made to expand, its tangential velocity will be changed on account of the condition which requires the circulation in space to be preserved.

If the air in the expanding ring finds itself with a greater angular velocity in space(regardless of sign) than that of the air which previously occupied its new position, the pressure gradient will be inadequate to balance the centrifugal force on the air and it will be thrown outwards.

The two cases (a) and (b) arise because of the two possible directions of rotation of the vortex in space and case (c) is the limiting condition in which the pressure gradient is exactly able to balance the centrifugal force but is unable to reduce any outward motion the ring of air may have.

IV. Representation of expanding or contracting vortices in the characteristic diagram

In order to study the development of a vortex in which exists instability of the nature discussed above, it is desirable to consider the representation on the r^2 . rv diagram of a contraction or expansion of a vortex.

Ignoring the vertical motion and frictional terms equation (2) may be written

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{uv}{r} = -lu$$

or

$$\frac{1}{r}\frac{D}{Dt}(rv) = -lu \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

where the differentiation D/Dt represents $\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}$ i.e. the rate of change in an expanding ring of air.

$$\text{or} \quad \frac{D}{Dt}(rv) = -lr\frac{Dr}{Dt} = -\frac{1}{2}\frac{D(r^2)}{Dt}$$

Thus for an individual ring of air

$$rv + \frac{1}{2}r^2 \text{ is constant} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

i.e. in the x . y diagram,

$$y + \frac{1}{2}x \text{ is constant,}$$

and it follows that the point representing the annulus in the diagram is displaced parallel to the line $y = -\frac{1}{2}x$.

From this it follows immediately that if the velocities v in a series of adjacent rings of air satisfy the conditions for instability of the type discussed, this instability within the series of rings will not be affected by their displacement outwards or inwards, in the absence of vertical motion.

Further, in the absence of vertical motion, the area between two rings must

remain constant.

Thus $\pi r_1^2 - \pi r_2^2$ is constant

or $x_1 - x_2$ is constant

and it follows that the representative points are equally displaced along the axis of x.

V. Effect of Dynamical instability on the formation of revolving storms.

It is now convenient to consider in some detail the effect of the presence or absence of dynamical instability on the development of a revolving storm about a vertical convection current.

We consider the first stage of the development of a storm as the formation of a large cumulonimbus cloud by convection. The air within the cloud (and also the descending air around the cloud) will be warmer and less dense than any undisturbed air at a distance. The isobaric surfaces will therefore be displaced upwards as indicated in Figure 3 until outflow from the column has taken place.

It will be seen that the pressure in, or over, the convectional cloud has been raised at a typical level A, and whether the circulation at this level is dynamically stable or not, an isallobaric outflow will take place, removing air from the column and causing the surface pressure to fall.

However when the convection cloud is fully developed no further warming of the air column takes place and pressure at level A ceases to rise.

If the circulation is dynamically stable, in the sense already discussed, a balanced vortex will have been established at all levels by the time pressure changes cease. This will be anticyclonic in the upper levels and cyclonic in the lower levels, and there will be no further tendency for outflow, or inflow or vertical motion.

The final state is illustrated in Figure 4.

If, however, the circulation at some level (or levels) aloft assumed to be A in Figure 4 is such as to be classed as dynamically unstable in the previous paragraphs, the development is different. Under these circumstances the outflow at level A will continue so long as the pressure excess in and near the cloud over the surrounding air is maintained. (It may even continue further owing to the dynamical inertia of the system).

Stage II in the dynamically unstable case is illustrated in Fig. 5; the outflow at level A having reduced the pressure excess at that level to zero. We will assume for the present that the vortex at level A were to return to its initial state with the same central pressure. The removal of air would then have produced a fall in pressure at the surface (greater than in the stable case) and, assuming that the vortex at the surface was dynamically stable, a cyclonic balanced vortex would be set up. Under these circumstances inflow at the surface and outflow aloft would cease were it not for the effect of surface friction.

The effect of surface friction in the lower layers of the atmosphere is to produce a flow of air across the isobars towards the low pressure. In the stable case Fig. 6 this will gradually cause the low pressure centre at the surface to fill, and the pressure at higher levels to rise still further. This latter rise will however be balanced by an increase in the anticyclonic circulation and a final state reached as illustrated. The outflow at the top and the inflow at the bottom will have ceased and the development of the cloud will follow the normal theory of convection in which the rotation of the earth is ignored.

The effect of surface friction is different when dynamical instability is present. This is illustrated in Figure 7. Again there is frictional inflow into the cyclonic vortex at the surface but so long as the vortex at level A remains dynamically unstable, as soon as pressure tends to rise at level A, air

is removed outwards and the pressure equalised. A continuous inflow at the surface, vertical motion in the core of the vortex and outflow at the level of dynamic instability is thus established.

VI. The effect of vertical motion on the stability of vortices.

There are two ways in which the vertical motion of the atmosphere in the model of the tropical storm affects the stability and instability of the upper vortex to which appeal is being made for the eviction of air. Firstly, as was pointed out in paragraph I, the existence of vertical motion introduces an additional term into equation (10) which has been the basis of the preceding discussion of stability and instability. Secondly, vertical motion will result in the lifting to the level of the unstable vortex of air with different rotation about the axis from that previously at this level. The distribution of angular velocity will thus be changed, and the stability or instability of the vortex may be affected.

Before discussing these two effects it is convenient to consider the work of Durst and Sutcliffe (1938) who rewrote equation (2) in the form

$$\frac{\partial v}{\partial t} + w\frac{\partial v}{\partial z} = -u\left(1 + \frac{v}{r} + \frac{\partial v}{\partial r}\right) \quad \ldots\ldots\ldots\ldots(18)$$

and used it as a means of determining u in terms of $\partial v/\partial t$ and $w\partial v/\partial z$. They have assumed that the term $1 + \frac{v}{r} + \frac{\partial v}{\partial r}$ is essentially positive, and found that under certain assumed conditions $w\partial v/\partial z$ is negative and numerically considerably larger than $\partial v/\partial t$. They have thus concluded that the vertical motion in a storm is one of the main factors in the eviction of air from it at high levels.

However in this present paper it has been assumed that $1 + \frac{v}{r} + \frac{\partial v}{\partial r}$ may be positive, negative, or zero. The effect of the "vertical motion" term in the stable case and cases (b) and (c) as classified above, is considered briefly below. Case (c) will be considered first because it indicates the inter-relation between $\partial v/\partial t$ and $w\partial v/\partial z$ and suggests why the numerical values assumed by Durst and Sutcliffe lead to false results.

Case (c). In this type of motion $1 + \frac{v}{r} + \frac{\partial v}{\partial r} = 0$

Thus equation (18) reduces to

$$\frac{\partial v}{\partial t} + w\frac{\partial v}{\partial z} = 0$$

and is indeterminate as a means of assessing u.

This may be interpreted as indicating that in "case (c)" motion the distribution of tangential velocity v is determined by the vertical advection of tangential velocity. This is not surprising and suggests that $\partial v/\partial t$ and $w\partial v/\partial z$ may be of the same order of magnitude in other cases also.

Durst and Sutcliffe were led to erroneous assessment of the magnitudes of $\partial v/\partial t$ and $w\partial v/\partial z$, because they regarded them as independent. In the very early stages of a storm the magnitude of the term $w\partial v/\partial z$ may be assessed by the method they adopted, i.e. assuming v increased uniformly from the base to the top of the storm, but as soon as the storm has started to develop the distribution of v will have been determined by w and we expect $\partial v/\partial z$ to be very small in the regions where w is large, and $\partial v/\partial z$ to be large at the top of the storm where w is zero. Thus Durst and Sutcliffe were led to over estimate considerably the value of the term $w\partial v/\partial z$.

In case (b) $1 + \frac{v}{r} + \frac{\partial v}{\partial r} < 0$.

Thus a negative value of $\partial v/\partial z$ and a positive value of w, as to be expected in a cyclone, will be associated with a reduction of u. However continuity conditions require that w can only remain positive so long as the mean values of u over higher levels of the storm is positive. It thus follows that the vertical motion effect cannot produce a negative value of the mean outflow

in the higher part of the storm, and, should it be able to reduce the outflow to zero since w must vanish simultaneously, the arguments of paragraph one indicate that the air will be again accelerated outward.

This problem is treated more rigorously below where it will be seen that the vertical motion terms may restrict the depth through which instability of the vortices may develop but cannot prevent it occurring through a finite layer.

In case (a) and in stable motion for which $1 + \frac{v}{r} + \frac{\partial v}{\partial r} > 0$ the

conclusions of Durst and Sutcliffe are valid, and the "vertical motion" term will cause an increase in the outflow u. This may be important in creating outflow from levels classified as stable in this paper, when cyclonic motion has been set up by the development of dynamic instability at a higher level.

We now return to equation (9) which can be written

$$\frac{\partial^2 u}{\partial t^2} = -(1 + \frac{2v_o}{r})(1 + \frac{\partial v_o}{\partial r} + \frac{v_o}{r})(u - \frac{1}{B}\frac{\partial}{\partial t}\left(\frac{\Delta}{\rho}\right)) - w\frac{\partial v_o}{\partial z}(\frac{2v_o}{r} + 1) \quad \ldots\ldots(9)$$

and consider the conditions under which this will continue to represent unstable motion although the vertical motion is not zero. So long as $\partial^2 u/\partial t^2$ always has the same sign as $(u - \frac{1}{B}\frac{\partial}{\partial t}\left(\frac{\Delta}{\rho}\right))$, the motion must be of an unstable type because positive values of $(u - \frac{1}{B}\frac{\partial}{\partial t}\left(\frac{\Delta}{\rho}\right))$ will always tend to increase and negative values to decrease.

This condition will be satisfied so long as the absolute value of the second term on the right hand side of equation (9) is less than that of the first term.

Thus for small perturbations in which $\frac{1}{B}\frac{\partial}{\partial t}\left(\frac{\Delta}{\rho}\right)$ is negligible, equation (9) will represent an unstable motion so long as

$$(\frac{2v_o}{r} + 1)w\frac{\partial v_o}{\partial z} \text{ is numerically less than } u(\frac{2v_o}{r} + 1)(1 + \frac{\partial v_o}{\partial r} + \frac{v_o}{r})$$

or

$$\left|w\frac{\partial v_o}{\partial z}\right| < \left|u(1 + \frac{\partial v_o}{\partial r} + \frac{v_o}{r})\right| \quad \ldots\ldots(19)$$

We shall now assume that there is a level at the top of the storm above which there is no horizontal outflow, and consequently $w_1 = 0$ at this level $z = z_1$ (say).

For symmetrical motion in cylindrical coordinates the equation of continuity reduces to

$$\frac{\partial(\rho w)}{\partial z} = -\frac{1}{r}\frac{\partial}{\partial r}(r\rho u)$$

Thus

$$\rho_1 w_1 - \rho w = -\int_z^{z_1} \frac{1}{r}\frac{\partial}{\partial r}(r\rho u)\, dz$$

Whence since r and z are independent and $w_1 = 0$

$$w = \frac{1}{\rho r}\frac{\partial}{\partial r}\left(r\int_z^{z_1} \rho u\, dz\right) = \frac{1}{g\rho}\frac{1}{r}\frac{\partial}{\partial r}\left(r\int_p^{p_1}(-u)dp\right)$$

$$= -\frac{1}{g\rho}\frac{1}{r}\frac{\partial}{\partial r}(r\bar{U}(p_1 - p)) = \frac{\bar{\rho}}{\rho}(z_1 - z)\frac{1}{r}\frac{\partial}{\partial r}(r\bar{U})$$

where $\bar{U}$ is the pressure-mean value of u through the layer p to p_1 and $\bar{\rho}$ is the mean value of the density in the layer p to p_1.

Substituting this value for w the inequality (19) becomes

$$\left|\frac{\bar{\rho}}{\rho}\,\frac{(z_1 - z)}{r}\,\frac{\partial}{\partial r}(r\bar{U})\,\frac{\partial v_o}{\partial z}\right| < \left|u\left(1 + \frac{\partial v_o}{\partial r} + \frac{v_o}{r}\right)\right|$$

or since $\frac{\bar{\rho}}{\rho} < 1$

$$\left|(z_1 - z)\,\frac{\partial v}{\partial z}{}_o\right| < \left|\frac{\left(1 + \frac{\partial v_o}{\partial r} + \frac{v_o}{r}\right)}{\frac{1}{ru}\frac{\partial}{\partial r}(r\bar{U})}\right| \quad \ldots\ldots\ldots\ldots (20)$$

The value of the expression $\frac{1}{ru}\frac{\partial}{\partial r}(r\bar{U})$ depends on the distribution of u both in the horizontal and in the vertical and cannot be assessed without making detailed assumptions about the distribution of v_o and u throughout the vortex. It however appears from its form that $\frac{1}{ru}\frac{\partial}{\partial r}(r\bar{U})$ will tend to the limit $\frac{1}{ru}\frac{\partial}{\partial r}(ru) = \frac{\partial}{\partial r}(\log(ru))$ as $z \to z_1$ and we confine our attention to an infinitely thin layer. It is also apparent that $\frac{1}{ru}\frac{\partial}{\partial r}(r\bar{U})$ will remain finite as $u \to 0$.

It thus appears that the inequality (20) determines a maximum thickness of the layer $(z_1 - z)$ in which dynamic instability can be developed, and so long as $\left|\left(1 + \frac{\partial v_o}{\partial r} + \frac{v_o}{r}\right)\right| > 0$ this will be of finite thickness.

In order to obtain some idea of the thickness of the layer in which dynamic instability may develop, we assume $\bar{U} < 2u$ and u does not increase more rapidly than r.

Then $\frac{1}{ru}\frac{\partial}{\partial r}(r\bar{U}) < \frac{2}{ru}\frac{\partial}{\partial r}(ru) = 2\frac{\partial}{\partial r}(\log(ru))$

$$2\frac{\partial}{\partial r}\log r^2 = \frac{4}{r}$$

Thus (20) becomes

$$\left|(z_1 - z)\,\frac{\partial v_o}{\partial z}\right| < \left|\frac{r}{4}\left(1 + \frac{\partial v_o}{\partial r} + \frac{v_o}{r}\right)\right|$$

Assuming that $\partial v_o/\partial z$ is independent of z this can be interpreted as requiring that the change in the tangential velocity v from the top to the bottom of the layer in which instability develops must be less than $\frac{r}{2}\left|\frac{\partial y}{\partial x} + \frac{1}{2}\right|$ where $\left|\frac{\partial y}{\partial x} + \frac{1}{2}\right|$ is the amount by which the slope of the characteristic curve in the $r^2 \,.\, rv$ diagram differs from that of the critical line $y = -\frac{1}{2}x$.

It may be noted, in passing, that there is an analogy here with the development of vertical instability as treated by the "slice" method (vide J. Bjerknes - 1938 Saturated adiabatic ascent of air through a dry-adiabatically descending environment). In the case of vertical instability, if the lapse rate only slightly exceeds the adiabatic, convection can only take place in narrow towers the area of which increases with the increase in lapse rate.

Similarly in the development of dynamic instability in vortices, if the critical condition

$$\frac{\partial v_o}{\partial r} + \frac{v_o}{r} < -1$$

is only just satisfied, the instability can be developed only in a relatively shallow layer, but as $\left|\left(1 + \frac{\partial v_o}{\partial r} + \frac{v_o}{r}\right)\right|$ increases the depth of the layer in which instability can develop increases also.

We now turn to the second effect of vertical motion on the stability of the vortex at the upper level of the storm, viz. that due to the changes caused

in the tangential velocity v.

In the model of a cyclonic storm which has been developed so far in paragraph IV and which will be extended in subsequent paragraphs, it has been assumed that most of the vertical motion in a tropical cyclone takes place in a central core(which may however have a small central eye of descending air as discussed later). This model largely fits with the observed facts and although it cannot be shown why upward motion should not be taking place into all regions of an unstable upper vortex, the following discussion will be confined to a consideration of the changes in the circulation in a vortex when air with a different angular velocity is introduced into the central core of radius r_o.

We have already seen in paragraph IV that, if dynamic instability exists in a certain annulus of the vortex, this instability will remain if this annulus is displaced outward and that the representative points on the r^2 . rv diagram are displaced parallel to the line.

This is represented in figure 8, which shows the representative line P_1Q_1 for an annulus displaying instability type (b), displaced to a position P_2Q_2.

It is clear from the diagram that, if air is introduced at the inner radius with velocity v represented by a point X, then the instability of the vortex as a whole will continue if X lies above P_1 because the slope of the line XP_2 will then be less (in the mathematical sense) than that of P_1P_2 which is -1/2.

On the other hand if the point X lies at X^1 below P_1 then the inner portion of the vortex will become stable.

Similar arguments may be applied in those motions classed as cases (a) and (c) in paragraph II above.

The results may be summarised as follows, when applied to the change in dynamic stability caused by upward motion in an inner core.

Case (a) An unstable vortex of type (a) will remain unstable if fed by an upward current in which cyclonic rotation increases or remains constant with height - it will become stable if the cyclonic rotation decreases with height.

Case (b) An unstable vortex of type (b) will remain unstable if fed by an upward current in which the cyclonic rotation decreases or remains constant with height - it will become stable if the cyclonic rotation increases with height.

Case (c) An unstable vortex type (c) will remain unstable if fed by an upward current in which the rotation remains constant, increases or decreases with height.

It has already been explained the initial development of convection phenomena tends to create a cyclonic vortex near the surface and an anticyclonic one aloft. Thus it is unlikely that a situation can be maintained in which cyclonic rotation increases with height. Instability of type (a) can therefore be disregarded as a factor likely to maintain the development of a revolving storm after the initial stages.

It is otherwise with unstable vortices of type (b) and (c), for so long as surface pressure is falling the cyclonic rotation of the air near the ground will increase and a decrease of cyclonic rotation with height will be maintained. When the deepening ceases, a state in which rotation is constant with height will be attained and this will be adequate to maintain the dynamic instability type (c) aloft.

VII. Critical conditions for the development of a tropical revolving storm.

On the basis of the theoretical results obtained above we may summarise the conditions which are necessary for the formation of a revolving storm as follows:-

(a) A lapse rate which will give rise to vertical instability and the development of cumulonimbus.

(b) A distribution of rotation at some level aloft such that

$$\frac{v_o}{r} \geqslant -\frac{1}{2} \quad \text{and} \quad \frac{\partial v_o}{\partial r} \leqslant -1 - \frac{v_o}{r}$$

(c) A distribution of rotation in the vertical such that cyclonic rotation decreases with height.

Condition (a) is normally satisfied over the ocean regions in which tropical cyclones form.

Whether condition (b) is satisfied is difficult to verify, but that anticyclonic wind shear of the same order as that required for critical conditions $v_o = -\frac{1}{2} r$ do frequently occur, can be seen from the mean upper winds at Rangoon and Port Blair (Andaman Islands) (Rangoon West 20 m.p.h. Port Blair East 5 m.p.h.) at 6 km. in March.

This represents a shear of 25 m.p.h. in 300 miles whereas the critical conditions $v_o = -\frac{1}{2} r$ in this latitude (15°N) require a shear of only 21 m.p.h. in 300 miles. It is to be noticed that the difference between the winds at Rangoon and Port Blair is that between two parallel wind streams and that a complete vortex does not exist in the mean circulation. It is, however, clear that an anticyclonic circulation of the critical magnitude is by no means impossible as a cause of cyclonic development.

Owing to the increase of l with latitude it is however apparent that the conditions for a dynamically unstable vortex are more likely to be satisfied in low latitudes than in high latitudes.

Condition (c) is likely to be satisfied frequently in the regions where tropical cyclones form and it may be that this need for a pre-existing low level cyclonic circulation is connected with the frequent formation of revolving storms on the I.T.F. and its junction with other air mass discontinuities.

The decrease in the frequency of storms towards the equator within latitude 10 degrees North or South is due to the fact that with very small values of l only a feeble cyclonic circulation can be set up near the surface by the vertical withdrawal of air. This is seen from equation (16) which shows that the velocity in a ring displaced from an original radius r_o is given by $vr - v_o r_o = l(r_o^2 - r^2)$. If the air is initially without rotation relative to the earth $v_o = 0$ and the value of v depends on l.

VIII. The further development of a revolving storm.

From the discussion in section VI it will be seen that the vortex at the upper level of the cyclonic storm where the dynamic instability exists, will gradually be changed in structure by the ascent of air with cyclonic rotation in the core of the storm.

It has already been shown for an individual ring of air rotating about the axis

$$rv + \frac{1}{2} r^2 = \text{constant} = r_1 v_1 + \frac{1}{2} r_1^2$$

where r_1 and v_1 are the initial values of r and v.

Thus as air is drawn from greater and greater distances into the surface low pressure (i.e. greater r_1) the value of v will increase; the limit being determined by friction. This air with strong cyclonic rotation will ascend in the core of the storm and ultimately move outward in the dynamically unstable upper vortex.

This upper vortex will therefore be transformed on the r^2 . rv diagram from one lying near the critical line $y = -\frac{1}{2} x$ to one represented by a line parallel to the critical line $y = -\frac{1}{2} x$ and passing through the point X (Figure 8) representing the rotation of the air raised from the surface near the centre.

The vortex in the upper levels will thus consist of a cyclonic vortex surrounded by an anticyclonic one. (A typical radius for the former in latitude 11°N corresponding to a wind velocity of 40 m.p.h. at 50 miles from the centre will be about 200 miles).

IX. The fall of pressure in a revolving storm.

There is always a marked centre of low pressure at the centre of tropical cyclones and the pressure there is usually from 10 to 60 mbs below that in the surrounding areas. Exceptional storms have been known to cause pressure falls of more than 100 mbs. It is desirable therefore to see whether the theory outlined above is capable of explaining pressure falls of the observed magnitudes.

It has been explained in paragraph VIII how the circulation of the dynamically unstable vortex at level A will be changed from an anticyclonic, or slightly cyclonic circulation, to an intense cyclonic vortex decreasing rapidly and becoming anticyclonic outwards. This latter vortex will correspond to a different pressure distribution at level A to that which originally existed. Owing to the increase of cyclonic circulation a fall of central pressure will be required and the vortex being unstable in the sense used in this paper, will continue to remove air outwards until this pressure difference is established.

This fall of central pressure at level A will cause a lowering of pressure both at levels above and below A. The fall of pressure above the storm will cause an inflow and descent of air above the storm as indicated in Figure 9. Assuming that the circulation at the higher levels is dynamically stable a balance will be reached when a cyclonic circulation has been set up above the storm, but only after the tropopause has been lowered and the air column been warmed adiabatically.

In order to assess the difference of pressure between the centre and the periphery of the cyclone (Pressure at P - Pressure at Q in Figure 9) we may write

$$\begin{aligned} \text{Pressure at P - Pressure at Q} &= (\text{Pressure P - Pressure S}) \\ &\quad - (\text{Pressure Q - Pressure R}) \\ &\quad + (\text{Pressure S - Pressure R}) \\ &= (\text{Weight of column SP - Weight of column RQ}) \\ &\quad + (\text{Pressure at S - Pressure at R}) \end{aligned}$$

The first term can be estimated from the difference in lapse rate between the original air column and a column ascending from the surface. The second term depends on the dynamics of the vortex at the level A and will be calculated below.

It will be seen when the formation of the "eye of the storm" is discussed that there is another factor likely to cause a lowering of central pressure viz. the centrifugal effect within the eye of the storm. This is difficult to assess numerically and is not likely to affect the order of magnitude of the results. It will therefore be omitted in the present paragraph.

Shaw (1922) assessed the difference in weight of the columns SP and RQ on the basis of balloon-sonde results at Batavia as about 80 mbs pressure, if the storm were to extend to the tropopause. This result is, perhaps, rather large as it depends on large differences in temperature in the levels between 8 and 15 km. A similar calculation based on a typical ascent for Addu Atoll (Indian Ocean)(9th February, 1945) during the cyclone season for the neighbouring ocean gives a difference of pressure of only about 26 mbs, although greater values might well occur on special occasions.

In this connection it is interesting to note that on 31st October, 1943, F/O. Rawlinson observed temperatures about 6° F higher (at 5000 to 10000 ft.) in the cloud system of a depression in the Bay of Bengal than in the air to the North of the cloud system. This is of about the same magnitude as assumed by Shaw for these levels in his computation for Batavia.

The difference in pressure between points S and R in figure 9 can be estimated as follows from the change in the circulation at the level A. We shall consider a vortex initially in the unstable state (case (c)) represented by the line PQ in the r^2 . rv diagram (figure 10) which is transformed into a state represented by the parallel line P_1Q_1 as described in section VIII. We shall determine the pressure gradient on the assumption that a steady state has been reached in which the outward velocity u can be neglected and we shall ignore the frictional and vertical motion terms.

Equation (2) then becomes

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = lv + \frac{v^2}{r}$$

The pressure difference between two rings r_1 and r_2 may then be obtained.

$$p_2 - p_1 = \rho \int_{r_1}^{r_2} (lv + \frac{v^2}{r})dr$$

$$= \rho \int_{r_1}^{r_2} (l\frac{v}{r} + \frac{(v)^2}{(r)})rdr$$

$$= \frac{\rho}{2} \int_{x_1}^{x_2} (l\frac{y}{x} + \frac{y^2}{x^2})dx \quad \ldots\ldots\ldots\ldots\ldots\ldots(21)$$

In the initial state

$$y = -\frac{l}{2}x$$

Thus

$$p_2 - p_1 = \frac{\rho}{2} \int_{x_1}^{x_2} (-\frac{l^2}{4})\,dx = -\frac{\rho l^2}{8}(r_2^2 - r_1^2) \quad \ldots\ldots\ldots\ldots(22)$$

The initial state is therefore associated with a pressure distribution

$$p = p_o - \frac{\rho l^2 r^2}{8}$$

and using a value of $\rho = 500 \times 10^{-6}$ gm/cm^3 corresponding to a level about 20000 ft, this gives a very small outward pressure gradient of about 1 mb. in the first thousand miles in latitude 11° N for the initial distribution of pressure.

In the final state of the vortex at level A let the tangential velocity be v_1 at the inner boundary of the unstable vortex (radius r_1) within which the vertical motion has been taking place.

The radial distribution of velocity may then be represented by the equation.

$$y = -\frac{l}{2}x + r(v_1 + \frac{lr_1}{2})$$

(the line P_1Q_1 in figure 10)

writing $\bar{v}_1 = v_1 + \frac{lr_1}{2}$

this becomes

$$y = -\frac{l}{2}x + r_1\bar{v}_1$$

Thus from equation (21)

$$p_1 - p_2 = \frac{\rho}{2} \int_{x_1}^{x_2} \left\{ l\,(-\frac{l}{2} + \frac{r_1\bar{v}_1}{x}) + (-\frac{l}{2} + \frac{r_1\bar{v}_1}{x})^2 \right\} dx$$

$$= \frac{\rho}{2} \int_{x_1}^{x_2} (-\frac{l^2}{4} + \frac{r_1^2\bar{v}_1^2}{x^2})\,dx$$

$$= \frac{l^2}{8}\rho\,(r_2^2 - r_1^2) + \frac{\rho\bar{v}_1^2}{2}(1 - \frac{r_1^2}{r_2^2})$$

The first term is the value of $p_1 - p_2$ in the initial state (see equation (22)) and thus it follows that assuming the pressure outside the storm remains constant there must be a fall of pressure at the centre of the vortex at level A of $\frac{\rho\bar{v}_1^2}{2}(1 - \frac{r_1^2}{r_2^2})$ during the development of the storm.

Putting

r_1 = 50 miles $\quad r_2$ = 200 miles

v_1 = 100 m.p.h. $\quad \bar{v}_1 = v_1 + \frac{1}{2}r_1$ = 102 m.p.h.

ρ = 500 x 10^{-6} gm/cm^3

the fall of pressure at the centre is approximately 3 millibars. This will produce a fall of pressure at the surface of approximately 6 mb. because of the reduction in density of the air column by the fall of pressure.

We may therefore conclude that a fall of central pressure in a tropical cyclone of the order of 20 to 30 mbs. may be caused by the change in density of the air by release of latent heat of condensation (this may even reach 80 mbs. if vertical motion extends to the tropopause). In addition a fall of 5 to 10 mbs. can be expected due to the change in circulation at the level from which air is drawn. These figures although approximate show that the theory outlined can explain the observed falls of pressure, and although it will be noted the major contribution is from the warming of the central air column, it is to be emphasized that this alone cannot be used to explain the fall of pressure in a tropical cyclone unless an explanation can be given as above to explain why the pressure at R (Figure 9) should not exceed that at S.

X. The formation of the "eye" of the storm.

It has been shown (equation (17)) that for an individual annulus of air drawn into the centre of a cyclonic storm

$$r_1 v_1 + \frac{1}{2} r_1^2 = C_1$$

and if v_1 is initially zero

$$r_1 v_1 + \frac{1}{2} r_1^2 = \frac{1}{2} R_1^2$$

(where R_1 is the original radius of the ring now radius r_1)

Until the region is reached where vertical motion is taking place the area between two consecutive annuli must remain constant.

Thus $r_1^2 - r_2^2$ is constant.

So that considering two annular rings for which

$$r_1 v_1 + \frac{1}{2} r_1^2 = \frac{1}{2} R_1^2$$

$$r_2 v_2 + \frac{1}{2} r_2^2 = \frac{1}{2} R_2^2$$

$$r_1 v_1 - r_2 v_2 = \frac{1}{2}(R_1^2 - R_2^2) - \frac{1}{2}(r_1^2 - r_2^2) = 0$$

Thus at any time the vortex at the surface is characterised by the relation rv = const.

This was pointed out by Durst and Sutcliffe (1938) who further observed that this relation cannot be continued up to the centre as it would require excessively large velocities in the innermost rings of the vortex. They therefore considered that the vertical motion must take place in an annulus a short distance from the centre and the relatively calm "eye of the storm" as observed in tropical cyclones then appeared as a necessary corollary.

It is implied in this treatment of the problem of the eye of the storm, that the very large values of v which would occur if there were no calm centre to the storm are prevented by frictional turbulence. If the turbulence concerned were that set up by vertical wind shear near the ground it is difficult to see why the wind velocities in the calm centre should be reduced to values below those observed outside. It is therefore postulated that the important factor is turbulence set up by the horizontal wind shear which increases rapidly towards the centre in a vr - vortex. This has been pointed out by Brunt - Physical and Dynamical Meteorology (2nd edition) p. 305.

If the horizontal turbulence is accepted as responsible for the calm eye of the storm, it is clear that it will act most rapidly near the centre where the velocity gradient is greatest, and that if there were no radial velocity the relatively calm "eye" so formed would grow outward. If an inward radial velocity was imposed on the air of the vortex the growth would be less rapid, because angular momentum would be carried inward by the inflowing air and greater destruction of momentum by turbulence would be necessary to produce the same size calm area.

We may thus expect that the calm "eye" of the storm will be smaller in those regions of the storm where inward motion is taking place, namely in the friction layers near the ground. Observations by Wexler (1945) on a flight into a tropical storm off the east coast of North America suggested a rapid increase in the diameter of the "eye" of the storm with height in the lowest 5000 ft. Although apparently still outside the central calm region and in a region of considerable upward motion, the observers noted that the cloud layer above them was very thin, a possible explanation being that the clear eye of the storm lay above them at no great height.

Durst and Sutcliffe (1938) have pointed out within the "calm" centre of the tropical storm the air will tend to move outward on account of the centrifugal effect. There will thus be a further fall of pressure at the centre of the calm area and descent of air in the eye of the storm will take place, presumably being ultimately balanced by the intensification of the cyclonic vortex aloft at the levels from which the descending air is drawn.

We are thus led to postulate as the final state development of a revolving storm the structure represented in Figure 11.

There is some evidence for the lowering of the tropopause (or at least for considerable warming of the upper troposphere) in the observations made by an American aircraft in the upper part of the central area of a Gulf of Mexico hurricane on 18th September, 1943. (These are reported by I.R.Tannehill in the bulletin of the American Meteorological Society September, 1945). This storm was rather weak and had already drawn extra-tropical air masses into its circulation and cannot be regarded as a typical tropical storm; however an almost isothermal layer was observed over the centre between 437 mbs. and the top of the ascent at 344 mbs. pressure (i.e. approximately 21000 to 27000 ft) and at the highest level the temperature was more than 20° F above that at radiosonde stations less than 300 miles from the centre. Indirect evidence for very warm air over a tropical storm in its later stages is contained in the observations of high level winds at Bermuda with the approach of such a storm as reported by Moorhead (1936). These show a remarkable increase (80 m.p.h.) in the westerly component between 32000 and 38000 ft. as the storm approached and assuming that the flow was geostrophic this would be evidence of very warm air aloft towards the centre of the storm.

XI. Downward flow in the lower layers of a cyclone.

The variation of wind with height in the layers near the ground has been investigated theoretically by Taylor and his analysis is reproduced by Brunt (Physical and Dynamical Meteorology 2nd Edition p.252). It is shown that the wind vector at any particular height is a fraction of the wind above the friction layer depending on the values of K and α_o, the coefficient of eddy diffusivity and the deflection of the surface wind into the low pressure. The angle of deflection of the wind into the low pressure, α, also depends on these two factors and not directly on the wind above the friction layer.

An increase in either α_o or K will result in an increase in the inflow across the isobars towards the low pressure. It is observed that with increase of wind velocity both α_o and K increase, and this is particularly true of winds over the sea where the wave height and consequently the surface roughness increases with wind velocity.

If we first assume that K and α_o are the same throughout the lower layers of a tropical storm and that the wind above the friction layer obeys the relation for a vr - vortex (vr = constant) it follows that, if the motion takes place according to Taylor's analysis, the inflow across any annulus in the friction layer is the same.

To show this we consider the inflow at any level at which the wind velocity = βv (where v is the wind above the friction layer) and is deflected at an angle α from the tangential direction.

The inflow across a ring of radius r is then

$$2\pi r \beta v \sin \alpha$$

But for a vr vortex, vr is constant for one level throughout the vortex and by assuming K and α_o constant we have assumed β and α constant for any fixed level. Thus the inflow at this level is constant throughout the vortex.

However as α and K are believed to increase towards the centre of a cyclone it follows that the inflow of air over each annulus will increase towards the centre. (Another factor tending in the same direction is that in a cyclonic vortex the pressure gradient is steeper than that based on the geostrophic wind equation on which Taylor's analysis is based).

In order to compensate for the increased inflow in the inner parts of the storm it follows that there must be a downward motion of air into the friction layer in the lower part of the outer region of the storm.

Such downward flow has been indicated in figure 11, but the rate of descent cannot exceed a few feet per minute, from this cause. Wexler (1945) reports a strong down current in this region of the storm and, although he does not quote any magnitude for it, a current which would be reported by an aircraft as strong could hardly be produced by the mechanism indicated above, in fact it is difficult to see how the accumulation of air caused by such a down current could be disposed of at all, if it covered the whole area of the storm between distances of 100 and 10 miles from the centre as Wexler's observations suggest.

XII. Fronts in the circulation of tropical cyclones.

It will be noted that throughout these notes the structure of a tropical cyclone has been assumed symmetrical despite the fact that a number of writers have postulated that (a) tropical revolving storms originate on fronts and in particular on the so called intertropical front and (b) that in a fully developed tropical revolving storm there are fronts, indeed several fronts, which can be distinguished by wind shifts and rainfall.

Within the storm area of a revolving storm the wind has been seen to be roughly inversely proportional to the distance from the centre. (vide Hong-Kong observations quoted by Durst and Sutcliffe (1938)), but the angle swept in time t by the air distant r from the centre is vt/r radians and thus is inversely proportional to r^2.

With such a distribution of wind any front within the storm area would be rapidly sheared out of existence; for with a velocity of say 12 m/s (26 m.p.h.) at 80 km.(50 miles) from the centre the air would circulate the centre in 13 hours, while in the air at 40 km.(25 miles) from the centre the air would have travelled four times round in that space of time.

The complete circulation of air around the tropical revolving storm is no fiction. (The brig Charles Heddle in February, 1845 ran before a cyclone in the Indian Ocean and circled it five times before she sailed into fair weather having suffered some damage.)

The diameter of most tropical storms is about 300 km. (200 miles) so that even on the edge air circulates about once a day. Thus it appears impossible that the energy of such a storm can be derived over a period of days from a frontal system in any manner similar to that of an extra-tropical depression.

It is suggested therefore that the formation of tropical storms on or near the Inter-tropical front is more likely to be due to the association of the front with a distribution of vorticity favourable to the initial development of a cyclone, than any inherent necessity in the mechanics of a cyclone for the existence of a frontal structure within it.

In their later stages it is well known that tropical cyclones draw extratropical air masses into their circulation and ultimately take on a structure of an extra-tropical depression, and that even while in the tropics they may ultimately produce changes in the pressure field outside the storm area which lead to the enhancement of airmass contrasts and the development or intensification of fronts. However, while within the tropics it seems most unlikely that such fronts can be traced or indeed exist within the relatively small area of strong winds.

References.

Shaw, Sir Napier	1922	Geophys. Memoir 2 No. 19
Brunt D and C.K.M. Douglas	1928	Mem. R. Met. Soc. 3 No. 22
Brunt D.	1921	Proc. Roy. Soc. (A) 99 397
Durst C.S and Sutcliffe R.C.	1938	Q.J.R.Met.Soc. Vol. 64 No. 273
Bjerknes J.	1938	Q.J.R.Met.Soc. Vol. 64 No. 275
Petterssen S.	1940	Weather Analysis and Forecasting.
Wexler	1945	Bulletin of the American Meteorological Society May 1945.
Tannehill I.R.	1945	Ditto Sept. 1945.
Moorhead H.B.F.	1935	Q.J.R.Met. Soc. Vol. 62 No. 263 p. 123

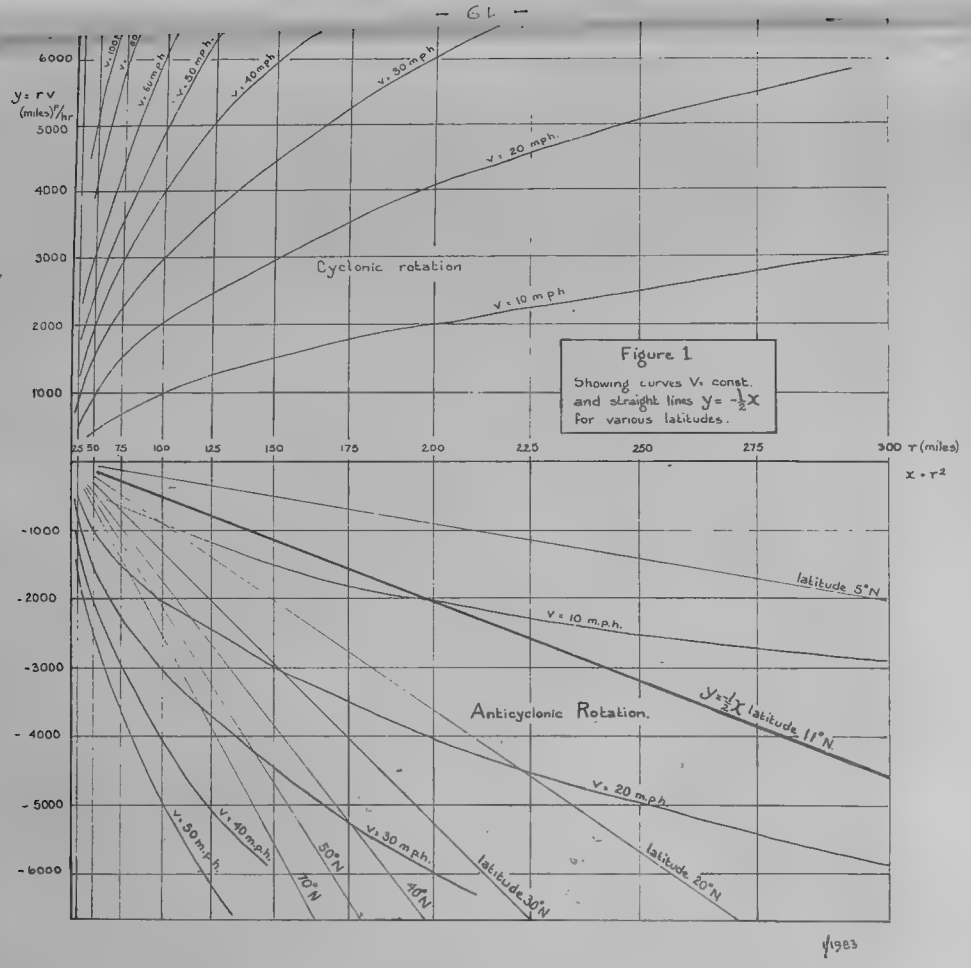
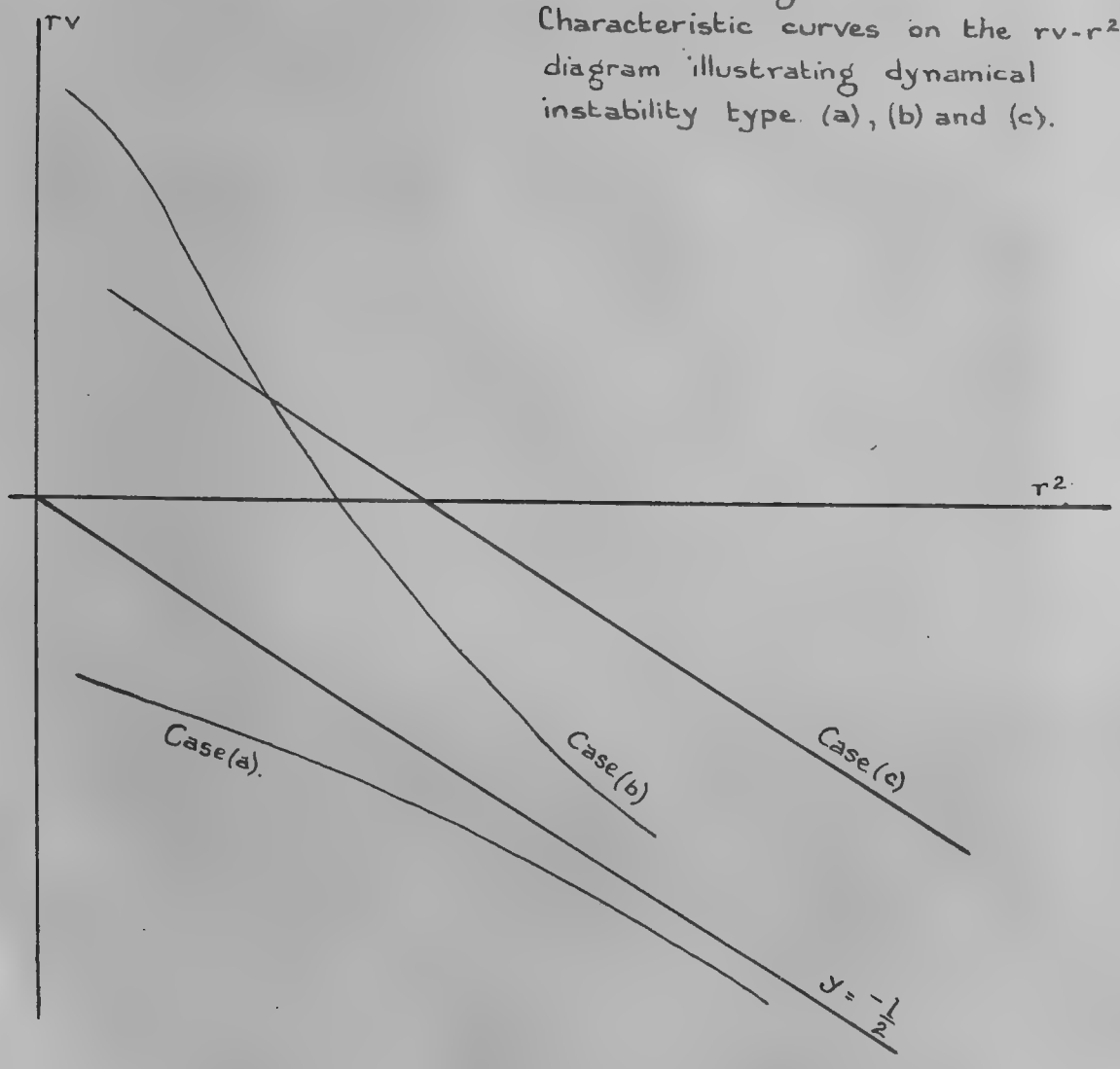


Fig 2.

Characteristic curves on the rv-r² diagram illustrating dynamical instability type (a), (b) and (c).



1/1983.

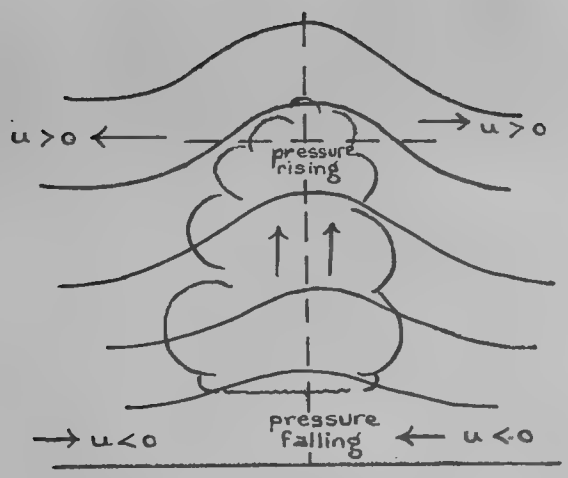


Fig 3. Stage I.

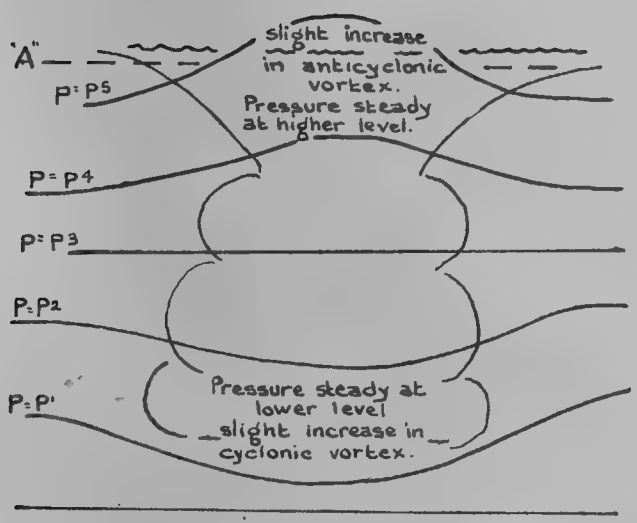


Fig 4. Stage II (stable case).

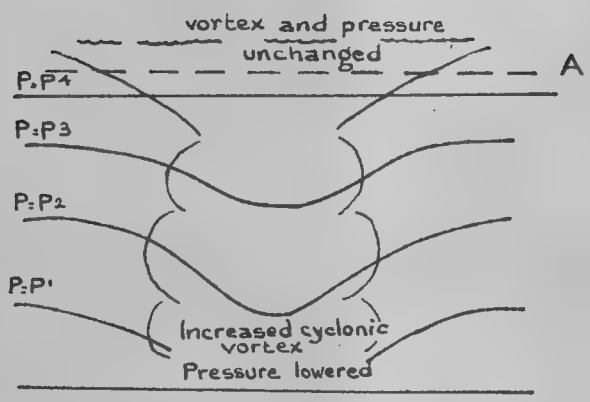


Fig 5. Stage II (unstable case).

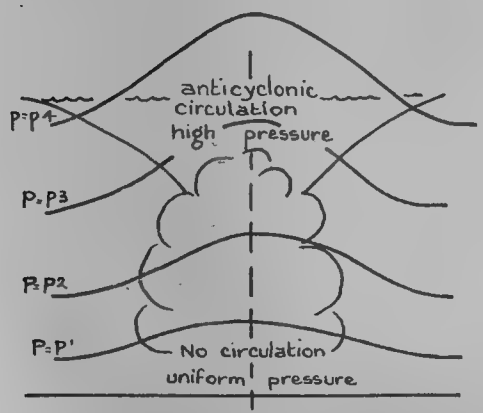


Fig 6. Stage III (stable case).

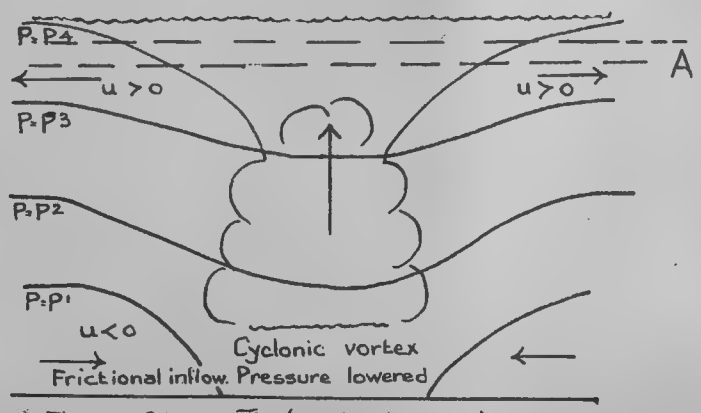


Fig 7. Stage III (unstable case).

1/1983.

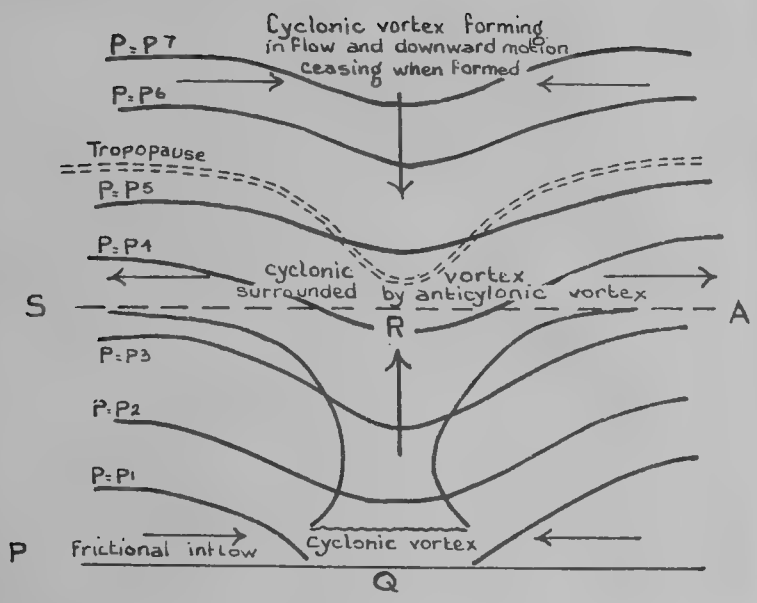


Fig 9. Stage IV (unstable case).

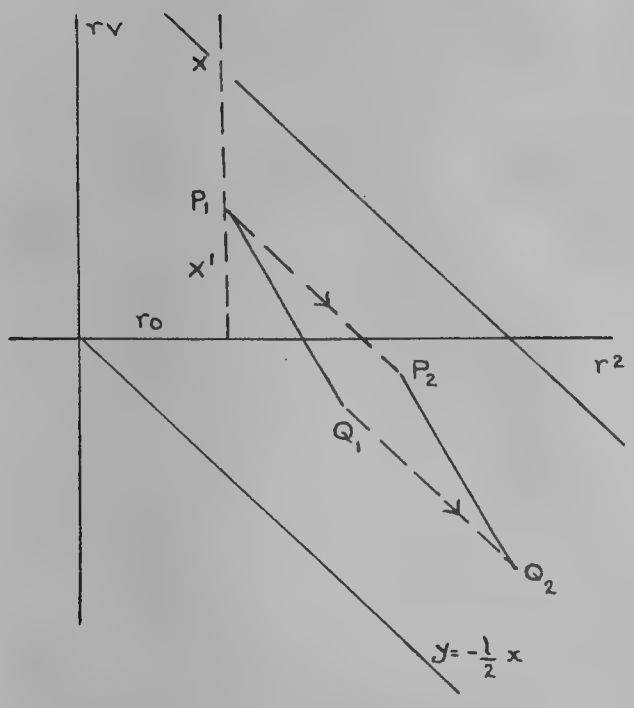


Fig 8. Illustrating on r^2-rv diagram changes in a vortex due to vertical motion.

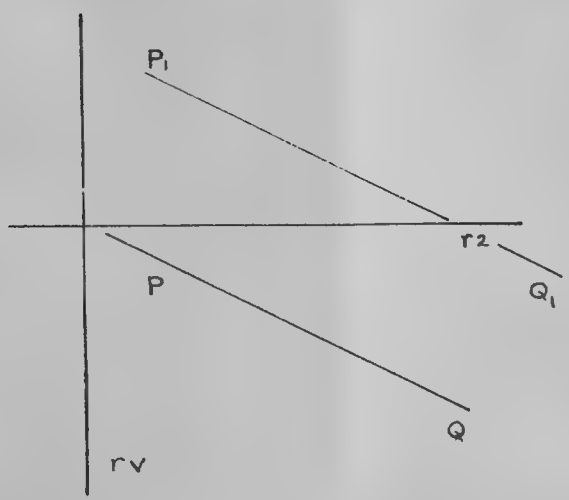


Fig 10.

1/1983

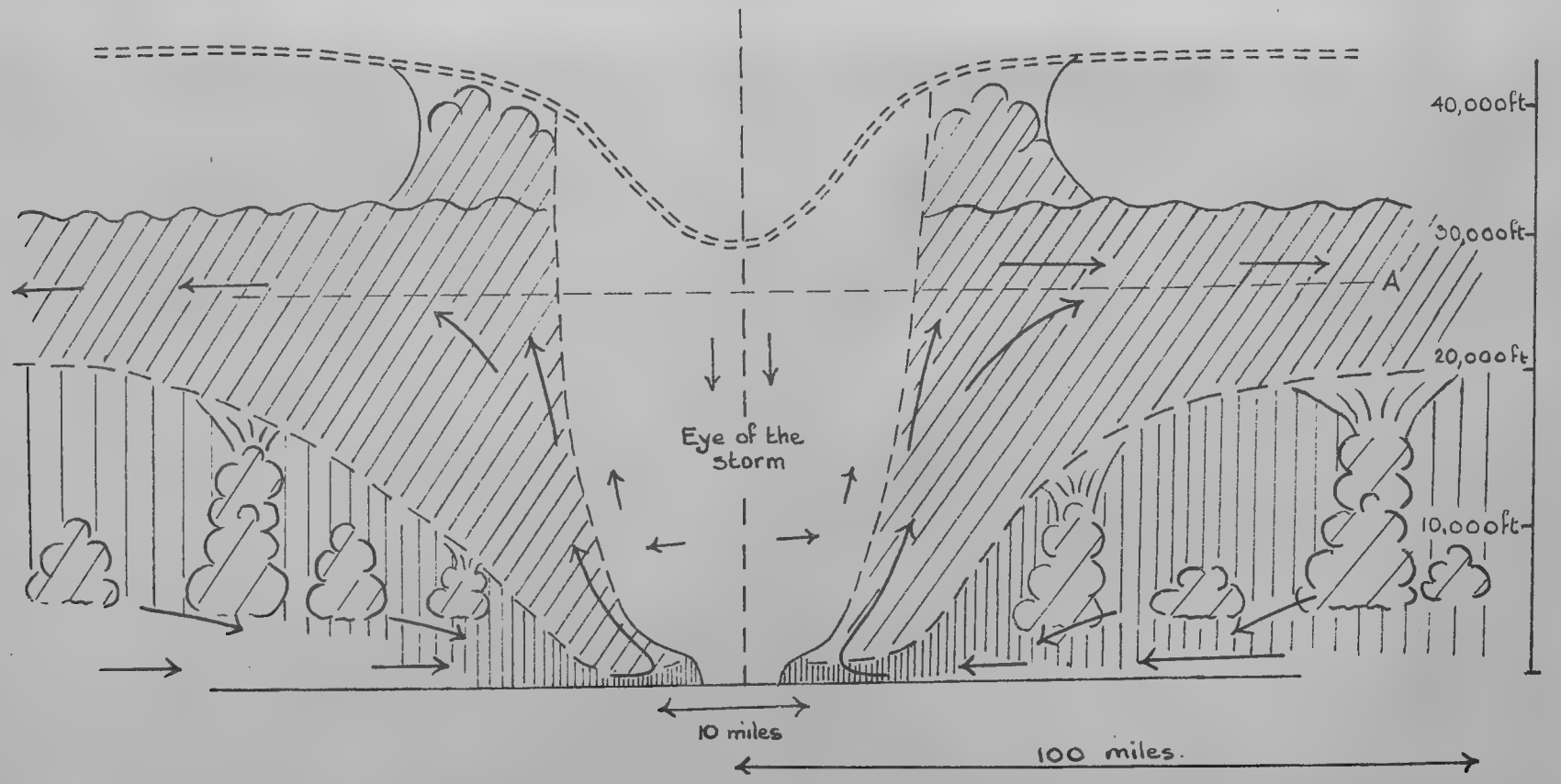


Fig 11. Final stage of development of a tropical storm.

1/1983.

47.~~979~~
667

Amendment and Additions to

M.R.P. 305.

Will you please make the following amendments and additions to M.R.P.305.

Amendments

Add Asterisk to M.R.Ps. - 270, 272, 273, 277, 280 and 305.

Additions

M.R.P.	Cat
306	C
307	U
308	U
309 *	U
310	U
311	U
312	U
313	C
314	U

G.147,442.

(OMITTED)

By

1.

2.

(a) Mazur

in R.& M. 109 and only a very brief description is needed here.

essentially of a suitable holder for a glass plate with a slot in the holder in front of the plate. This slot is opened for

The instrument is mounted on the aircraft with the slot facing forward into the natural wind. Thus, when the slot is opened,

in the air. Mazur used two kinds of coating, a mixture of vaseline and oil and a thin layer of lig. The oil mixture

er droplets and the plate must be covered with a

e formation of small pits as a result of the

are examined through a

aircraft on the

two methods, of course, give only a mean size for the drops.

/The

Water drops at

with the axis perpendicular to

with the cylinder and so form a deposit

cylinder. This deposit

In part the

Bruce

Im Sommer, 1926, Met. Zeit., Sonnblick-Ver, 55, 1926, pp. 25-28.

at leisure some hours later, Diem photographed his plate while in the aircraft - normally within about 15 seconds of exposure.

data available consist of:-

Cloud Type

Number of droplets per c.c.

particular cloud.

series.

In several cases two samples were obtained in one cloud but in only one

/For

Height
Cloud Type

The liquid water content of the cloud (gms/m^3) as deduced from the

100. This left only 19 of the original 59 samples.

4. Presentation of Results

values

6. Method of Reducing Diem's Data

7. Tabulated Results

shows:-

(b) Cloud type
(c) Temperature
(d) Number of samples

cumulative volume (D_{90} - D_{10}

computed from (e) and the calculated volume of N drops. Thus (c) and (f)

(d) is omitted. The values of L in brackets were obtained by the deposition

available. There is no information available as to the number of drops per unit volume.

The quantity $(D_{90} - D_{10})$ has been included to indicate the range of diameters of the drops which together constitute the bulk of the liquid content of the cloud L.

value of the average diameter.

8. The Large Drop Problem

It will be seen from Table 1 that on some flights Mazur measured drops

the D_{100} column of Table 1 we find an entry of 112 μ against flight No. 29. This means that the upper limit of the step of the diameter series which

from the table the lower limit of this step was 75 μ. Normally such drops

classed as cloud particles. With the method of presentation adopted their inclusion involves certain difficulties. It is clear that one or two

cumulative volume curve. But the number of the large drops which were caught in the sample was so small that it could not be regarded as a reliable statistical guide to their frequency of occurrence. On the other hand in any problem involving consideration of the volume of liquid water in the cloud, the importance of the few large drops may well outweigh that of all the smaller drops.

the presence of these large drops. For example flights numbered 11 and 14

11	2000 ft. to 2800 ft.	3.9°C	14	717	1478	68	-	-	-	-	-	-
14	3400 to 4000 ft.	2.4°C	100	607	750	197	55	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

clear from the D_{100} column of Table 1 that such clouds are not all of

on this problem.

9. Variation Within the Cloud

/Table

Table 3

Variation within the Cloud

[illegible] No.	[illegible] type	No. of Samples	D_{50}		D_v	
			[illegible]	σ	[illegible]	σ
12	Sc.	5	[illegible]	[illegible]	[illegible]	[illegible]
20	"	5	[illegible]	[illegible]	[illegible]	[illegible]
24	"	5	[illegible]	[illegible]	[illegible]	[illegible]
26	"	5	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		-	-	[illegible]	-	[illegible]
21	St	3	[illegible]	[illegible]	[illegible]	[illegible]
22	"	3	[illegible]	[illegible]	[illegible]	[illegible]
23	"	3	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		-	-	[illegible]	-	[illegible]
16	Cu	3	[illegible]	[illegible]	[illegible]	[illegible]
18	"	3	[illegible]	[illegible]	[illegible]	[illegible]
27	"	3	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		-	-	[illegible]	-	[illegible]
11	Large Cu	3	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		3	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		3	-	[illegible]	-	[illegible]
9	A.St.	3	[illegible]	[illegible]	[illegible]	[illegible]
19	A.Cu.	3	[illegible]	[illegible]	[illegible]	[illegible]

It is clear from these figures that Cumulus cloud is more homogeneous [illegible] before precipitation within the cloud occurs even large cumulus clouds may [illegible]

It is to be noted that the values of σ (D_{50}) and σ (D_v) for the cumulus clouds and for the first of the large cumulus clouds (flight No.11) are all less than the corresponding figures obtained on any other flight except No.12.

The figures for D_{50} in Table 3 show that as the mean value over several samples increases the variability (i.e. the standard deviation) also increases. This may be due to the fact that the larger drops are so few in number that the number caught in the sample is inadequate as a guide to their frequency. The effect is not evident in the relation between σ (D_v) and D_v. In this connection it is worth noting that although Mazur does not state the volume of cloud sampled a comparison of his figures for the total number of drops sampled and the number of drops per cc. indicates that the sample usually consisted of about 2cc.

/10.

10. Variations between Clouds of the Same Type and of Different Types.

Table 4

Data from	Flights					
	11					

Table 5

Variation between clouds of similar Type and which

Data From	Cloud Type	No. of Flights	D_{50}		D_V		$(D_{90} - D_{10})$	
			Mean	σ	Mean	σ	Mean	σ
Mazur	Cu.	8	12.8	3.7	9.6	2.5	10.4	7.7
	St.Cu.	6	14.0	3.7	9.8	2.6	15.7	9.6

Conclusions

/The

that the curves are almost identical with regard to the distribution of

substantial agreement if large drop clouds are omitted and that cloud type is not a major factor in determining drop size distribution.

11. Effect of Temperature

From tables 1 and 2, D_V, D50 and $(D_{90} - D_{10})$ have all been plotted

temperature is not a controlling factor in determining D_V, D_{50}, $(D_{90}-D_{10})$.

12. Relation between Liquid Water content, number of drops per unit volume and D_V.

against the liquid water content of the cloud. It is clear that there is

increasing liquid water content. Since the curve is not a straight line

that for the most part an increase in liquid water content is accounted for

On the whole the agreement between Diem's and Mazur's results in this

13. Drop Size distribution as function of liquid water content.

are inadequate to permit subdivision with regard to both cloud type and liquid

liquid water content. For this purpose two groups only were formed from the results of each investigator viz. clouds with liquid water content L less

to fill in the gaps in Table 2. Such values are enclosed in brackets.

was decided to include this in the smaller water content group. One of

for various groupings of the clouds. Examination of this table leads to the

/(1)

and 6 have been plotted. These are regarded as giving reliable guides to

curves (16.5 -10.5 = 6.0) differs but little from the difference in the

15. Conclusions

In any discussion of the size distribution of cloud particles it is

size as drizzle or rain drops, shall be included within the definition of

be greater than that of the smaller particles. If such drops are included in

to the size of the biggest water drops in the cloud. The vertical air

cloud particles and rain drops.

The difficulty outlined above originates in the fact that there is

travelled by the air in which the cloud forms. It is by no means clear

16. Summary

cloud.

it seems likely that the number of drops per unit volume also

per cubic metre and with liquid water content between 0.2 and 0.5 gms per cubic metre.

constant with regard to height, temperature and liquid water

with time.

Flight No.	Cloud	Temp. °C	No. of Samples	Total No. of Drops N	Liquid Water L gms/cu.m	Drop per cu.m m	Min	For 5 gms/cu.m V	Max.	Vol. Diam.	Vol. or C%
							D_0	D_{50}	D_{100}	V	
5	St Cu	2.7	3	1846	.38	239			45	5.6	8.4
7	"	1.4	4	2086	.26	246			39	12.6	1.
8	"	7.8	5	2527	.15	237			39	10.7	9.
24	"	-2.0	5	3055	.06	314			45	7.2	55.
26	"	3.7	6	5027	.32	395			75	11.6	-1.
20	"	1.8	8	6798	.41	704			75	10.4	7.
4	"	6.1	3	828	.82	193			100	0.1	0.
12	"	14.6	5	2091	.02	306			18	8.1	5.
25	"	-5.0	2	1269	.10	606			30	6.9	4.
11	Large C	3.9	5	2277	.07	214			18	8.4	3.
14	" "	2.4	6	1798	.42	156			75	17.3	9.
13	Cu	5.1	1	461	.23	216			18	12.6	3.
16	"	7.8	5	3257	.24	350			24	10.9	0.
17	"	1.9	4	1267	.25	149			75	14.7	0.
18	"	2.2	2	1086	.07	255			30	8.3	8.
19	"	?	6	2291	.23	180			112	13.4	5.
18	"	7.4	5	1833	.11	175			30	10.5	0.
17	"	3.0	3	3190	.05	661				5.3	5.
6	"	-5.7	4	2372	.49	419				13.1	7.
10	"	-4.6	2	1570	.12	553				7.6	
			7								
			1						75	14.8	
			2			57			24	10.4	7.
										5.8	60.
			1			371				10.1	143.
			2								

				Water L gms/cu.m	Min. D_0	For 50% Cum.Vol. D_{50}	Max. D_{100}	Vol. Diam.	for 80% Vol.
				(.07)	2	6.9	10		
					2	6.5	9		
				(.18)	2	11.6	20		
				.20	4	18.9	30		
				.42	6	17.2	22		
				.23	2	26.6	38		
				(.26)	3	12.3	18		
				(.11)	2	10.0	16		
				(.06)	2	14.0	20		
				(.13)	2	7.5	13		
				(.24)	3	16.1	24		
				.16	3	10.8	17		
				.15	2	18.9	36		
				.14	3	17.0	26		
				.11	2	16.0	22		
				.27	3	26.3	34		
				.32	3	24.2	38		
				.16	2	8.8	19	8.1	10.9
				(.21)	3	24.7	34	17.1	20.2

Table 6

Percentage Cumulative Volume

5	3.1	12.1	14.7	17.4	21.1	25.0	27.9	28.7	30.9	15.6	0.28
6	2.9	10.2	12.5	16.9	24.6	33.3	41.9	45.3	52.5	14.1	0.26
1	0.5	2.1	2.3	2.9	6.0	13.8	16.8	16.9	16.5	1.6	
3	0.6	2.0	2.2	2.9	5.3	11.0	13.7	14.1	14.8	2.7	
4	0.7	1.4	2.5	7.8	14.0	18.4	24.6	27.9	22.8	1.8	
5	1.2	3.7	5.3	5.5	5.3	6.1	7.6	7.8	8.0	3.4	
6	0.9	2.8	4.0	7.1	11.9	16.4	21.5	23.9	27.9	2.7	
7	0.8	2.9	3.9	6.3	10.7	15.6	20.1	21.9	24.8	3.5	

2 Diem's data	12 Flights
3 Mazur's	24 Flights
4 Mazur's data	13 Flights
5 Diem's data	7 Flights
3+5 Mazur's + Diem's data	20 Flights
1 Mazur's	44 Flights

G.13831

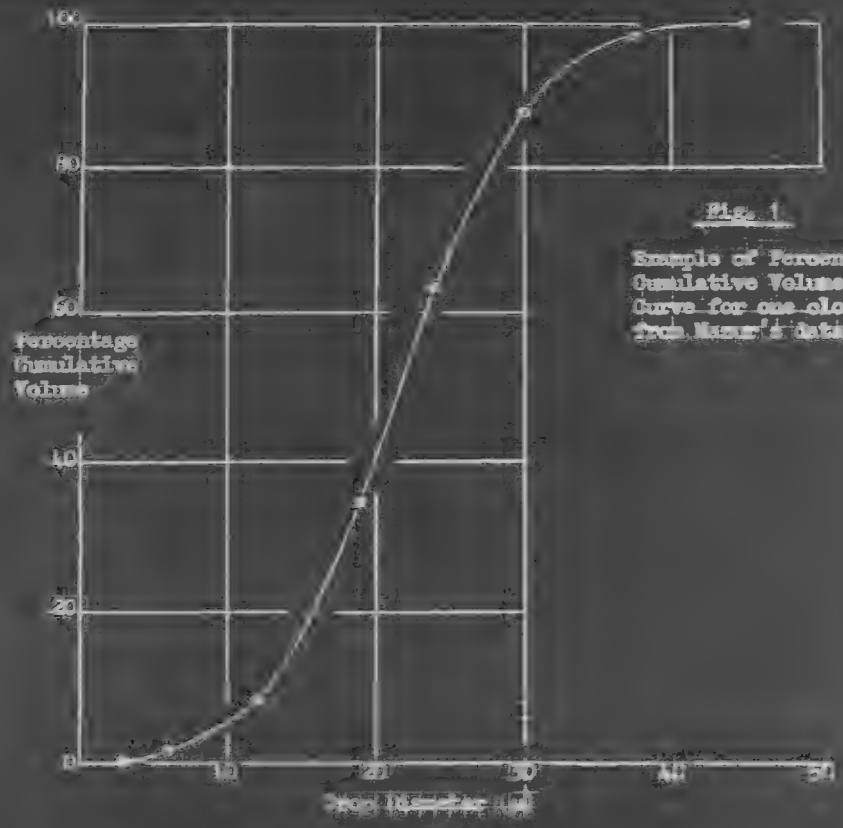


Fig. 1

Example of Percentage Cumulative Volume Curve for one cloud from Mazur's data.

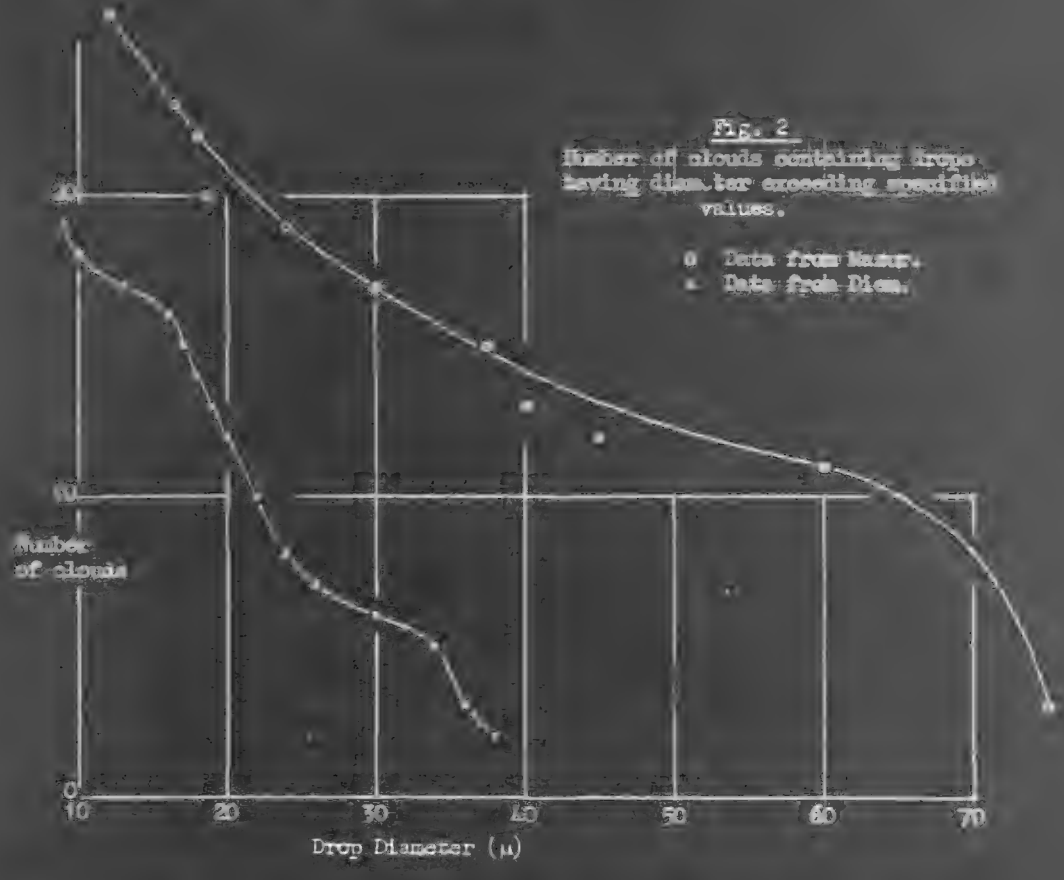


Fig. 2

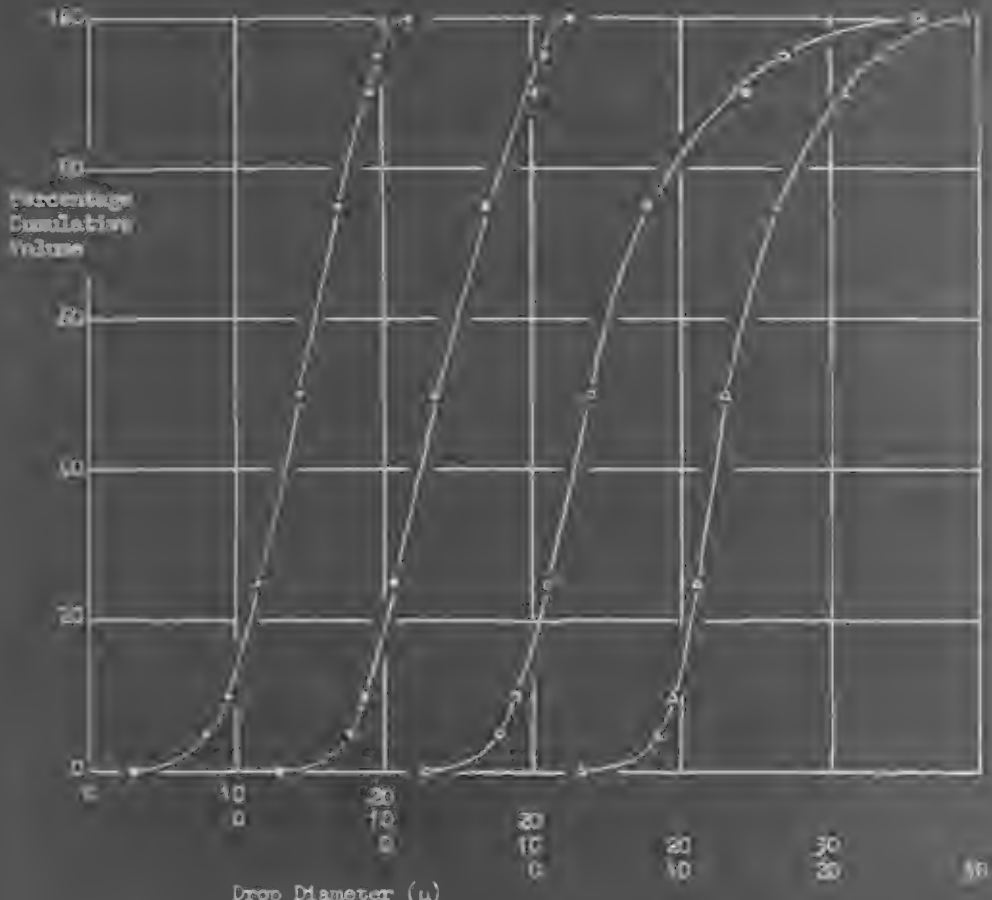
Number of clouds containing drops having diameter exceeding specified values.

o Data from Mazur.
x Data from Diem.

Fig. 3

Percentage Cumulative Volume Curves.

- Diem. A. St.clouds
- [illegible]] omitting clouds containing drops with diameter exceeding 30μ.



Note. To aid clarity the origin of the diameter scale has been moved one inch (= 10μ) to the right for each curve in succession.

Fig. 4

Liquid water content of cloud L plotted against number of drops per cc. n. Mazur's data only

• clouds having drops with diameter > 50μ

o clouds having no drops with diameter > 50μ

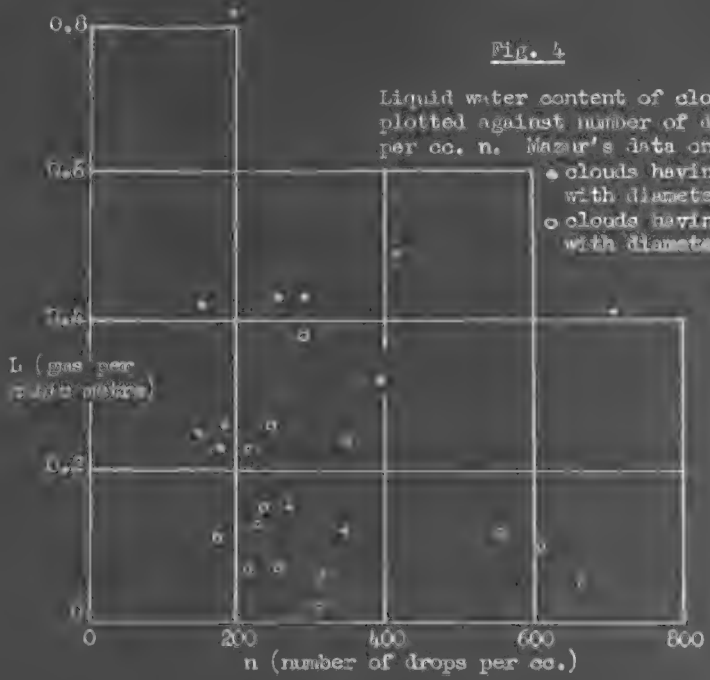


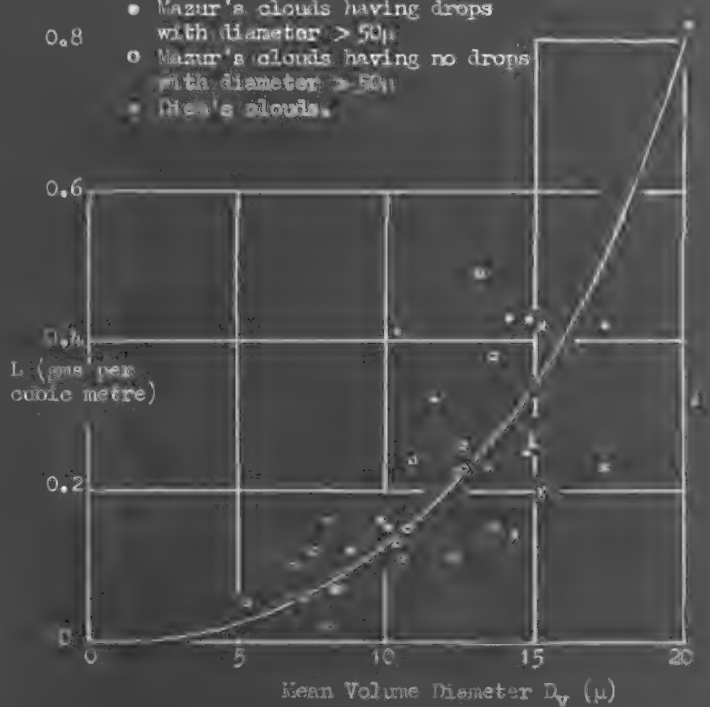
Fig. 5

Liquid water content of cloud L plotted against mean volume diameter D_v.

• Mazur's clouds having drops with diameter > 50μ

o Mazur's clouds having no drops with diameter > 50μ

+ Diem's clouds.



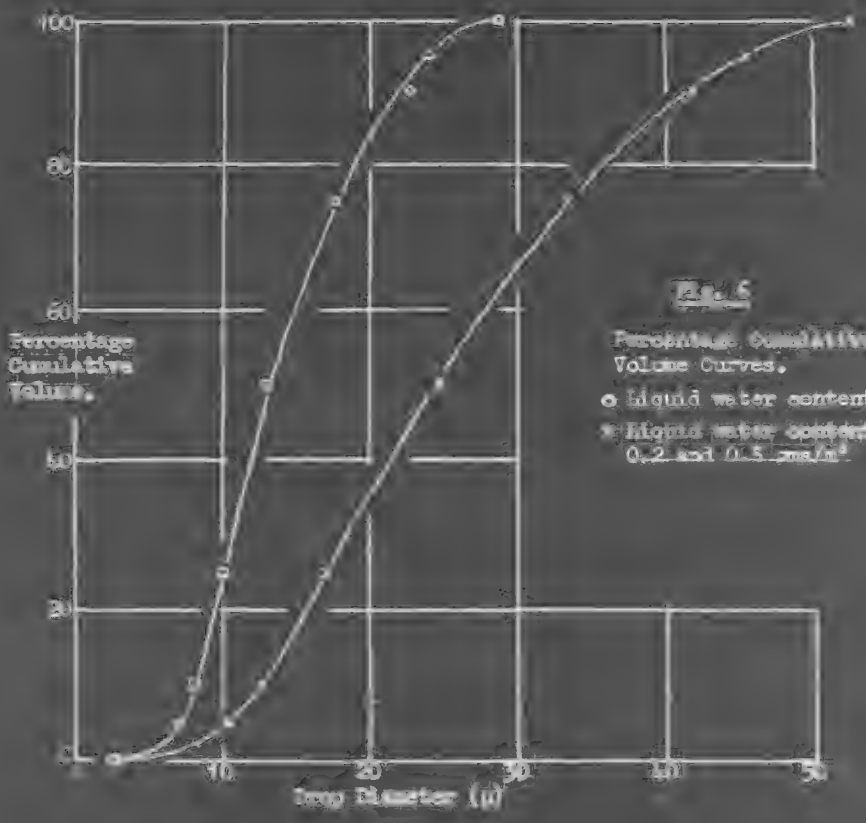


Fig. 6

Percentage Cumulative Volume Curves.

o Liquid water content ≤ 0.2 gms/m³

× Liquid water content between 0.2 and 0.5 gms/m³

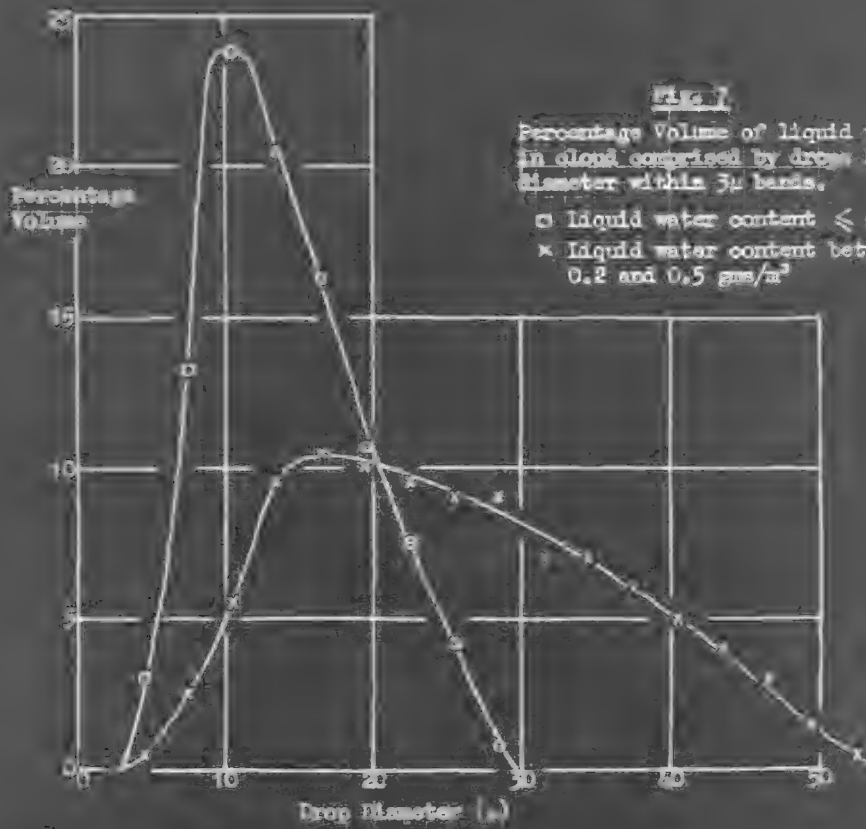


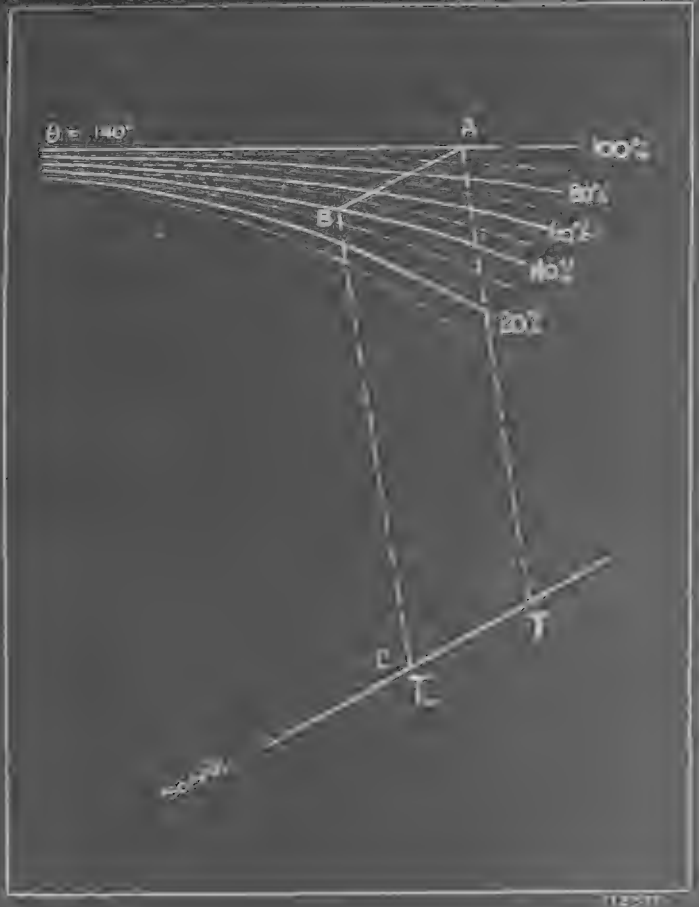
Fig. 7

Percentage Volume of liquid water in cloud comprised by drops having diameter within 3μ bands.

□ Liquid water content ≤ 0.2 gms/m³

× Liquid water content between 0.2 and 0.5 gms/m³

MAPS 318, 319, 320, 328, 329


A
100%
B
80%
60%
40%
20%
T
T_L

M.F.P. 318.

30th December, 1946.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

THE PRESENT POSITION OF WEATHER FORECASTING

IN

TROPICAL REGIONS

By

A.G. FORSDYKE

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

PRESENT POSITION OF WEATHER FORECASTING IN TROPICAL REGIONS.

CONTENTS.

The uses of weather forecasts in the tropics.

1. -
2. Aviation forecasts.
3. Forecasts for shipping.
4. Forecasts for agriculture.
5. Forecasts for the general public.

Tropical forecasting in general.

6. -
7. The reliability of the observations.
8. The frequency of reports.
9. -

The relation between the wind and the pressure distribution.

10. -
11. Practical investigations.

Theoretical considerations.

12. The equations of motion.
13. The geostrophic wind and gradient wind.
14. Stokes' solution.
15. Accelerated motion.
16. The effect of friction.
17. The flow of air across the isobars.
18. Convergence and divergence.

Upper air and convectional phenomena.

19. The co-ordination of upper air observations.
20. Convectional phenomena.
21. Synoptic aspects.

Frontal zones and discontinuities.

22. General.
23. Classification of fronts.
24. The intertropical trough & intertropical convergence zone.
25. Tropical waves & surges.
26. The polar front in the tropics.

Tropical revolving storms.

27. -

Conclusion.

28. Summary of problems for investigation.
29. Some suggestions regarding forecasting technique.

References.

30. -

THE PRESENT POSITION OF WEATHER FORECASTING IN TROPICAL REGIONS.

THE USES OF WEATHER FORECASTS IN THE TROPICS.

1. The tropical regions occupy nearly one half of the earth's surface and their economic importance, though not comparable with that of some areas of temperate latitudes, is very great, and is likely to increase with progressive development and

[illegible] not only support large populations, but also supply the

by important sea and air routes. In general, therefore, meteorology in the tropics is important in connection with agriculture and communications, though, in contrast to temperate regions, there is less emphasis on the latter owing to the lower density and frequency of traffic. Because of this there has been a tendency in the past for meteorological services in the tropics to incline towards the climatological rather than the synoptic aspects of meteorology. The war however, has led to considerable developments of the synoptic services to meet the needs of operational flying, and the necessity for these services, though now diminished, is likely to continue in view of the probable growth of civil aviation. Apart from agricultural and aviation requirements some use is also made of weather forecasts by shipping interests and the general public.

2. Aviation forecasts.

Though over mountainous areas and some coastal regions heavy cloud and rainfall occur frequently, it may be said broadly that weather conditions adverse for flying are rare in the tropics in comparison, say, with NW Europe. Both over sea and land the predominating conditions are fair or fine, with well broken cloud at a safe height, good visibility and light or moderate winds. The occasions on which the meteorological service can be the means of saving lives or aircraft by advising cancellation or diversion of flights are comparatively few. When adverse conditions do arise however, they are attended by special hazards. Diversionary landing grounds are limited in numbers and often poorly equipped, and for some trans-ocean flights, using isolated islands as stopping places, they are non-existent. Many routes involve long flights over imperfectly charted inhospitable country, lacking well defined landmarks, where forced landings result in hardship and suffering, and often danger to life. Over mountain areas it is extremely risky for a pilot flying above cloud to come down to check his position; with modern radio aids to navigation this should not in general be necessary, but in the tropics the worst weather is usually associated with violent thundery conditions which make radio communication difficult if not impossible; under these conditions accurate forecasts are invaluable.

The main hazards to aircraft in the tropics are thunderstorms with their dangerously turbulent clouds, the obscuring of hills and landing grounds by low cloud or fog, dust storms in arid areas, and, over the sea, occasional squalls and tropical revolving storms. As has been said above, these adverse conditions are comparatively infrequent and limited in extent, but when they do occur they are usually violent and intense and are all the more dangerous when they are encountered suddenly by pilots who are accustomed to flying in mainly fair weather.

/In

In East Africa, the tropical region known to the present writer, low cloud over and around the destination airfield arises principally from widespread afternoon thunderstorms, and makes landing hazardous at a particular airfield on perhaps five days a year. It is important that warnings of these conditions be sent out in good time to allow incoming flights to be cancelled or diverted. The alternative landing grounds are usually at considerable distances, say one to three hundred miles, and the bad conditions rarely affect more than one of them simultaneously; long distance aircraft usually carry enough petrol to reach them, so that the meteorological advice required is usually with a view to diversion rather than cancellation of these flights. At night the thunderstorms often spread out into large areas of light rain with overcast conditions, though except in hilly areas the cloud base is not, as a rule, dangerously low.

Early morning fog and low stratus occur in many tropical areas, and though they have not greatly affected flying in the past, except to cause delays in starting, with the development of night flying and dawn landings they come more into the picture. Hitherto in East Africa little success has been achieved in forecasting these conditions from the data available on the previous evening. As a rule they are present only for an hour or so following sunrise but in particular orographic situations, and perhaps also in association with frontal zones, they may occasionally, as on the high ground around Nairobi, persist until the afternoon. They are, however, more localised than the thick cumulonimbus associated with widespread afternoon thunderstorms, and an aircraft can, as a rule, climb above them.

Convective processes in the tropics are very intense and it is hazardous for an aircraft to fly into a cumulonimbus cloud at any time; by day these clouds can usually be avoided, but not always by night; therefore forecasts for night flying should include accurate information regarding them particularly when fog or stratus is likely to force aircraft to fly below them.

In certain areas tropical revolving storms give rise to very dangerous conditions to aircraft in flight, and even at great distances may cause swells which prohibit the take-off and landing of flying boats. Really severe cyclones are infrequent, though when they do occur their effects are catastrophic. Moderate cyclones are fairly frequent and give rise to stormy weather about as violent and widespread as that associated with deep winter-time depressions near the British Isles. The forecasting of tropical storms, is, therefore, of the greatest importance for aircraft.

Upper winds are usually light or moderate and are not liable to rapid variations either in space or time, so that they do not, as a rule, give rise to flying hazards. Improved forecasting of upper winds would, however, yield a useful return in that reserves of fuel now carried might be partly replaced by pay-load. Except for squalls which are usually short-lived, the surface winds at most airfields present no difficulties to pilots of aircraft, and in East Africa it has not been usual to include them in forecasts. They are important however at a few airfields, e.g. Nakuru, Kenya, where the topography gives rise to strong katabatic winds across the runways, for the hills which cause the katabatic flow do not permit aircraft to take off into the wind. One or two serious accidents to flying boats have occurred owing to landing down wind, but these were not in any way due to meteorological advice.

Seasonal and diurnal effects are relatively much more marked in the tropics than in temperate latitudes, not because they are necessarily greater in themselves, but because they are less masked by the irregular changes. It is therefore comparatively easy to forecast the general character of the weather for a long flight; it is on the other hand extremely

extremely difficult to forecast either the [illegible] weather or the detailed sequence of weather at a particular airfield, and for the latter, the forecaster at the airfield, equipped with local knowledge, has a great advantage over his colleague at a central office. Some airfields become unserviceable after heavy rain, a condition of which a local forecaster may be able to give effective warnings.

Synoptic charts supplied to pilots of long distance aircraft are not of great value in the tropics. Rarely can they indicate satisfactorily the movement of bad weather associated with zones of convergence, for there is reason to believe that this movement is often discontinuous. In low latitudes little significance attaches to isobars, and well defined pressure systems with their characteristic weather are rare. The synoptic chart accompanying a forecast is therefore little else than a pictorial presentation of actual weather reports, and, on account of the pronounced diurnal variation of weather a chart based on the latest reports prior to the flight may be misleading; a chart based on conditions at stations 24 hours previous to the time at which the aircraft will be in their vicinity would be more informative.

The writer has no experience of the forecasting of dust storms, but they appear, as a rule, to be associated with fronts, which can be detected on the synoptic chart. They may render landing of aircraft impossible for some hours when they affect an airfield.

3. Forecasts for shipping.

Generally in the tropics, forecasts are not so useful to shipping as in temperate latitudes on account of the infrequency of bad weather. Tropical revolving storms with their attendant gales or hurricane winds and high seas are the greatest danger, and it is one of the functions of the meteorological service to give timely warnings of these storms so that ships may avoid them. Squalls of limited extent and of short duration are fairly frequent near the Equatorial Convergence Zone and other "fronts", but in the present state of knowledge they cannot be forecast accurately; as a rule they do not constitute a danger to shipping being too transient and localised to raise high seas. Gales, apart from their association with tropical storms, are rare and fogs are almost entirely restricted to certain well known areas, the western cold water-coasts of the continents, where they are an important climatic feature.

4. Forecasts for agriculture.

In areas like East Africa where the rainfall is uncertain and the rate of evaporation very high, accurate forecasts covering a few days or a week are extremely valuable, both for guides to planting, since the success of a crop is to a large extent dependent upon adequate moisture soon after planting. Seasonal forecasts would be of even greater value; they would prevent the waste caused by planting seeds over large areas for which subsequent drought causes total crop failure. The forecast for 24 hours is not of great value for these operations.

Other weather elements such as temperature, frost, wind and sunshine have little effect on agriculture except insofar as they

/are

are features of the climate of a region and are dependent to a large extent upon altitude; synoptically the only element that really matters is rainfall.

Forecasts for the general public.

The interest of the general public lies mainly in the question as to whether or not it is likely to rain, for in warm climates no other meteorological element whose changes or incidence are irregular gives rise to such inconvenience or discomfort. Heavy rain may render road travelling difficult if not impossible owing to mud and washaways, and in the rainy season at least, it is advisable to take meteorological advice before starting on a long journey. In certain areas tropical storms may cause loss of life and great damage, and their occurrence is therefore always a matter of concern to the whole community. Local and regional winds [illegible] Mediterranean or West Africa, and the monsoon are important [illegible] modifying effects associated with their [illegible]

TROPICAL METEOROLOGY IN GENERAL.

It is sometimes stated that forecasting in the tropics requires a different technique from that employed in temperate latitudes, but this is only partly true. The same basic dynamical and thermodynamical laws apply to the atmosphere and the same influences are operative everywhere, though in differing degrees in different regions. Generally, the problems of forecasting are less amenable to mathematical or quasi-mathematical treatment in the tropics than in temperate latitudes; there is for example, no simple approximate relation between the wind and the pressure distribution, and many of the manifestations of weather are instability phenomena and hence incapable of precise prediction. In temperate latitudes forecasting is based largely on the association between isobaric forms and the types of weather which accompany them; in the tropics well defined pressure systems are usually absent, and the isobars as a rule have no significance in relation to weather. Barometric tendencies, after correction for the regular oscillation are often not much greater than the errors in the barometric readings themselves, especially when considerable corrections have to be applied on account of the reduction to a standard level, and their usefulness in forecasting is still uncertain, though they have been of assistance in detecting feeble troughs or wave-like disturbances moving over a group of stations near to sea level. In temperate latitudes convergence and divergence occur in association with certain features of the isobaric and isallobaric fields, and with fronts; the connection is so close that the convergence or divergence is tacitly assumed in the anemuriss situations. In the tropics on the other hand, convergence and divergence come into the picture as it were in their own right, and must be estimated from the observed wind fields. The synoptic chart must therefore be supplemented by upper wind charts. Finally, the absence of well defined pressure and frontal types with their more or less regular movement does not permit interpolation or extrapolation on the synoptic chart into areas from which observations are lacking, and in the tropics there are often very extensive,

/7.

(Plate 6 on following
Fig 1 — see Book)

The most noteworthy contributions to the subject so far made have been by the Institute of Tropical Meteorology of the University of Chicago and by the Tropical Research Centre of the Royal Australian Air Force Meteorological Service, but most of the tropical meteorologists have made some contribution. The American writers deal chiefly with fronts in the tropics in a general way; the Australian writers on the other hand have confined themselves in the main to the S.E. Trades and the regions to the westward of Australia, but they have tested their own conclusions critically and produced supporting evidence in the form of [illegible] on the synoptic charts. The outstanding piece of work on tropical meteorology is known as "Publication No. 251C, Weather in the Indian Ocean, Part III, Aids to Forecasting." Though some of its conclusions in [illegible] have been proved wrong - sufficient for any reader so inclined to attempt re-analysis of the charts with which it is illustrated.

THE RELATION BETWEEN THE WIND AND THE PRESSURE DISTRIBUTION.

10. The main difficulty in this problem arises from the weakness of the geostrophic force in low latitudes; the terms in the equations of motion which express the acceleration due to changing pressure distribution or curvature of the isobars become comparable in magnitude to the geostrophic term and must be taken account of in attempting to solve the equations. It is not to be expected therefore, that there will be ever that approximate relation between the wind and the instantaneous isobars which holds in middle and high latitudes and is expressed by the geostrophic wind equation. Observation shows, in confirmation of this, that near the equator the isobars afford no guide to the surface and upper winds as regards either their speed or direction. Instances of the wind blowing more or less directly across the isobars from low to high pressure are by no means infrequent.

11. Practical Investigations.

A necessary fist step is to determine from observations the extent to which the winds depart from geostrophic in low latitudes, and a comprehensive investigation into the laws of [illegible] pressure gradient and wind force (?) should be carried out for these regions. Preliminary investigations into the relation between the mean pressure gradient and the mean wind direction have been made by Jameson (3) for Ceylon, and more recently for an area in the South Atlantic (4). In the former it is difficult to allow for the effect of the high ground on the pressure distribution and the wind flow, but in the latter no such disturbing factor is involved. Jameson obtains the following mean values for the angle (γ) between the wind direction and the isobars.

Latitude (S)	5°	10°	15°	20°
γ	45°	41°	32°	31°

This investigation should be extended to include the relation between the barometric gradient and the wind force, and should be carried out for other tropical areas. The SW Pacific would appear to be a suitable region for this investigation, utilising the observations at the numerous island stations for which the pressures can be corrected to sea level without much error. Observations now available in this country for similar work are those from St. Brandon and neighbouring islands in the western Indian Ocean for parts of the years 1944-5. St. Brandon (16°S, 59°E) is a very small flat island with an excellent exposure for winds from all directions, and the gradient could be determined from the pressure there and at the other islands.

/4

A brief examination of the East African charts and of the charts in M.O. 451C suggests that in the Mauritius area (in the trade wind belt of the southern Indian Ocean), where the pressure distribution is almost symmetrical about the surface wind from the geostrophic direction is very little greater than in temperate latitudes. The speed of the surface wind speed to the geostrophic appears to be less than in temperate latitudes though not markedly so. These conclusions are in agreement with those obtained theoretically in para. 16 below. The comparison is not strictly [illegible] which are not restricted to cases of steady pressure distribution. Nearer the equator the investigation is more difficult, for the [illegible] appear to be [illegible]

Theoretical Investigations.

12. The equations of motion.

The winds associated with a given distribution of pressure are theoretically determinate by means of the general equations of motion which relate the forces acting on the air on the one hand to the accelerations on the other. If it is assumed that the motion is horizontal, the equations are

$$\frac{du}{dt} - [illegible] = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2}$$

$$\frac{dv}{dt} + 2\omega u \sin\phi = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu \frac{\partial^2 v}{\partial z^2}$$

x,y,z are the coordinates, the axes forming a right-handed system, x and y being horizontal and z vertical, positive upwards,
t is the time,
u,v,w are the components of velocity of the air,
ω is the angular velocity of the earth's rotation, $7.29.\ 10^{-5}$ sec^{-1}
ϕ is the latitude, the equations hold if ϕ is reckoned positive in the northern and negative in the southern hemisphere,
ρ is the density of the air,
ν is the kinematic viscosity of the air.

The terms on the right hand side express the forces acting on the air due to the pressure gradient and internal friction respectively. The [illegible] du/dt, dv/dt express the ordinary accelerations, d/dt denoting differentiation following the motion. The terms in $\omega \sin\phi$ express the fact that air moving freely (and therefore unaccelerated relative to axes fixed in space) is accelerated relative to the system of axes rotating with the earth to which the motion is referred. The apparent deviating force which arises from the acceleration is the Coriolis force, or [illegible].

13. The geostrophic wind and gradient wind.

It is not possible to obtain a general solution of the equations of motion as they stand, but in temperate and high latitudes an approximate solution, which is justified by its agreement with observation, is obtained [illegible], when

(a) that the motion is unaccelerated,
(b) that the pressure distribution does not vary with time and that the gradient is uniform, the isobars then consist of parallel straight lines, uniformly spaced,
(c) that friction can be neglected; the solution may be taken as applying to the atmosphere above the level at which the frictional drag of the earth's surface is effective,
(d) that the Coriolis force does not vary with latitude, or alternatively that the motion is confined to a small range of latitude over which $\sin\phi$ may be treated as constant.

The equations of motion may then be reduced and combined into the well known [illegible],

$$V = \frac{1}{2\omega \sin\phi}\frac{\partial p}{\partial n}$$

where V is the velocity of the air and $\partial p/\partial n$ the pressure gradient. The motion is parallel to the isobars and directed so as to keep the lower pressure to its left in the northern and to its right in the southern hemisphere.

The solution can be extended to the case of steady motion in a circular path, leading to the gradient wind equation,

$$\frac{1}{\rho}\frac{\partial p}{\partial n} = 2\omega V \sin\phi \pm \frac{V^2}{R}\cot\theta$$

where R is the radius of the path and the angular radius of the earth circle path. The upper sign applies to cyclonic and the lower to anticyclonic motion.

The immediate point of interest is now the solution in the tropics in low latitudes. The geostrophic wind equation becomes invalid, for the pressure gradients are small and often indeterminate, the geostrophic force is also small on account of the factor $\sin\phi$, and the accelerations can no longer be neglected in comparison with them. In steady motion the gradient wind equation may continue to be valid, the acceleration due to curvature of the path balancing that due to pressure gradient, and indeed such a balance is probably achieved in a tropical revolving storm, but in general the air does not move in the tropics in such regular fashion, and it has also been shown that the acceleration apart from that due to curvature, is still important. Both equations are complicated by the fact that in low latitudes the variation of the Coriolis force with latitude is most rapid. Any solution of the equations of motion which is to be valid in the tropics must, therefore, take account of the acceleration and should also allow for the variation of the Coriolis force with latitude.

14. Grimes' Solution.

A solution which is valid in the tropics was obtained by Grimes (5) and discussed and extended by Crossley (6). This solution is subject to certain restrictions, the most important of which are

(a) that the motion is [illegible],
(b) that the motion is independent of longitude, $\partial u/\partial x = \partial v/\partial x = 0$,
(c) that the horizontal motion is non-divergent $\partial u/\partial x + \partial v/\partial y = 0$.

[illegible] and y [illegible] the [illegible] and [illegible]

Particular solutions of these differential equations, which satisfy initial conditions specified values of the velocity and vorticity in a given latitude. These conditions however, determine not only the stream lines, but also the pressure distribution, whereas the practical problem is to determine the motion from the specified pressure distribution and the initial velocities. The isobar and stream-line patterns thus obtained do indeed conform to the [illegible] pressure and flow patterns of [illegible] regions of the [illegible] during [illegible], but the above restrictions limit the solutions to mean conditions. The restriction to steady motion is contrary to observations in many parts of the tropics. Crossley (loc. cit.) has shown, however, that on account of the small assumed variations of pressure the solutions apply only to mean motion and not to instantaneous motion. In his extensions to Grimes' work, Crossley has also removed restriction (b), but the solution thus obtained still determines both the motion and the pressure distribution.

The restriction to non-divergent motion implies that the vertical velocity w is zero, unless the assumption of constant density is only made to apply to the velocity and pressure fields determined from the solution, but either evades the important question of convergence and divergence, which should be determined from the horizontal flow. The vertical velocity is usually omitted on the ground that its inclusion renders the equations of motion intractable. It is thought, however, that it may be possible to remove the restriction $\partial u/\partial x + \partial v/\partial y = 0$ while still assuming that the terms involving w in the equations of motion are negligible, and Crossley (loc. cit., para. 8.3) in fact does so, towards justifying this assumption. If this is so, the problem becomes that of determining u and v from the given pressure distribution, and hence the corresponding non-horizontal divergence. The physical mechanism

that the pressure field and the inertia of the moving air determine a field of flow which may involve convergence or divergence, but that for simplicity of treatment the resulting vertical velocities may be neglected in the dynamical equations. It is with this in view that the investigations outlined below have been pursued.

15. Accelerated motion.

It is possible to integrate the equations of motion taking account of a time-variation of pressure in a few special cases provided that the following simplifying assumptions are made:

(a) that the variation of Coriolis force with latitude can be neglected,
(b) that the pressure field is delineated by straight isobars uniformly spaced, but the gradient and direction of the isobars may vary with time. At any instant the motion is everywhere the same so that the space derivatives in the acceleration terms do not appear in the equations of motion.

These assumptions impose limits on the validity of the solutions obtained, but they serve to indicate the nature of the motion in special cases and bring out some important differences between the motions appropriate to high and low latitudes. Assumption (b) means that the accelerations are given by $\frac{du}{dt}$ and $\frac{dv}{dt}$ only, and the equations of motion are then by reason of the partial to ordinary differential notation, the solutions may be taken as applying to the air as seen as it moves under the influence of the same pressure field, say to motion within a 5-degree square. Then with velocities of up to say, 30 m.p.h. the solutions could apply over a period of an hour. Higher degrees of approximation can, however, be found only by trial.

The equations of motion have been discussed in many papers, notably by Brunt and Douglas (7) and Sutcliffe (8), and some of the results given below are merely adaptions to specific pressure variations of the more general results given by these writers.

The simplified equations of motion have been used by a number of writers, notably by Gold (9) in relating the air movement to the diurnal variation of pressure, and by Sellick in connection with periodic variations of pressure, in an unpublished paper shown to the present writer in 1944.

Let the pressure field be represented by

$$p = p_0 - A(t)y$$

where p_0 is constant and A is a function of t only. The isobars are then straight lines parallel to the axis of x and uniformly spaced, but the gradient may vary with time.

Writing α for $2\omega \sin \phi$ and substituting for p we obtain the equations of motion in the form

$$\frac{du}{dt} = \alpha v , \qquad \frac{dv}{dt} = -\alpha u - \frac{A(t)}{\rho} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In which, since there is only one independent variable t, ordinary and not partial differentials have been written.

(a) General solution

The general solution is the sum of the Complementary Function

$$u = P \cos \alpha t + Q \sin \alpha t$$
$$v = -P \sin \alpha t + Q \cos \alpha t$$

in which P and Q are arbitrary constants, and of the Particular Integral,

$$u = F(t) , \qquad v = \frac{1}{\alpha} \frac{dF(t)}{dt}$$

where $F(t)$ is the particular integral of the differential equation

$$\frac{d^2f}{dt^2} + a^2 f = -\frac{A(t)a}{\rho} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The complete solution is therefore,

$$\begin{aligned} u &= P\cos at + Q\sin at + F(t) \\ v &= -P\sin at + Q\cos at + \frac{F'(t)}{a} \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

where $F'(t) = dF(t)/dt$.

The constants P,Q are determined from the initial conditions, say $u = u_o$, $v = v_o$ at $t = t_o$.

$$\begin{aligned} P\cos at_o + Q\sin at_o &= u_o - F(t_o) \\ -P\sin at_o + Q\cos at_o &= v_o - \frac{F'(t_o)}{a} \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$$\begin{aligned} P &= u_o - F(0) \\ Q &= v_o - \frac{F'(0)}{a} \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

(b) Constant pressure field.

$A(t)$ is constant and will be written simply as A.
$F(t) = -A/\rho a$ and $F'(t) = 0$.

Let u_g, v_g be the components of the geostrophic wind corresponding to the given pressure distribution.
Then

$$u_g = -\frac{A}{\rho a} = F(t), \qquad v_g = 0 = F'(t)$$

Further let $u' = u_o - u_g$ and $v' = v_o - v_g = v_o$ so that u', v' are the ageostrophic components of wind at $t = 0$.
We obtain from (5)

$$P = u' \qquad Q = -\frac{v'}{a}$$

and the complete solution is therefore [X]

$$\begin{aligned} u &= u'\cos at + v'\sin at + u_g \\ v &= -u'\sin at + v'\cos at \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Superposed on the geostrophic motion is an oscillatory motion in which the amplitude of the velocity variation is $\sqrt{(u'^2 + v'^2)}$ and the period is $2\pi/a$, i.e. one half-pendulum-day. The following table shows the approximate values of the period, in hours, for different latitudes.

Lat.	0°	5°	10°	15°	20°	25°	30°	60°
Period	∞	138	69	46	35	28	24	16

In temperate and high latitudes where the departures from the geostrophic wind are on the average small, it is to be expected that the oscillatory motion will also be small. Brunt and Douglas (7) omit the oscillatory terms from their analysis, stating that no such oscillations have ever been observed. The Meteorological Glossary, on the other hand, states in the article on "Isallobaric Wind" that the harmonic oscillation would be

X These equations have been given in a slightly different form by Hesselberg. Ref. No. 30.

[illegible] is doubt because it could be destroyed very quickly by frictional effects, but important discrepancies which have been found when attempts have been made to relate observed values of the pressure gradient and acceleration [illegible] into the boundary existence of the initial conditions.

The view is here put forward that under the conditions specified the catastrophic wind is completely expressed by the oscillatory terms which are therefore an essential part of the solution.

In the tropics the departures from [illegible] are known to be very large at times, and in estimating the wind from the pressure distribution the oscillatory terms must be taken into account. Owing to the large value of the period $2\pi/\alpha$ the periodicity of the motion corresponding to these terms will not at first be apparent, but simply diminishing components $u'\cos \alpha t$, $v'\cos \alpha t$. When sufficient time has elapsed to show the periodic nature of the motion the pressure distribution will have changed and the initial conditions will no longer hold.

It is suggested that the equations (5) [illegible] relating to winds to the pressure distribution in the tropics. It is assumed that over a limited area the pressure gradient is uniform and that it remains constant over a period of say, 6 to 12 hours; it is further assumed that the pressure gradient can be estimated from the synoptic charts. The [illegible] wind can therefore be evaluated.

Let the wind be measured at a particular instant, taken as $t = 0$. Then u' and v' can be obtained, and the numerical solution follows on substitution of the appropriate values in the equations (6)

It is to be noted that the calculated winds depend upon the pressure gradient which is assumed constant, and the measured wind at a particular instant. The wind at some previous instant may be used to calculate the present wind, or the measured present wind may be used to calculate the wind at any future instant within the period for which the solution is valid. The calculation would, however, be [illegible] as well as [illegible] conditions.

The assumptions made in applying this solution are similar to those made in the case of the catastrophic wind equation, the principal one being that the pressure gradient is constant and uniform.

A trial should now be carried out to see how the results obtained by [illegible] of this solution compare with observation.

(c) Solutions for particular [illegible] of $\lambda(t)$.

Exact solutions of the equations (1) can be obtained for any form of the function $\lambda(t)$ which permits the particular integral of the equation (2) to be found. Among the forms which yield solutions are the polynomial, exponential and circular functions. Solutions will therefore be possible for pressure variation which can be [illegible] provided that these variations [illegible] the [illegible] of the [illegible] of the motion.

(i) It is assumed that the pressure gradient changes smoothly in a finite time from the constant magnitude B to the constant magnitude C without change of direction. The variation is then simply expressed by

$$p = p_0 + \left\{ \frac{B + C}{2} + \frac{C - B}{2} \tanh \beta t \right\} y \qquad \ldots\ldots\ldots\ldots (7)$$

where β is constant. At $t = -\infty$ this expression is [illegible] $p = p_0 + B$, from which, as t increases, p [illegible] asymptotically, reaching the maximum rate of change at $t = 0$, and then increasing further it [illegible] asymptotically approaches the value $p = p_0 + C$. This variation is shown in Fig. 1, in which time varies along the horizontal and pressure gradient along the vertical axis.

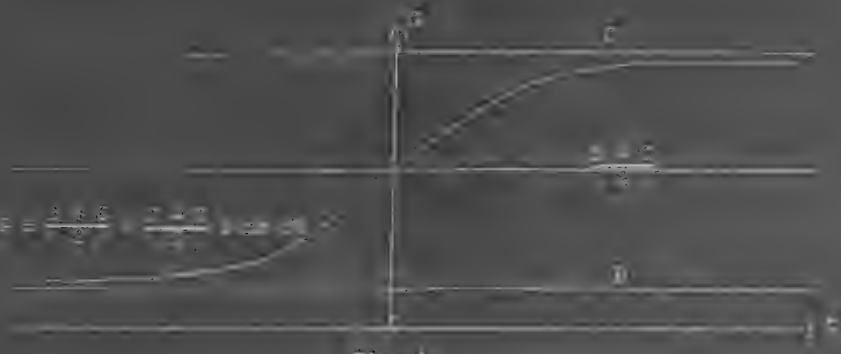


Fig. 1.

On account of the factor tanh βt the differential equations are not integrable, so that this example will not be considered any further.

(ii) A similar variation of pressure, for which the differential equations [illegible],

$$p = p_o + (B - \frac{B-C}{2} e^{\beta t})y \text{ for } -\infty \leqslant t \leqslant 0,$$
$$p = p_o + (C + \frac{B-C}{2} e^{-\beta t})y \text{ for } 0 \leqslant t \leqslant +\infty \quad \ldots\ldots\ldots\ldots(8)$$

The pressure is continuous at $t = 0$ and so also is $\partial p/\partial t$, but $\partial^2 p/\partial t^2$ and [illegible] discontinuous.

[illegible] $t = -\infty$ and [illegible]. The rate of change of pressure is determined by β, being rapid if β is large and gradual if β is small. The curve of pressure gradient against time is similar to that in Fig. 1.

[illegible] geostrophic, i.e.

$$u_o = -B/\rho a, \quad v_o = 0 \quad \text{at} \quad t = t_o = -\infty$$

For $t \leqslant 0$, applying (2) above, we obtain

$$F(t) = -\frac{B}{\rho a} + \frac{(B-C)a}{2\rho(a^2+\beta^2)} e^{\beta t}, \quad F'(t) = \frac{\beta(B-C)a}{2\rho(a^2+\beta^2)} e^{\beta t},$$

and therefore, $F(t_o) = -B/\rho a$, $F'(t_o) = 0$
Hence from (4),

$$P \cos at_o + Q \sin at_o = u_o - F(t_o) = 0$$
$$-P \sin at_o + Q \cos at_o = v_o - F'(t_o) = 0$$

so that $P = Q = 0$.

[illegible] (3),

$$= -\frac{B}{} + \frac{(B-C)a}{} e^{\beta t}, \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

$$v = \frac{\beta(B-C)}{2\rho(a^2+\beta^2)} e^{\beta t}$$

For $t \geqslant 0$ we obtain similarly,

$$F(t) = -\frac{C}{\rho a} - \frac{(B-C)a}{2\rho(a^2+\beta^2)} e^{-\beta t}, \quad F'(t) = \frac{\beta(B-C)a}{2\rho(a^2+\beta^2)} e^{-\beta t}.$$

The initial conditions are now obtained by putting $t = 0$ in (9), giving

$$u_o = -\frac{B}{\rho\alpha} + \frac{(B-C)\alpha}{2\rho(\alpha^2+\beta^2)}, \qquad v_o = \frac{\beta(B-C)}{2\rho(\alpha^2+\beta^2)}$$

$$T = \frac{(C-B)}{\rho} \frac{\beta^2}{\alpha(\alpha^2+\beta^2)} \quad (t = 0$$

The solution for $t \geq 0$ is therefore, from (3),

$$u = \frac{C-B}{\rho} \frac{\beta^2}{\alpha(\alpha^2+\beta^2)} \cos \alpha t - \frac{C}{\rho} - \frac{(B-C)\alpha}{2\rho(\alpha^2+\beta^2)} e^{-\beta t}$$

$$v = \frac{B-C}{\rho} \frac{\beta^2}{\alpha(\alpha^2+\beta^2)} \sin \alpha t + \frac{\beta(B-C)}{2\rho(\alpha^2+\beta^2)} e^{-\beta t} \qquad \ldots\ldots\ldots\ldots(10)$$

The solutions (8) and (10) exemplify certain features of the motion which are discussed from a more general point of view in (iii) and (iv) below.

(iii) The oscillatory motion. [X]

The amplitude is, from (10), proportional to the total change of pressure gradient. It also depends upon the relative values of α and β; if β is large in comparison with α the amplitude is large, and vice versa; near the equator, where α is small β must be very small if the oscillatory motion is to be negligible. It would be easy, though somewhat laborious, to calculate tables showing the amplitude for series of values of [illegible].

The sine and cosine terms do not appear in the solution for $t \leq 0$, suggesting that if the motion is initially geostrophic and the pressure variation is smooth the oscillations will be absent. It appears that the oscillations arise as a result of the discontinuity at $t = 0$.

Suppose that at $t \leq 0$ we have a non-oscillatory solution given by

$$u = \frac{F_1(t)}{\rho} \qquad \ldots\ldots\ldots\ldots(11)$$

where $F_1(t)$ denotes the appropriate form of $F(t)$ given by (2). For $t \geq 0$ let the appropriate form of $F(t)$ be $F_2(t)$ so that the solution is

$$u = P \cos \alpha t + Q \sin \alpha t + \frac{F_2(t)}{\rho}$$
$$v = -P \sin \alpha t + Q \cos \alpha t + \frac{F_2'(t)}{\rho\alpha} \qquad \ldots\ldots\ldots\ldots(12)$$

The initial conditions are obtained by putting $t = 0$ in (11), thus,

$$u_o = \frac{F_1(0)}{\rho}, \qquad v_o = \frac{F_1'(0)}{\rho\alpha}$$

[illegible], therefore,

$$P = \frac{F_1(0) - F_2(0)}{\rho}, \qquad Q = \frac{F_1'(0)}{\rho\alpha} - \frac{F_2'(0)}{\rho\alpha} \qquad \ldots\ldots\ldots\ldots(13)$$

At $t = 0$ there is a change in the function which expresses the variation of the pressure gradient with time. The equations (13) show that this change will not introduce oscillations provided that the Particular Integral $F(t)$ and its first derivative $dF(t)/dt$ are continuous at the change. This is a particular case of a general conclusion reached by Brunt and [illegible].

If the changes of pressure which occur in nature are smooth and represented by continuous functions, and there is no reason to suppo[illegible] that in general they are not, the oscillatory motion is not present. The

[illegible]

[illegible] of pressure may, as in example (i), involve unintegrable equations, but it may be presumed that the particular integral, though not calculable, is continuous. On this view therefore the oscillations arise only in the normal expressions for pressure because of the artificial discontinuity introduced into $\partial^2\zeta/\partial t^2$ in order to make the equations integrable.

On the other hand, the sudden change of pressure gradient across an isobar may perhaps be regarded as real discontinuities and may therefore [illegible]

(iv) The isallobaric motion.

[illegible] pressure distribution (8);

$$u_g = -\frac{B}{\rho\alpha} + \frac{B-C}{\rho\alpha}e^{\beta t}, \qquad v_g = 0$$

From (9) we obtain, writing $u' = u - u_g$, $v' = v - v_g$,

$$u' = \frac{C-B}{2\rho}\frac{\beta^2}{\alpha(\alpha^2+\beta^2)}e^{\beta t}, \qquad v' = \frac{\beta(B-C)}{2\rho(\alpha^2+\beta^2)}e^{\beta t} \quad \ldots\ldots\ldots(14)$$

[illegible]

$$u_g = -\frac{B}{\rho\alpha} - \frac{B-C}{2\rho\alpha}e^{-\beta t}, \qquad v_g = 0$$

Write

$$\bar{u} = u - \frac{C-B}{\rho}\frac{\beta^2}{\alpha(\alpha^2+\beta^2)}\cos\alpha t$$

$$\bar{v} = v - \frac{B-C}{\rho}\frac{\beta^2}{\alpha(\alpha^2+\beta^2)}\sin\alpha t$$

i.e. $\bar{u}$ and $\bar{v}$ are the component velocities given by (10) omitting the oscillatory terms. Then, if $u'' = \bar{u} - u_g$, $v'' = \bar{v} - v_g$ we obtain from (10)

$$u'' = \frac{B-C}{2\rho}\frac{\beta^2}{\alpha(\alpha^2+\beta^2)}e^{-\beta t}, \qquad v'' = \frac{\beta(B-C)}{2\rho(\alpha^2+\beta^2)}e^{-\beta t} \quad \ldots\ldots\ldots(15)$$

In (14) and (15) u'' and v'' are the components of velocity parallel and perpendicular respectively to the isobars, and also to the isallobars. They arise on account of the time-variation of pressure and hence may be called the components of the isallobaric type. It will be remarked that the conditions of the oscillation given by (10) [illegible] closely related to the components of geostrophic departure at the initial instant as in the case of the constant pressure field in (b).

From (14) and (15) we obtain $u'/v' = \beta/\alpha$, [illegible] latitudes, where α is small, the component along the isobars is very small in comparison with the cross component. This appears to be the case with [illegible] be considered that the changing pressure gradient gives the initial impulse to the cross-component of wind, and it is the action of the [illegible] force on this wind which produces the component along the isobars. The apparent anomaly arises from the fact that we are here considering relative to moving winds, but departures from [illegible] winds.

If in (i) we write $u = u - u_0$, $V = v - v_0$ where u_0 and v_0 are the components of wind at $t = t_0$ [illegible] and V are the components of total wind change from t_0 to t.

We then have

$$U = \frac{(B-C)\alpha}{2\rho(\alpha^2+\beta^2)}e^{-\beta t}, \qquad V = \frac{(B-C)\beta}{2\rho(\alpha^2+\beta^2)}e^{\beta t}$$

giving $U/V = \alpha/\beta$ and showing that the [illegible] component of the wind is the greater by would be expected. The isallobaric component of the wind because the velocity is zero [illegible] to the original pressure gradient.

(v) Periodic change of pressure gradient.

(vi) Variation of direction of the pressure gradient.

curved isobars.

6. The effect of friction.

(b) Taylor (10) using observations made by Dobson (11) has shown that

special cases:

(c) Extended friction.

and ... assembly and not the molecular viscosity, which is effective. The ratio of the two coefficients of viscosity varies with the degree of turbulence of the atmosphere; it is usually of the order 10^5 to 10^6.

... friction at the earth's surface also comes into play, exerting a drag which is always in the sense opposite to the wind and is proportional to the square of the velocity. The mathematical expression of this fact is the boundary condition which is employed in solving the equations of motion when the eddy-viscosity terms are included. The equations were first solved for steady motion by Ekman in 1905 and have been discussed by G.I. Taylor, Ekman and others. The following outline treatment follows that of Taylor (12) but with special consideration of the probable effects of friction in the trades in connexion with temperate latitudes.

The differential equations for horizontal motion under the influence of the pressure distribution $p = p_0 + Ay$, where A is constant, are

$$\frac{\partial u}{\partial t} = \omega v + \nu \frac{\partial^2 u}{\partial z^2}, \qquad \frac{\partial v}{\partial t} = -\omega u + \nu \frac{\partial^2 v}{\partial z^2} - \frac{A}{\rho} \quad \ldots\ldots(16)$$

... have been ignored.

The solution for steady motion $\partial u/\partial t = \partial v/\partial t = 0$, given by Taylor, is

$$v = e^{-pz}(V_0 \sin(\theta - pz) + V \sin pz)$$

where V is the speed of the geostrophic wind corresponding to the pressure gradient ... the surface ... the angle at which the surface wind cuts the isobars and p is equal to $\sqrt{(\omega/2\nu)}$.

By assuming that the frictional drag at the earth's surface acts in the direction exactly opposite to that of the wind, Taylor obtains

It follows that θ cannot, in steady motion, exceed 45°. The fact that values of θ in excess of ... observed in the trades may be taken as ... unsteady.

If it be further assumed that the drag is equal to $k\rho V_0^2$, where k is a ... coefficient ...

$$\frac{\sin\theta}{1 - \sin 2\theta} = \frac{kV}{\sqrt{2\omega\nu}} = \frac{f}{\sqrt{\omega}} \quad \text{(say)} \quad \ldots\ldots\ldots\ldots(17)$$

This enables us to investigate the variation of θ, and hence of V_0/V, with latitude, for given conditions ... the atmosphere.

For moderate ... over Salisbury Plain (lat. 51°), Dobson (11) ... found, ...

$$\theta = 20°, \qquad \frac{V}{V^0} = 0.6$$

From these figures, f in equation (17) can be evaluated, and using the value thus obtained we may ... between ... and ... Solutions graphically, we then obtain the following values of θ and V_0/V for various latitudes.

Latitude	...	...	...	...	51° (Salisbury Plain)
θ	...	...	...	...	...
$\frac{V}{V^0}$	0	0.40	0.46	0.52	0.55	0.60

These results indicate that under the assumption of steady motion the modification due to latitude of the frictional effect is fairly small, except quite near the ..., at the ... a good deal smaller

(d) Special solution for accelerated motion, with friction.

This solution is

$$u = \sum_{n=1}^{\infty} e^{-\nu q_n^2 t}(C_n \cos q_n z + D_n \sin q_n z) + \sin \alpha t \sum_{n=1}^{\infty} e^{-\nu q_n^2 t}(P_n \cos q_n z + Q_n \sin q_n z) \quad \ldots\ldots(18)$$

with a similar expression for v. The symbols have the same meanings as in (c) with, in addition, $q_n = 2n\pi/h$, n being a positive integer and h the height of the layer considered (conveniently taken as the lowest level at which in steady motion the wind direction is geostrophic).

By putting $t = \theta$ we have

$$u_o = 0 = V(1 - \sin\theta\, e^{-pz}(\cos(\theta - pz) + \sin(\theta - pz))) + \sum_{n=1}^{\infty}(C_n \cos q_n z + D_n \sin q_n z) \quad \ldots\ldots(19)$$

The term in V is the solution for steady motion. C_n and D_n can be

v the constants P_n, Q_n can be evaluated in a similar way, the expressions

Though equation (18) does not represent a motion which is likely to

oscillatory motion dies out in the layer affected by friction. It is

We have $q_1 = 2\pi/h$.

It has been shown by Taylor (10) that $h = (\theta + 3\pi/4)\sqrt{2\nu/\alpha}$. Now

Take $\nu = 5 \times 10^{3}$ value for Salisbury Plain, Shaw, Manual of

Lat.	5°	10°	20°	51° (Salisbury Plain)
t_1 (hours)	6.25	3.02	1.50	0.63
t_1 (hours)	20.8	10.0	5.0	2.1

initial value, (so that $e^{-\nu q_1^2 \tau} = 0.69$); similarly $t_{\frac{1}{10}}$ is the period in which the amplitude is reduced to one tenth of its initial value.

The table shows that in low [illegible]

[illegible] in a few hours.

These conclusions apply only from the surface up to the height h to

treatment is that h has been arbitrarily selected, which would appear

investigation required, and the further discussion of the numerical values arising from the equation (18) are not considered appropriate to this

---

From the foregoing it is clear that the tendency for the wind to blow across the isobars is greater in the tropics than in temperate

pressure, and the resulting transfer of air tends to diminish the pressure

difficulty, since the change of pressure at any level is due to the accumulation or loss of air integrated from that level to the upper limit of the atmosphere. It is insufficient to consider the flow across

because they have been obtained on the assumption that the motion is

is everywhere zero, and the pressure cannot change as a result of the

of the time, or in any forms which lead to integrable differential equations.

18. Convergence and divergence.

(The word "divergence" is, where the context permits, used in its

divergence.)

in the tropics. That they are of basic importance in the tropics, where they arise simply as characteristics of the field of flow, has already been indicated in para. 6. It is the belief of the writer that many if

convergence. There is however no obvious reason why convergence should be concentrated in narrow zones, and it is considered therefore that the

The problem is to calculate the divergence from the field of

field. It is sufficient for this purpose to assume incompressible; the equation of continuity is then

$$\frac{\partial w}{\partial z} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) .$$

which relates the vertical motion to the horizontal divergence. If further, it is assumed that u and v do not vary with height in a layer from $z = z_1$ to $z = z_2$, we obtain on integration through the layer

$$\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)$$

If the layer is next the surface so that $z_1 = 0$, $w_1 = 0$, we have

$$w_2 = - z_2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$

(b) Stream-line analysis.

A little progress towards the solution of the problem has been is shown by a short arrow and the speed is written in figures beside it, drew the lines so that, roughly, their distance apart is inversely proportional to the speed of the wind. When this is done some of the isopleths of wind direction (isogonals) and of speed (isovels), the little easier.

meeting at a pronounced angle, but much more difficult when their stream-lines.

stream-lines have other uses. The main one is the interpolation of stream-lines filling a role similar to that of isobars in the higher

(c) Mapping the field of divergence.

(1) Resolve the [illegible] and respectively, and plot the values of u and v on separate charts (Figs. 3 and 4). These are isopleths of u and v in the equator, [illegible] kilometres per hour and the isopleths are drawn at intervals of 10 knots.

(2) Calculate the values of the gradients of u and v at many points on [illegible] points on Figs. 3 and 4 where the prevailing air currents which may be estimated are the maximum of simplicity. At these points draw arrows perpendicular to the isopleths to show the gradient directions, and calculate the gradient numerically by measuring the intercepts of the isopleths on the arrows. In Figs. 3 and 4 the magnitude of the gradient in kilometres per hour per 100 [illegible] is given beside each gradient arrow.

(3) Calculate the components of the gradients of u and v in the E and N directions respectively, thereby obtaining the values of [illegible] beside the arrows in Figs. 3 and 4.

(4) On separate charts, Figs. 5 and 6, plot the values of $\partial u/\partial x$ and $\partial v/\partial y$ at the positions of the mid-points of the gradient arrows of Figs. 3 and 4, and draw the isopleths.

(5) By gridding the isopleths of Figs. 5 and 6, plot the values of $\partial u/\partial x + \partial v/\partial y$ and draw the isopleths, Fig. 7.

In resolving into components in (1) and (3) it is sufficient to take the direction to the nearest 10°. The work may be shortened by using a "components table" constructed for intervals of 1 unit in velocity or velocity gradient, and 10° in direction from 0° to 90°.

If the arrows in Figs. 3 and 4 are drawn for exactly the same points in the charts, the values of $\partial u/\partial x + \partial v/\partial y$ for these points could be obtained by simple addition. But it often happens that the most suitable points for measuring velocity [illegible] according to (2), the isopleths of $\partial u/\partial x$ or $\partial v/\partial y$ give a large number of points for [illegible] the field of $\partial u/\partial x + \partial v/\partial y$.

Fig. 8 shows the precipitation reported from about 1200 on the day for which Fig. 7 was constructed. The following are points of interest in relation to the [illegible] indicated by letters in Fig. 8.

A. Kenya and N. Tanganyika. There is a marked area of convergence on [illegible] and widespread precipitation [illegible] Fig. 8. This [illegible] have been [illegible] the convergence [illegible].

B. N. Kenya and Somaliland. A few scattered showers perhaps associated with the E.E. extension of [illegible] convergence [illegible] moist convergence [illegible] in the neighbourhood of B [illegible] in the [illegible] between the Gulf of [illegible] and [illegible]. The air was however, too dry to give much precipitation.

C. Eritrea and Red Sea. Moderate convergence but the air was too dry for much precipitation.

D. Palestine and Transjordan. Moderate convergence and fairly general precipitation around the [illegible].

E. Eastern Madagascar. [illegible] right. These were orographic showers over the high mountains [illegible] inland, [illegible] this area.

F. Southwestward of [illegible] associated with a cold front which moved into the area from the S.W. [illegible] precipitation occurred [illegible].

G. South Arabia. [illegible] of an [illegible] cyclone [illegible] the air was very

[illegible] that there were only [illegible] of precipitation.

Figs. 5 to 7 refer only to the winds at 2 km. and are therefore representative only of the layer from 1.5 to 2.5 km. For a complete picture of the field of divergence the same computation should be carried out for the winds at the other levels and the total divergence calculated by adding the results.

The process outlined above leads to an estimate of the divergence at the time of observation of the winds. It is a problem for investigation whether, using the methods of para. 15, the variation of the divergence with time could be estimated, using the observed winds and the pressure distribution.

(d) The units of divergence.

The units of divergence have been discussed by Kindle (14) from whose paper the following remarks are mainly taken. The dimensions of divergence are $M^0L^0T^{-1}$ in a system of units in which the unit of length is the same as for the velocity as for the velocity gradient. In the c.g.s. system the unit is the sec^{-1}, and the values of divergence ordinarily significant range from 10^{-8} to 10^{-4} sec^{-1}. For practical purposes a more convenient unit is the kilometre per hour per 100 kilometres; it is equal to $2.78.\ 10^{-3}$ c.g.s. units, and has been used in constructing Figs. 3 to 7. The following table, showing the ranges of values of the divergence, in c.g.s. units, and the type of weather with which they are usually associated, is taken from Kindle's paper. The equivalent values in km./h./100 km. are shown in the second column.

Divergence

c.g.s. units.	km./h./100 km.	Approximate weather.
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
$+10^{-8}$	$+3.6.\ 10^{-3}$	Very light subsidence.
$+10^{-6}$ to $+10^{-5}$	$+3.6\ 10^{-1}$ to $+3.6$	Moderate subsidence.

This table applies to the tropical regions, the third column being based on the average values of humidity in those regions.

According to this table the maximum values of the convergence in Fig. 7 correspond to something intermediate between light and heavy [illegible]

(e) Errors arising from inaccurate wind measurement.

Suppose that the wind measurement is subject to a maximum error δV, and that the velocity gradients are calculated from observations at stations uniformly spaced in a square pattern, the side of each square being d hundreds of kilometres. The maximum error in

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \quad \text{is then} \quad \frac{4\delta V}{d}.$$

Now δV may be taken as 5 km./h. Therefore the maximum error in the calculated divergence is

$$\frac{20}{d} \text{ km./h./100 km.}$$

With the existing distribution of pilot balloon stations d is

almost always 100 k.m. or more, and the maximum error in the divergence, [illegible] this compares with the maximum value of 8 k.m./h./100 k.m. found in the example in (c) above, from which it would appear that the proposed method for mapping the field of divergence is subject to too large errors to be of any use. Further consideration shows, however, that this conclusion is incorrect. As far as u only is concerned, the maximum error in [illegible] can only arise if the observed values of u have the maximum error at the two upwind corners of the square and the same error but of opposite sign at the two downward corners, with simultaneously a similar distribution of errors in v; and generally for a square of side d hundreds of kilometres and a 100 k.m. network of observing stations, the maximum error, $\frac{20}{d}$ k.m. /h./100 k.m., can only arise if, for u, the observed values at the $(d + 1)$ points on the upwind side of the square each have the maximum error while on the opposite side they have the same error but of opposite sign, with simultaneously a similar distribution of errors in v. But in fact the errors in u and v would be random ones to which a smoothing process would be applied in calculating the mean values along the respective pairs of opposite sides of the square, and this would in general reduce the error in the calculated divergence to a small fraction of its maximum possible value.

We conclude therefore that though the maximum possible errors in the calculated divergence are of the same order of magnitude as the maximum values found for the divergence in the example of (c) above, the distribution of divergence found in the example does in its main features at least approximate to the true distribution owing to the fact that in the smoothing process inherent in the drawing of the isopleths of u and v observational errors are largely eliminated. But whether the method is a useful one or not can only be decided by [illegible]

UPPER AIR AND CONVECTIONAL PHENOMENA

19. The coordination of upper air observations.

Upper air temperature observing stations are at present so widely [illegible] as India and the southern part of the U.S.A. it is not possible to construct upper air charts and atmospheric cross section diagrams as in temperate latitudes. Though the observations are of great value individually they cannot yet, in general, be coordinated. It is unfortunate that, since the relation between the wind and the pressure distribution in the tropics is not a simple one, the observations of upper winds afford little assistance in constructing thickness lines and contours of the isobaric surfaces, which have therefore to be based almost entirely upon the heights and thicknesses computed from the available U.A.T. reports. A large increase in the number of radiosonde stations in the tropics is therefore necessary, and if real understanding of the atmospheric circulation is to be obtained the network must be at least as dense as that of temperate latitudes. If, as seems likely, it is not possible to establish a network of this general density, it would probably be better, at least in the initial period, to concentrate the available radiosondes in fairly close groupings over limited areas rather than to spread them thinly over the whole of the tropical regions, in order to see whether there are significant variations of upper air conditions over short distances, which the more widely spaced stations would fail to detect. An outstanding problem for investigation is that of the circulation above the subtropical anticyclones and the equatorial low pressure belt, which are maintained by high level convergence and divergence respectively.

In India, where there is a fairly close network of U.A.T. observing stations, some work has been done on the construction of charts of upper air contours. In a recent unpublished paper by Ratcliffe, it has been concluded that there is a tendency, as might be expected, for bad weather to be associated with "cold pools" between the 1000 and 500 mb. levels; but the problem of forecasting the development and movement of the cold pools has still to be investigated. In temperate latitudes use is made of the concept of the geostrophic transport of thickness lines (15) in this connection, [illegible] latitudes where geostrophic conditions very rarely obtain. It might be possible however, to estimate the movement of the cold pools, using streamline charts if necessary, in conjunction with the equations (6) of para.15. Some unpublished work by Schove in West Africa shows that "cold troughs" moving from E to W between Nigeria and Sierra Leone are fairly common during the winter months when constant NE winds prevail, indicating that under steady conditions the cold pools may be expected to move with the [illegible].

The effect of high ground on the contours of the upper air pressure surfaces has an important practical application in connection with altimeter corrections for aircraft. It has been examined, very roughly, in East Africa by working out the mean heights of the 500 mb. surface for three stations, Mombasa, at sea level, Nairobi, altitude 5,400 feet,

almost in a straight line, of total length about 550 miles, with Nairobi [illegible] period examined, it was found that the average height above sea level of the 500 mb. surface was, at Nairobi about 600 feet, and at Soroti about 500 feet above that at Mombasa, suggesting that the upper level pressure surfaces [illegible] fall with the ground surface. This is because over the high ground the insolational heating is applied at a considerable height above sea level, and the lowest few thousand feet of air are, during the day, a good deal warmer than the air at the same level over the sea or plains. From this it appears that high ground may have a marked influence on the air flow not only at its own level, but for a considerable height above. The high upper level pressure over the East African plateau, together with the effects of the plateau itself are probably the cause of the rapid change in direction of the SE monsoon along the East African coast, which is frequently S at Mombasa (4°S) and SW at Mogadishu (2°N).

20. [illegible]

Most of the rainfall of the tropics is convectional in origin, or, when primarily due to other causes, it is increased owing to convectional effects. Individually, particularly at inland stations, the tephigram is the greatest single aid to local forecasting; in East Africa it is of greater assistance than the synoptic chart, especially when forecasting the day's weather from [illegible] greater than the saturated adiabatic, with high humidity, at least up to the freezing level, is required for the outbreak of general precipitation, though light precipitation is often observed when the cloud tops do not reach the freezing level. On many occasions when the lapse-rate is quite unstable the weather remains fair because there is insufficient moisture for precipitation; the importance of accurate values of humidity in the upper air cannot [illegible] amounts of water vapour condensed for a given degree of cooling at the high temperatures prevailing, convectional clouds are very dense and the precipitation from them heavy in comparison with temperate latitudes; on the other hand, many cumulus clouds which would give precipitation in temperate latitudes do not do so in the tropics because they do not develop to the high altitude of the freezing point.

In forecasting convection the usual criteria for instability are applied to the tephigram, but it has been found better to use the mean water vapour content in the lowest 100 mbs. than to use merely the water vapour content of the surface air; i.e., it should be assumed that the ascending air parcel is a thoroughly mixed sample, representative of a thick layer of air. It has hitherto been assumed that the effect of thorough mixing is to bring about a uniform distribution of potential temperature, and for this reason [illegible] levels has been used in conjunction with a curve of mean water content for the same layers, to find the condensation level for

/convection

turbulence cloud. In view however of a recent paper by Priestley and Swinbank (16), which shows that in addition to ordinary mechanical turbulence, which transfers heat in the direction of decreasing potential temperature (normally downwards) there is an effect due to convective turbulence, which arises from buoyancy effects due to small-scale horizontal variations of temperature, and which may transfer heat upwards; the idea of turbulent mixing applied in this way to the tephigram requires further investigation.

On many occasions in East Africa the ascent curves gave lapse-rates slightly in excess of saturated adiabatic, and there was enough moisture present to lead to convection if the ordinary parcel method criterion held good, but precipitation did not usually follow. This was attributed to the stabilising effect of the descending environment, as discussed by Bjerknes (17). It was found possible to allow for this, very roughly by assuming that the whole of the environment was lowered by an amount corresponding to an increase of pressure of 30 mbs. A new temperature curve was constructed by drawing, from each of a number of points on the original, a dry adiabatic to the pressure line higher by 30 mbs. The new environment curve drawn through the points thus obtained, gave a better estimate of convection that the original curve. It must however be emphasized that the range of 30 mbs. was purely empirical.

If the lapse-rate is markedly unstable, and the air moist, convection follows the normal diurnal sequence, reaching its peak in the late afternoon, and the resulting precipitation is often widespread, the individual storms merging to form a continuous rain area. With slight to moderate instability the afternoon convection is often restricted to the hills, and over the plains and plateau areas there is then little cloud, presumably owing to the compensating subsidence. During the night, however, the rain frequently extends to the plains also. It is thought that the precipitation over the hills cools the lower layers of air there and a katabatic outflow is set up, which, reaching the plains, undercuts and starts convection in the air which has been made unstable by the full day's insolation. Grimes (18), has mentioned similar effects in India and Burma, and has also remarked that in the Ganges Valley the outflowing cold air from below a thunderstorm causes the storm to be regenerated in the air surrounding it, giving rise to an appearance of continuous movement of the storm. Precipitation at night in convective situations may also be increased owing to the instability arising from radiation from the cloud tops, and also by the delayed realisation of the latent instability brought about by the insolation of the preceding day in the manner described by Dexter (19); further investigation is required into these processes. Another feature of interest is that in a slightly stable situation, the cumulus cloud which forms during the forenoon decreases in the afternoon as warmer and drier air from near the surface takes part in the convective process. Finally there is the question of forecasting the low cloud or fog at dawn from the tephigram of the previous evening; no success has yet been achieved in this problem since it is not known how the temperature and humidity in the lowest layers vary during the night. The investigations into turbulent and radiative cooling at night, at present being carried out in this country, should throw much light on this problem.

/20.

21. Synoptic aspects.

The tropical regions are characterised by the absence of well defined changes of air mass type and therefore the markedly characteristic tephigrams which are associated in temperate latitudes with the various air masses are not much in evidence. Instead, the tephigrams tend, on the whole, to be characteristic of the locality and season, and for the most part the day to day fluctuations are small, though, as will be mentioned below, they are important in certain localities. Experience in East Africa and the western Indian Ocean indicates that there are four main types of tephigram, to which brief consideration will now be given.

(i) Over the sea, in the trade wind belts, the tephigram usually shows an inversion or isothermal layer, outside which the lapse-rate is approximately saturated adiabatic; the air is moist below the inversion, but dry above. The height of the inversion, or, more significantly, the depth of the moist layer, increases from about 4,000 feet in latitude 20° - 25°S to 10,000 or 15,000 feet in latitude 10° - 15°S, and nearer the equator the inversion is usually not found at all. With a shallow moist layer only small cumuli are formed, from condensation tops of trade cumuli and cumulonimbi where the moist air is deep, hence the depth of the moist layer is a useful guide in forecasting. At Mauritius (20°S) and in S. Madagascar (25°S) the inversion or isothermal layer sometimes disappears completely with a surge of cold air from S or SW, a considerable fall of temperature occurring in the upper levels without much change at the surface, suggesting the northeastward movement of an upper cold front (para.25b). In lower latitudes very little change of upper air temperature has been observed in association with these surges.

(ii) Over the land in the outer part of the tropical regions the tephigrams are characterised by extreme dryness. On hot days the lapse-rate becomes dry adiabatic up to 12,000 or 15,000 feet, and though violent bumpiness and sometimes duststorms accompany these conditions, precipitation is rare.

(iii) Over the sea in the equatorial low pressure belt the tephigrams show a lapse-rate about equal to or slightly greater than saturated adiabatic, and moist air at all levels, which conditions persist almost unchanged from day to day. Wave-like perturbations moving from an easterly direction (para. 25a) in this area appear to have little or no effect on the ascent curves, which are therefore of little assistance in detecting these disturbances.

(iv) The Nairobi ascents have provided a good series of data for an inland station almost on the equator, but they exhibit some peculiar features and it is not known to what extent they are typical of equatorial stations in other parts of the world. Though in general the day to day changes of upper air temperature are small, there are sometimes large and rapid changes at the high and medium levels, a moist unstable distribution becoming transformed into a dry stable one or vice versa within 24 hours. So far it has not been possible to forecast these changes. It is difficult to see

/how

how they could be brought about by advection; nevertheless, this should be investigated by endeavouring to trace back the air at each level to its source region, using stream-line charts constructed at as frequent intervals as the observations permit. A more likely explanation however, is that the changes are brought about by the vertical motion resulting from horizontal convergence or divergence. A widespread rainfall is invariably accompanied by unstable moist ascent curves. It is important therefore, that the effect of these on the lapse-rate and humidity in the upper air be investigated. To this end the method of calculating the divergence discussed in para.18 might be used; and Austin (20) has shown how to calculate the stretching or contraction of an air column from the divergence; the associated precipitation can then be assessed if the original humidity distribution in the upper air is known.

FRONTAL ZONES AND DISCONTINUITIES.

22. General.

Though frontal ideas have become well established in relation to the weather of temperate latitudes and form the basis of analysis and forecasting in those regions, their extension to the tropics is still only tentative. Theoretical considerations supported by climatological observations have in the past led to the generally accepted view that, broadly speaking, the air of the tropics consists of a vast, almost homogeneous, warm, moist mass, within which the only factors operative for the production of cloud and rain are radiation and orographic and convectional effects, modified by the diurnal variation of insolation. The area between latitudes 30°N and S comprises one half of the earth's surface, and the greater part of it consists of ocean, the variations of whose surface temperature are, in the main, slight and gradual. Over this extensive, almost uniform surface the air movement is comparatively slow, often very slow, and not only is it usually impossible for appreciable discontinuities of air temperature to be set up in the region, but those from outside tend to be quickly smoothed out. The whole air mass acquires a high temperature, and, for the most part, in the lower layers at least, a high moisture content. It would appear therefore that the copious rainfall and heavy cloud formation which ensue when this air is subject to convection resulting from its movement over heated land areas, to orographic uplift when it meets the numerous and substantial mountain barriers and to the uplift due to convergence between the trade winds of the N and S hemispheres, could be readily explained in terms of air mass properties without having recourse to frontal ideas.

It is, however, a matter of observation that the weather of many parts of the tropics does not always follow a regular diurnal sequence, nor does it have a fixed distribution from day to day, as it would if it were the result of the action of the above processes on a homogeneous air mass. One day is rarely the exact counterpart of its predecessor, though there is on the whole far less variation than in higher latitudes. For example, in the Kenya Highlands during the SW monsoon, thick low cloud with rain or drizzle, commencing during the night, may on some occasions persist until midday or later; on others they may disappear in the early part of the forenoon, while on many days during this season the weather remains continuously fair. In India the tendency of the SW monsoon to occur in "bursts" of increased wind associated with much cloud and heavy rainfall, followed by quiet relatively fair periods has been described by many writers.

/The

The present writer's experience in flights over the western Indian Ocean has shown the existence of belts of disturbed weather which, there is good reason to believe, change their position and intensity from day to day. Reports from other regions also support the view that superposed on the general constancy of tropical weather there is a variable element, ill-defined but none the less real. It is natural to attempt to relate this changeability of weather to horizontal variations of characteristics within the air mass and in particular to consider whether these variations can ever become so sharp as to constitute narrow transition zones or possibl fronts.

Fronts may be expected to occur within the tropics either as a result of their intrusion from higher latitudes or of fronto-genetical processes within the tropical air mass. In the first of these processes the polar front is swept into the tropics while an intense anticyclone builds up in the cold air behind it. To begin with, the front exhibits a marked discontinuity of temperature, and though on moving into the tropics it is subject to frontolysis on account of surface influences and subsidence of the cold air, it should be possible, given a sufficiently close network of observing stations, to achieve continuity in tracing it from chart to chart. On account of the masking effect of diurnal variation and other influences the surface observations are in many cases non-representative; the effect on pressure tendencies has already been mentioned; at coast stations the general wind flow is much affected by pronounced sea breezes; and daytime temperatures depend almost entirely upon the cloudiness. In the second process there is little or no difference of temperature and humidity between the front-forming air masses, but the weather is caused by the uplift resulting from convergence between them; the front may be regarded as dynamical as opposed to thermal. The formation and persistence of a front of this type depends upon the field of flow; the zone of convergence is a feature simply of the flow pattern and it has no permanent existence in the sense of being associated with particular particles of air or with the boundary between two distinguishable air masses. Examples of convergence zones are the intertropical trough between the trade winds of the N and S hemispheres, and the surges of the trades. It may happen that the thermal and dynamical causes of fronts are simultaneously present; for example the equatorward movement of an old polar front is often associated with a surge in the general current; it is to be expected that under these circumstances the front will be more active than normally.

A third type of zonal disturbance is the wave-like perturbation or trough, lying more or less at right angles to the general air stream and appearing as a line of small sinuosities in the isobars. Little is known of the causes or origin of such troughs, but they appear to be similar to the minor perturbations which travel along [illegible] of temperate latitudes.

In recent years, as a result of the increased attention given to tropical meteorology during the war and the improved synoptic reporting networks, fronts in low latitudes have been the subject of study of most of the tropical meteorological services. Though differing on points of detail most of the papers which have been written are in agreement on general principles. The present state of knowledge is best summarised in a paper published by the University of Chicago, Institute of Tropical Meteorology (21).

23. Classification of fronts.

Early in 1945 an American committee deliberated on the standardisation of tropical nomenclature and received the benefit of J.Bjerknes' participation. This Committee recommended the following definitions. Comments on them have been inserted in [illegible].

I. Semi-permanent features.

Intertropical Trough. The trough of low pressure found in the vicinity of the equator between the subtropical high of the N and S hemispheres. It is orientated approximately E-W and is usually found within 200 miles of the equator. The trades of both hemispheres flow without marked convergence.

Intertropical Convergence Zone. The intertropical trough is called the intertropical convergence zone where the trades of the two hemispheres meet at a pronounced angle. The zone is 50 - 200 miles wide and usually 5° or more from the equator. The air on the equatorial side is trade wind air from the opposite hemisphere. If the convergence zone is 5° or more from the equator this wind turns to westerly as it approaches [illegible]

Monsoon Convergence Zone. A narrow zone, usually within 200 miles of the equator, extending E-W, where the monsoon changes rapidly from an easterly to a westerly component.
(The use of the term "trough" is not without objection. In extra tropical latitudes a trough of low pressure is a term associated with bad weather and usually with activity, indeed convergence is frequently associated with a trough of low pressure, and it is generally comparatively narrow. A preferable term for the "Intertropical Trough" would be Doldrums or Equatorial Low. The Monsoon Convergence Zone requires evidence of reality before it is accepted).

II. Transverse Waves and Surges.

Waves in the Easterlies. These are found in regions where the easterlies are at least 15,000 to 20,000 feet deep. They are non-frontal wavelike perturbations of this current, with trough and crest lines orientated about normal to the current; and they move with the current. In some cases, there is a shallow wedge of Westerlies under the deep easterlies.

Waves in Circumpolar Westerlies. The upper level waves in the circumpolar westerlies intrude into the tropics if the westerlies themselves have extended into low latitudes. The westerlies overlap the outer parts of the trade wind zone, with the base of the westerlies sloping up towards the equator. Waves in the upper westerlies produce eastward moving perturbations in the underlying easterlies.

Waves in Equatorial Westerlies. [illegible] Equatorial Westerlies is found especially in the East Indies and the Indian Ocean, extending up to at least 15,000 or 20,000 feet, but not always based on the surface. Non-frontal wavelike perturbations occur in the Westerlies and move eastwards.

/Monsoon

Monsoon Surges. Surges in the monsoon currents (primarily in the Indian Ocean and W. Pacific) are convergence zones, and travel in the direction of the monsoon current from one hemisphere into the other.

III. Shear-Lines and Surges.

Frontal Shear Lines or Surge-Lines. The remnants of cold fronts that have intruded into the tropics are referred to as frontal shear-lines or surge-lines of the trade. The density-discontinuity across the old front has largely disappeared but a wind-shear between the trade wind air and the air of more recent polar origin remains. The frontal shear-lines do not extend high into the atmosphere. The surge-lines do not have an anticyclonic circulation above them and extend into the high atmosphere. Thus, shear-lines and surge-lines merely represent two different transformations of the cold front.

Other Shear-Lines or Surge-Lines. Shear-lines or surge-lines as defined above, but whose origin cannot be traced directly to the polar front.

24. The intertropical trough and intertropical convergence zone are essentially the same. By some writers they are known as the equatorial trough and equatorial front respectively. They lie along the zone of low pressure separating the subtropical high pressure belts. Into this zone, which is usually very wide and ill-defined, the two trade streams flow. Cloud and precipitation will develop if either (a) the two air streams differ appreciably in temperature in which case a true front will form with the warmer air overrunning the colder, or (b) there is true horizontal convergence between the air streams.

REFER TO FIGURE 9 AT END.

It is often difficult to judge from the synoptic chart whether true convergence is actually taking place in this situation. Fig.9 illustrates the apparently most favourable case, in which anti-cyclonic cells of the N and S hemispheres are situated opposite one another. The winds are indicated by the arrows. A cursory examination would lead to the conclusion that at A, near the eastern ends of the high pressure cells the air streams are converging, at B they are flowing side by side without convergence and at C they are diverging. If however, the wind between A and B is sufficiently strong, air is removed from the vicinity of A at the same rate at which it arrives and no convergence takes place at A. Similarly if near C the westward velocity is less than that at B, there may actually be convergence in the vicinity of C. Whether the air flow

/is

is convergent or divergent in any locality could only be judged from an accurate chart of stream lines, or calculated from a close network of upper wind observations.

The equatorial belt of low pressure is often ill-defined and the few isobars that appear at all do have little systematic arrangement and change their pattern from chart to chart. Within this belt the air movement is often very sluggish over a wide area only on the boundaries of which are the true trades found. Such an area is called an Area of Doldrums and on the charts it appears as a loop formed by bifurcation of the intertropical trough. Grimes (22) states that with a Doldrum area the main frontal activity occurs where the trades meet the stagnant air, and he distinguishes between the cases in which the trade overruns and undercuts the Doldrum air; the former has little effect on the weather but the latter is associated with disturbed conditions.

The variation of pressure along the trough is an important factor affecting convergence. If in Fig.9. the pressure is higher at C than at B, air moving from B to C will be retarded and convergence will take place. If pressure is relatively high to the eastward of A this effect will be increased. Walter has stated that there is reason to believe that the inter-monsoonal rainfall of British East Africa tends to be heavier when pressure is relatively high in the intertropical trough over West Africa and South India.

When the axis of the trough recedes from the equator into the N. or S. hemispheres, the trade wind from the opposite hemisphere crosses the equator and subsequently, owing to the reversal of the Coriolis force, it acquires a westerly component which tends to increase with increasing latitude. The best known example is the SW monsoon of the Indian Ocean, when the intertropical trough is displaced far to the north on account of the deep summer low which develops over N. India. Other examples are in the NW monsoons of N. Australia and N. Madagascar, and the SW monsoon of W. Africa. At the trough itself it would appear that there is a marked tendency for convergence between the trade to show that bad weather does occur frequently in this situation. Grimes (loc. cit.) states that in the main [illegible] a line, rarely a surface of discontinuity is formed between the opposing currents. He states that in the summer of the N. hemisphere the SW monsoon advances, undercutting the air of the NE trades and therefore forming a cold front. It advances northwards in a series of surges which alternate with periods of southward recession, when it has the nature of a warm front. In East Africa and the Sudan the north-easterlies are usually found to overlie the south-easterlies. It would be expected that, in general, the air from the other hemisphere would, in virtue of its longer journey in low latitudes, behave as the warmer mass and therefore over-run the other, but this is not usually [illegible] Grimes and Sverdrup, the slope is [illegible] for the lowest 5,000 ft., above which the surface lies nearly horizontally. In East and West Africa and the Netherlands East Indies however, a more usual value of the slope is 1 in 200 or 1 in 300, and it persists often up to 6 kms.

The pressure distribution is of little or no assistance in [illegible] of the intertropical convergence zone, the stream line chart is the best aid, the frontal zone usually being located as the line of separation between winds having northerly and southerly components respectively. Over wide areas however, the

/surface

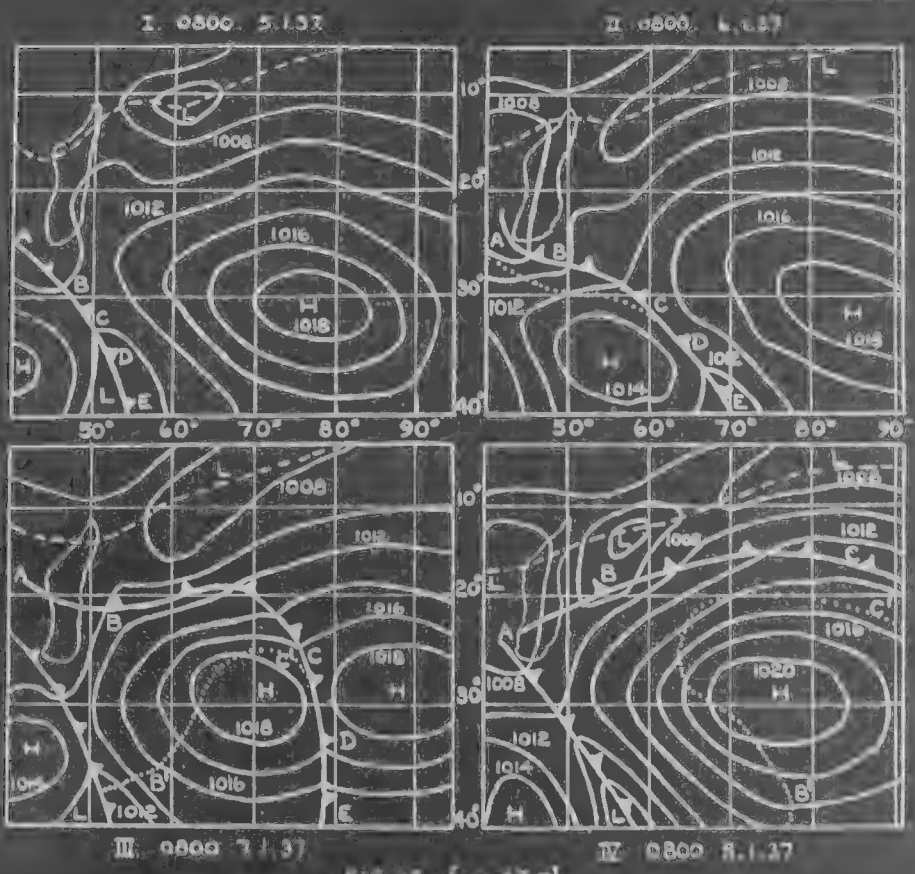


FIG. 12 (---L.T.F.)

Alignment of the Polar Front in the S.W. Indian Ocean

H33-42
no book

7. The reliability of the observations.

The work of many of the native observers in tropical countries is surprisingly good, especially the pilot balloon observing, but naturally it does not approach the high standard demanded at home. In particular, the observations of cloud type and height often seem very uncertain and the forecasters can have little confidence in these. It is evident that some observers fail to keep that close watch on the weather between synoptic hours which skilled and efficient observing demands. Instrumental observations are on the whole quite good, better than those for which the exercise of judgment is necessary. But it does appear that, given the necessary initial training supplemented by fairly frequent visits to stations by meteorological officers, the native observer can attain a highly satisfactory standard. In the tropics, as elsewhere, the observations must be of the highest accuracy obtainable.

8. The frequency of reports.

In some tropical areas forecasting has in the past been severely handicapped owing to the synoptic reports not being made at frequent enough intervals. In the British Isles reports are not required hourly. Partly, on account of the more regular diurnal variations, reports of this frequency are not necessary in the tropics, the absence of definite systems and progressions makes extrapolation much more difficult and it is considered that the interval between synoptic reports should not be greater than 3 hours. The pronounced diurnal variation of weather may itself be adduced as an argument for a high frequency, for reports made only twice daily may consistently miss important features of the weather and give no indication of the variations between the widely separated synoptic hours. For example, observations made daily at 0900 and 1500 (Local Time) only, miss the early morning fog or low cloud which most regularly breaks or thins at about sunrise but disperses before the time of the morning observation. Climatological summaries based on these reports are very misleading.

9. The main problems of tropical forecasting have been outlined by Hare (1); they are as follows:-

(a) The relation between the wind and the pressure distribution, including the estimation of convergence and divergence.

(b) Moist air and convectional instability.

(c) Frontal zones and discontinuities.

(d) Tropical revolving storms.

(e) Forecasting for agriculture.

The number of papers, published and unpublished, on tropical meteorology is very great and it is clear from a perusal of many of them that the general problem has been attacked piecemeal

/The

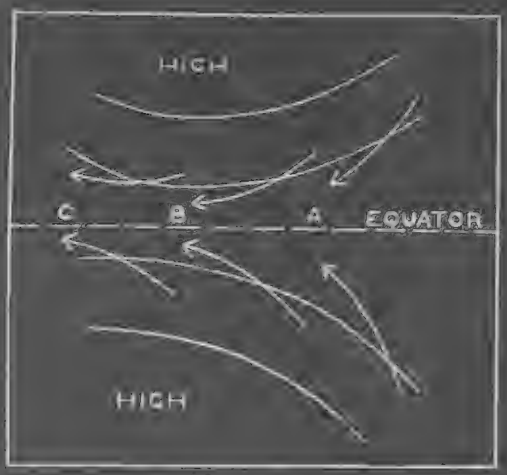


FIG. 9

Inter-tropical Convergence Zone

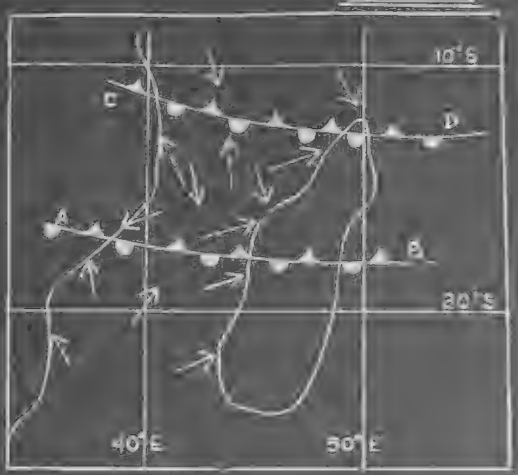


FIG. 10

Discontinuous movement of Inter-tropical convergence Zone

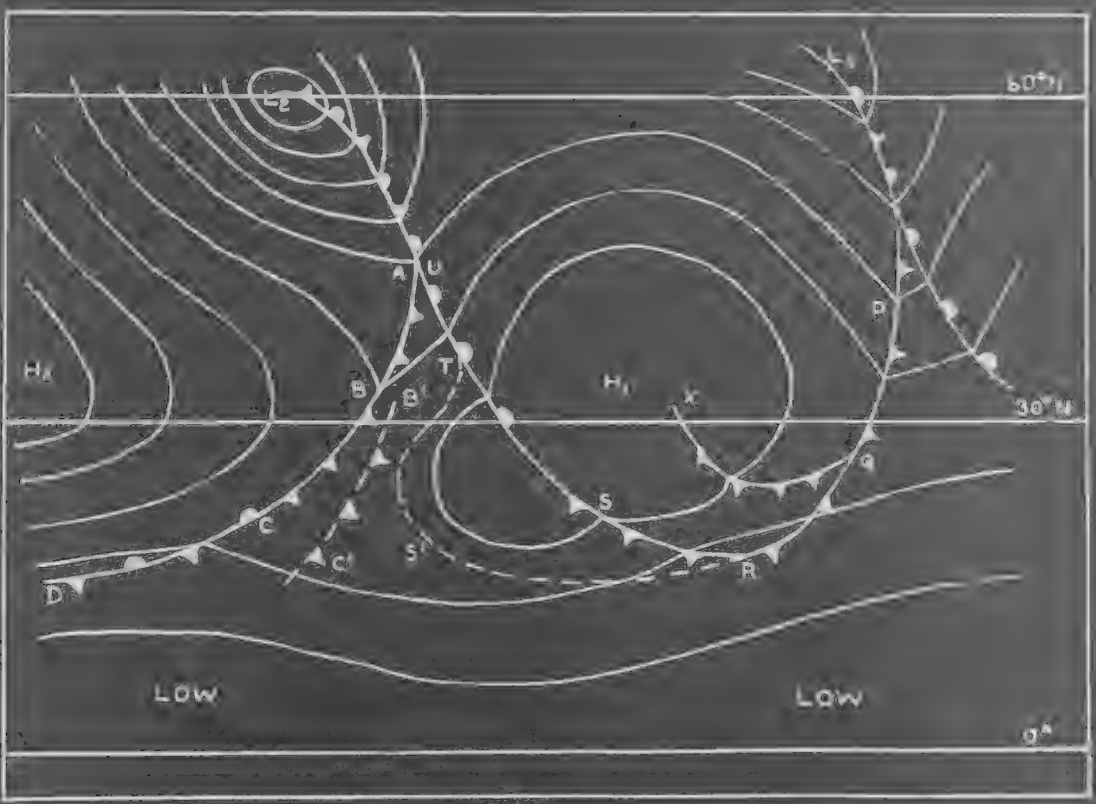


FIG.11

Penetration of Polar Front into the Tropics

/1000

FRONTAL TROUGH
DRY AIR
LOCAL
DRY AIR)
WIDESPREAD AND
HEAVY
FRONTAL
TROUGH
NO
(OBSERVATIONS)
12.00Z
Precipitation at Station, or
in sight of Station ( ), at
12.00Z or within previous
hour

FIG. 8

Mult. E 4 11 44
Isopleths of $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$
obtained by gridding
Isopleths of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$

FIG. 7.

1/66

FIG. 6.

FIG. 5.

A.M. 24.11.44
-14 Values of ψ
Isopleths of ψ
Gradient of ψ
Values of ∂ψ/∂y

FIG. 4

1/2560

A.M. 24.11.44
Values of u
Isopleths of u
Gradient du/dy
Values of du/dy

FIG. 3.

/Ext

30°
15°
0°
15°
30°
15°
30°
45
60°
AM 24 NOV 1944
Winds at 2 Km
Speeds in Km/hr
Streamlines
Zones of Convergence

FIG. 2

30. REFERENCES

1. Hare, F.K., Problems of forecasting in low latitudes, M.R.P. 259, 1945.

2. Gold, E., Barometric gradient and wind force, M.O. No.190, 1908.

3. Jameson, H., On a diurnal variation of the barometric gradient over Ceylon. Ceylon Journal of Science, Section E, Vol.1. Part 2, 1928.

4. Jameson, H., The flow of air in intertropical regions in relation to the isobaric gradient. M.R.P 279, 1946.

5. Gromes, A., The movement of air across the equator. Memoirs of the Malayan Meteorological Service, No.2, 1937.

6. Crossley, A.F., The motion of air in equatorial latitudes. Conference of Empire Meteorologists, 1946, Memorandum [illegible].

7. Brunt, D. and Douglas, C.K.M., The modification of the strophic balance for changing pressure distribution. Mem. Roy. Met. Soc., Vol III. No.22, 1928.

8. Sutcliffe, R.C., On development in the field of barometric pressure. Q.J.Roy. Met. Soc., Vol. 64, 1938.

9. Gold, E., The relation between periodic variations of pressure, temperatures and wind in the atmosphere. Phil. Mag., Jan, 1910.

10. Taylor, G.I., Proc. Roy. Soc. A, Vol XCII, 1916, p.198.

11. Dobson, G.M.B., Q.J.Roy. Met. Soc. Vol. XL, 1914, p.123.

12. Taylor, G.I. On eddy motion in the atmosphere. Phil. Trans. A. Vol. CCXV, 1915, p.1.

13. Eyre, J. (Miss), and others, Streamlining; a recommended method for [illegible] Meteorological Memoranda, No.7, 1945.

13A. Solot, S.B., The Meteorology of Central Africa, U.S.A.A.F. 19th Weather Region, Accra, 1943.

14. Kindle, E.C., An application of kinematic analysis to tropical weather, C.W.O., U.S.A., 1945.

15. Petterssen, S., and Priestley, C.H.B., The preparation of upper [illegible] charts, M.O. [illegible] 1945.

16. Priestley, C.H.B., and Swinbank, W., The vertical transport of heat by turbulence in the atmosphere. M.R.P. 265, 1946.

17. Bjerknes, J., The saturated ascent of moist air through a dry [illegible] Q.J.R. Met. Soc. Vol. 64, 1938.

18. Grimes, A., Notes on forecasting in India and Burma,

Vol. 70, 1944, p.129.

20. Austin, J.M. Cloudiness and precipitation in relation to frontal lifting and horizontal convergence. Papers on Physical Oceanography and Meteorology, M.I.T., Vol. IX, No.3, 1943.

Institute of Tropical Meteorology, Puerto Rico, March,

22. Grimes, A., Forecasting in the equatorial zone. India

23. Daily Synoptic Series, Historical Weather Maps, Northern

24. Forsdyke, A.G., Note on the movement and alignment of

25. M.O.451 (c) Weather in the Indian Ocean, H.M.S.O. London, 1942.

26. Durst, C.S., and Sutcliffe, R.C., The importance of vertical motion in the development of tropical revolving storms, Q.J.Roy. Met. Soc. Vol. 64, January,1938.

28. Deppermann, Rev. C.E. Outlines of Philippine frontology. Manila 1936

29. Guldberg, C.M. and hn H., Studies on the movement of

30. Hesselberg, Th., Über oszillatorische Bewegung der Luft. Ann. Hydr. Berlin, 1915, P.311.

I wish to express my thanks to Mr. C.S.Durst, B.A., for the of this report.

surface winds are often the only wind data available; hence it is usually the surface front which is so placed, whereas observations shows that the worst weather often occurs up to 200 miles away from the surface front. It may be assumed that the front will move towards the side on which the winds are weaker, but it is difficult to estimate the speed of the latter from the observed winds. Often the front appears to jump.

REFER TO FIGURE 10 AT END.

For example, in the Mozambique Channel, the front may be located in the position AB (Fig.10). A few hours later the front may be several hundred miles to the northward, say in the position CD, the winds between AB and CD having changed from northerly to southerly in the interval. Between AB and CD the full arrows represent the winds while the front is in the position AB; the dotted arrows when it is at CD. This is not a case of ordinary frontal movement, for with the wind speeds obtaining it would be quite impossible for the front to move the large distance between AB and CD. The front at AB usually dissolves, while a new one is formed at CD, and the air which was at first flowing southwards between AB and CD now returns northwards. Similar occurrences have been noted over British East Africa. It is not at present possible to forecast them, though on one occasion at Nairobi in November 1944 a few hours' warning was given by a rapid thickening of high and later of medium cloud - indicating that on this occasion convergence was first manifest at high levels. This cloud sequence was a most unusual one, and it cannot be said, that in East Africa, rain is often preceded by anything like the pre-frontal cloud sequence of temperate latitudes.

It is not clear from the definition in para. 23 exactly what is meant by the Monsoon Convergence Zone; presumably what is intended is the zone of maximum curvature of the trajectory as air crossing the equator is diverted owing to the reversal of the Coriolis force. There does not appear to be any observational evidence for the existence of bad weather along this zone.

25. Transverse waves and surges. Examples of a number of classes of wave like perturbations or troughs, the existence of which has only come to be recognised during the last few years. Minor perturbations of this type have long been recognised as travelling along "straight isobar" systems of temperate latitudes, and it is reasonable to expect that they will occur in the tropics also, but little is known regarding their origin.

(a) Easterly waves are shallow troughs travelling in a general east to west direction in the broad easterly currents which occur on the equator-ward sides of the sub-tropical anticyclones. They have been described in detail chiefly for the West Indies region, principally

/ in

in papers published by the University of Chicago Institute of Meteorology (1). The troughs [illegible] move with the general current. The wind changes at the trough are relatively pronounced, but there is [illegible] surface.

Ahead of the trough there is divergence with fine weather; behind it for a distance of up to 300 miles there is convergence, with moist air extending up to high levels, and an area of bad weather. At upper levels the trough is displaced to eastward of the surface trough; at 25,000 feet the average displacement is about 60 miles. There is some evidence that the pressure trough is more pronounced at 10,000 feet than at the surface and sometimes a closed circulation is found there.

Easterly waves in the Caribbean occur chiefly in summer, in the deep easterlies; they are also said to occur in both hemispheres in the western Pacific. In the western Indian Ocean minor disturbances moving from E or ESE have been found crossing the area from Mauritius to Madagascar and British East Africa. They occur chiefly in winter (S.H.) when the SE trade crosses the equator; it is believed that the perturbations may also cross the equator, and, [illegible] Affect India. They have been detected from observations of wind changes, cloud and precipitation which were found to occur at Mauritius, Eastern Madagascar, the Comore Islands and the coast of British East Africa at intervals which suggest that they move with general current; as a working rule it has been assumed that they move with the mean speed of the winds at 1 and 2 kms. The wind change [illegible] the backing to NE which occurs behind the trough. The pressure changes are small and often masked by the regular diurnal variation; if however, the pressures are corrected for the normal diurnal changes appropriate to the station and season, the troughs can be detected and followed. At Mombasa in 1944-5 considerable success was achieved by Fulwood (R.A.F.) and Coles (Naval Met. Service) in forecasting the movement of these troughs by graphing the departure of pressure from normal for various stations at 3-hourly intervals. It was not however, possible to forecast changes in the intensity of the troughs, and a number which were forecast failed to appear, through filling up. The bad weather zones are usually narrow, often less than 50 miles across; in their central portion heavy cumulonimbus clouds with showers occur; on the outer boundaries there are often altostratus and stratocumulus.

The troughs appear to occur more frequently in the western Indian Ocean, where three or four may pass a station within 24 hours, than in the Caribbean. On a flight from Seychelles to Mauritius in October 1944, the present writer passed through four of those zones. These had previously moved across Mauritius and had been included in the official analysis, the positions forecast being very close to those actually experienced. In East Africa these troughs were at first considered to be old polar fronts which had penetrated the sub-tropical high pressure belt; this point will be further discussed in para. 26. Marsden and Fairley in a paper as yet unpublished have described in detail two cases of similar disturbances affecting the Lesser Antilles in winter, and they tentatively identify them with old polar fronts which have moved round the Azores anticyclone.

The well known West African torrado may be a similar type of perturbation intensified by the frontal effect arising from the wide differences between the physical properties of the air masses involved,

(also

which are drawn respectively fro the Sahara and the Gulf of Guinea, and brought into juxtaposition along the West African coasts. This equatorial area is exceptional in that it is a meeting place of air streams having the widely differing characteristics necessary for the formation of a true front.

(b) Waves in the circum-polar westerlies occur at the surface on the leward sides of the subtropical high pressure belts and at higher levels they extend to low latitudes where the westerlies overlie the trades. It is tempting to identify these with the upper parts of cold fronts associated with the depressions of middle latitudes. At Mauritius they are frequently associated with nimbostratus cloud and rain which spread from SW above the lower current from ESE. They should be indicated on the charts as upper fronts. It is possible, therefore, to have in the same part of the chart a lower front moving from an easterly, and an upper front moving from a westerly point, and crossed lower and upper fronts may occur. In the present state of knowledge however, he would be a bold forecaster who included crossed fronts in an official analysis.

(c) The outstanding example of a current of equatorial westerlies is the SW monsoon of the N.Indian Ocean, and the waves within it appear as the monsoon surges of India. They may perhaps be identical with the westward moving disturbances of the S. hemisphere described in (a) above, which have travelled with the general current. Elsewhere the areas covered by the equatorial westerlies are comparatively small and there are no well known examples of perturbations occurring in them, though they possibly occur in the NW monsoon of N.Australia, originating as cold fronts in the outflow of air from the winter anticyclone of Siberia.

(d) In any broad current in the tropics monsoon surges are likely to occur as a result of strengthening of the subtropical anticyclone by the advection of a cold anticyclone from high latitudes. Apart from the weather and cloud associated with them, they could only be identified with the aid of a detailed stream line chart. Failing this, an increase in the surface wind upstream, especially if this appears to take place along a line across the stream, should be regarded as indicating a probable surge.

26. The polar front in the tropics.

The shear-lines and surges of para. 23 III above are mainly associated with the intrusion of the polar front into the tropics. Some consideration has been given to this question in Australia and New Zealand where the cold front, lying roughly N-S between adjacent cells of the subtropical anticyclones is known as the Meridional Front. In the N. hemisphere little attention seems to have been given to the manner in which cold fronts of temperate latitudes can penetrate into the tropics, passing the barrier of the subtropical high, and in the present notes this question will be considered to some degree.

REFER TO FIGURE 11 AT END.

The left hand half of Fig.11 represents the intrusion of a cold front into low latitudes as it is usually shown on the synoptic charts. L_2 is the main depression and BCD the associated cold front. An intense cold anticyclone H_2 spreads southwards behind the cold front, while the subtropical anticyclonic cell H_1 recedes eastwards ahead of it. The trailing portion of CD of the front is pushed far to the southward by the cold current flowing round the anticyclone H_2, and it takes up a position parallel to the stream and separates the polar from the tropical current; these flow side by side and the difference of temperature between them is soon smoothed out, partly owing to the surface influences, but partly also to subsidence in the polar current. A change of velocity only is left to mark the frontal line; such a lines is called a shear-line, and the weather associated with it is usually feeble.

If a polar anticyclone spreads towards the tropics, it is to be expected that there will be a marked tendency for subsidence in the cold current associated with it, for the undercutting by the cold air of the warm air which it displaces is hindered less and less by the decreasing Coriolis deviation; the potential energy of the cold mass is quickly dissipated in low latitudes because the small Coriolis force allows the mass to collapse owing to the more or less direct outflow near the surface. The cold current is comparatively narrow in middle latitudes and may be pictured as a powerful jet, whose energy is frittered away when it spreads out freely into the vast tropical regions.

At BC(Fig.11) the front at the surface is almost stationary or moving very slowly SE and its frontal characteristics have almost disappeared. Aloft, however, in this region, the westerlies of middle latitudes are still present and the upper part of the cold front B'C' is carried on ahead of the surface front (para.24b). To westward of D the front is not usually shown on the charts drawn in temperate latitudes, it being assumed that in this region it has completely disappeared.

It is, however, of interest to consider, if only from a kinematical point of view, what the alignment of the front near D would be westward of D. The right-hand half of Fig.11 shows the cold front PQR of the previous depression L_1 to eastward of the subtropical high H_1. TU is the warm front of the depression L_2 and kinematically it is a continuation of the front PQR. If H_1 was built up entirely in the polar air, the link between R and T would be via S' and lie round the boundary of the polar anticyclone. The frontal alignment would then be PQRS'TU. If however, as seems more likely, H_1 built up across the front, linking up with the former subtropical high pressure cell, the frontal linkage would be similar to RST. In this case the segment RS would move from an easterly direction. Secondary cold fronts such as SX would move to eastward, and the westward movement of some of the perturbations discussed in para. 24a might be thus accounted for.

An examination of a number of weather charts of the northern hemisphere (23) confirms the possibility of the type of development described above. The published charts do not carry the analysis beyond the stage shown in the left-hand part of Fig.11, but in a number of cases it is possible to make a reasonable extension to the analysis, obtaining configurations similar to that shown on the right of Fig.11. The corresponding frontal alignments for the southern part of the western Indian Ocean have been discussed in an unpublished note by the present writer (24). The examination of charts for the northern hemisphere has shown, however, that the intrusion of cold fronts from middle latitudes into the tropics occur too infrequently to account for the large number of westward moving troughs that were observed in the western Indian Ocean, so that polar outbreaks are not the only cause of the latter, most of which ought therefore to be regarded merely as perturbations in the easterlies, concerning the origin of which little is known at present.

The preceding discussion emphasizes the necessity for extending the working charts of the tropics well into temperate latitudes, for it is in those regions that the advancing cold fronts can be most easily picked up. This has been done in a series of charts of the Indian Ocean, which have been analysed in great detail

REFER TO FIGURE 12 AT END.

/in

in the Meteorological Office in collaboration with the Naval Meteorological Service (25). Cold fronts from temperate latitudes are followed up to the time of fusion with the Inter-tropical Convergence Zone, a typical sequence being shown in Fig. 12, I - IV which are copies from the original publication. It is of interest to consider the movement of the cold front ABCD as shown by successive charts, particularly that of the segment ABC, which is seen to be always towards the N and NE, and in II and III is definitely against the direction of the flow of the air. If the displacement of the front is estimated from the isobaric field, even after allowing for a component of airflow across the isobars from higher to lower pressure, it is seen that in I and II, AB should move towards W or SW while C should remain stationary, while the large northeastward movement of BC shown between II and III appears to be quite impossible. Few upper wind observations are available for the area concerned, but over southern Madagascar they are shown in the original publication as northeasterly up to 4 kms. at least, throughout the period, and it is likely that the winds over the whole area to eastward of Madagascar would be from an easterly point up to that height also. A suggested alternative alignment of the front is shown by the dotted line B'C' in diagrams III and IV (the isobars would of course have to be altered slightly in order to fit the amended frontal position, but in any case, they are largely estimated). The segment of the front immediately to northward of the anticyclonic centre would be moving towards the west. The upper part of the front, just to northward of C would, possibly, continue to move towards E in the upper northward westerly current.

Only in very favourable circumstances can these fronts retain their characteristics into low latitudes, as, for example, over the Gulf of Mexico and off south China, in winter: the extensive cold land masses of North America and East Asia allow the cold air to penetrate to quite low latitudes without undergoing appreciable heating. In Australia, cold fronts moving from the south have been traced right across the continent, but except in the south they are much enfeebled, and difficult to detect. Well marked cold fronts have been encountered by aircraft just to the southward of Madagascar, but they have been almost undetectable a few hundred miles farther north. It is probably true to say that the polar front has little direct significance on moving into the tropics, but is has an indirect significance in that it may be associated with a surge of cold air which may give rise to convergence and therefore to disturbed weather.

The writer has studied in some detail a case of a surge of cold air which affected S and E Africa and the SW Indian Ocean from 12th to 17th May, 1944; and since the effects noted are, in the main, typical of these surges, a summary of the principal features may well be given here. The surge was associated with the development of a pronounced anticyclone which had spread from high latitudes over S Africa, while the cold air spread northwards over the Transvaal, the Rhodesias and Madagascar. On 13th May a vigorous cold front, having the characteristics to a marked degree of a cold front of temperate latitudes was encountered in about latitude 27°S, on a flight from Tulear (S Madagascar) to Durban, and for some 400 miles behind the front the usual squally showery conditions were experienced. During the following 3 days the cold air spread northwards almost to the equator. But the

/cont

front locally dissipated rapidly, but though its passage was marked by a fall of upper air temperature and a shower at Tulear, it had little or no effect on the upper air temperature in the north of Madagascar. Over Madagascar the surface temperatures were somewhat lower in the cold air than they had been at the same times on the days before its arrival, and there was an increase in shower activity well behind the front; over the Rhodesias, while there were a few showers and thunderstorms near the front, the cold air was accompanied by much stratocumulus and a marked fall of humidity, dew points on the plateau falling below 20°F. locally, with dry bulb temperatures about 45°F - 50°F. Generally the indications were that over the whole area the cold air was subsiding strongly. But perhaps the most interesting feature was the surge of pressure. The pressures at a number of stations, after correction for the normal diurnal variation have been plotted against time. All showed a rise of pressure, which, as would be expected, decreased with decreasing latitude, but the surprising feature is that the rise of pressure took place almost simultaneously at all the stations during the two days 13th - 14th May. Whether this is a usual feature of such surges, and if so, what are its implication, is a question for further investigation.

TROPICAL REVOLVING STORMS

27. The general characteristics, tracks, localities and times of occurrence of tropical revolving storms are well known, having been described by many writers, so that they will not be referred to further in this report. The causes of their formation however, constitute one of the outstanding unsolved problems of meteorology. All theories are agreed that a deep moist unstable air mass is essential; but this is a frequent condition in many tropical areas, and it would appear that in addition some form of trigger action, which is of comparatively rare occurrence, is necessary to start off a storm. No theory is adequate which does not explain the fall of pressure at the centre of the storm, and in this respect the most satisfactory explanation so far put forward is that of Durst and Sutcliffe (26), the gist of which is that the air ascends near the centre of the storm, retaining its horizontal velocity, and in the upper levels is evicted from the central regions because the geostrophic and cyclostrophic forces together outweigh the reduced pressure gradient. With regard to the synoptic aspects of the question, a number of writers, notably Grimes (27) and Depperman (28) are of the opinion that the storms always develop at the equatorial convergence zone, usually at a point of bifurcation, due to the presence of a third air mass and known as a triple point. Some American writers (21) state that disturbances with closed circulations may form on most of the fronts described in paras. 24 - 27 though many of them do not develop to storm intensity. It is considered, however, that little definite is yet known of the conditions pre-disposing to storm formation, and it is only possible to say, under these conditions, that a risk is present. One would like to see a series of properly analysed charts showing clearly the participating fronts and air masses in a case of storm formation. Grimes (27) has put forward the further view that a cyclone only forms when there is a surge across the equator, of air, which in the hemisphere of origin had sufficient anticyclonic vorticity, which is conserved as cyclonic vorticity in the other hemisphere.

The main difficulty which confronts the forecaster using synoptic charts is, however, that the storms frequently form in areas devoid of observing stations, and their development and existence can only be

/ inferred

inferred from observations [illegible]. The signs of the storms are similar to those of temperate [illegible] depressions - increasing wind with cyclonic circulation, increasing cloud through the sequence high, medium and low, and falling barometer; in addition there is a pronounced swell travelling directly outwards from the centre, which can often be detected at a range of 1,000 miles. Once the existence of a storm is known the forecasting problem is not especially difficult, using well known methods, and the fact that the storm is usually slow-moving enables adequate warning of the risk to be given, though it is not possible to forecast for long in advance the exact track of the central portion, which constitutes the main danger.

28. Summary of problems for investigation.

References are to the relevant paragraphs in this report.

A. Relation between wind and pressure distribution.

(i) Practical investigation on lines of Gold's "Barometric Gradient and Wind Speed" for low latitudes, extension of Jameson's investigation to include other seasons and other areas (crossing the equator) and to give details of the actual gradient and wind velocity calculated for each square. Para. 11.

(ii) Solution of the equations of motion allowing for the variation of the Coriolis force - extension of Grimes' solution. [illegible] by Guldberg and Mohn (29) Para. 14.

(iii) Solution of equations for accelerated motion with constant uniform pressure field and its application to forecasting wind. Para. 15(a)

(iv) Solution of equations for accelerated motion by expressing the pressure variation as exponential or circular functions (Fourier Series) of the time. Para. 15(c)

(v) Investigation as to the reality of the oscillatory motion arising from the equations of acceleration. See also paper by Hesselberg on Atmospheric Oscillations (30) Para. 15(c)

(vi) The effect of friction; decay of atmospheric oscillations (See also paper by Hesselberg (loc.cit.) in which it is assumed that the friction is proportional to the velocity; Further investigation of the solution for unsteady motion and its extension to include initial departures from appropriate solution for steady state. Para. 16.

(vii) Investigate the effect of the movement of air across the isobars on the pressure distribution Para. 17.

(viii) Development of technique for mapping quickly the field of convergence and divergence. Para. 18.

(ix) Applications of (iii) and (iv) to calculate future convergence or divergence. Para. 18.

B. Upper air and convection.

(x) Extension of temperate latitude methods of upper air analysis to the tropics. Para. 19.

(xi) The effect of high ground on the upper level pressure distribution and its effect on the flow Para. 19.

(xii) The effects of convective turbulence on the U.A.T. distribution. Para. 20.

(xiii) The effect of the descent of the environment on convection estimated from the tephigram. Para. 20.

(xiv) Nocturnal spreading of thunderstorms from hills to plains; radiation from cloud top, and delayed realisation of latent instability Para. 20.

(xv) Turbulent and radiative cooling of the lower layers at night: forecasting of early morning fog and low stratus. Para. 20.

(xvi) Effects of advection and divergence on the upper air temperature. Para. 21.

C. Frontal zones and discontinuities.

(xvii) Calculation of convergence and divergence in connection with the formation, activity and disappearance of convergence zones. Para. 24.

(xviii) The detailed structure of easterly waves and their classification into convergence zones and true fronts. Para. 25.

(xix) Study of synoptic charts in connection with the instrusion of the polar front into the tropics; study of pressure surges. Para. 26.

D. Tropical storms.

(xx) Further investigation into the dynamics and thermodynamics of tropical storms and other phenomena such as line squalls. Para. 27.

(xxi) The synoptic conditions associated with tropical storms and depressions generally, by examination of particular examples. Para. 27.

29. Some suggestions regarding forecasting technique.

In consideration of the points discussed in this report and the problems listed in para. 28 the following suggestions are made regarding forecasting technique in the tropics.

(i)

(i) Working charts. The synoptic chart should be retained; it still is the best method of representing actual weather and surface winds. Isobars are necessary in the equatorial regions in connection with the problems (iii) and (iv) of para. 28, but otherwise they have little significance; in extra-tropical areas, to which the working charts should extend, they are the basis of the chart. Areas and zones of convergence and divergence should be indicated on the surface chart, but the details in respect of upper levels should be reserved for subsidiary charts.

(ii) Subsidiary charts. These should show the winds and streamlines at standard upper levels, and the charts for the appropriate levels might well show the heights of standard pressure surfaces computed from the upper air temperature data. Isopleths of the divergence should also be drawn on these charts, which could be on a considerably smaller scale than the main working chart.

(iii) Fronts. It is necessary to distinguish between 3 types of discontinuity:

(a) [illegible] areas,
(b) ordinary surface fronts.
(c) [illegible] fronts.

These should be marked on the charts; the standard mode of representation would suffice for (b) and (c) but a new one is required for (a).

(iv) Pressure and tendencies. This question has been discussed in the Appendix to Memo. No. 46/6, Conference of Empire Meteorologists 1946. It is considered that 6-hourly pressure tendencies are of value in tracing short-period disturbances such as easterly waves, but before they can be used the semi-diurnal pressure variation must be eliminated. Therefore, it is better to use differences from normal (for the time of day and season) rather than actual pressures. At high level stations the errors involved in the reduction of pressures to a standard level are so great that unless the stations are all close to the standard level their pressures cannot be coordinated; therefore, for tracing travelling disturbances differences from normal of station level pressure should be used.

It is of the utmost importance in view of the slight pressure gradients of the tropics and the large effect produced on the wind by a small change of pressure, that barometric readings be of the highest accuracy attainable.

(v) Prebaratic charts should be drawn where possible; they should include fronts, and zones and areas of convergence and convection (the latter based upon the convergence, moisture and lapse-rate) but not isobars.

(vi) Detection of thunderstorms by spheric or radar methods.

In the tropics thunderstorms constitute the principal hazard to aircraft in flight, and since many of the routes are over ocean or thinly populated land areas, where observing posts are few, many extensive thunderstorms are missed by the reporting networks; in addition, communication between many reporting stations and the central office is difficult and restricted to a few routine periods. The advantages of a number of suitably located sferic or radar stations for the detection of these thunderstorms are obvious.

/30.

Amendments to M.R.P. 318.

The following amendments should be made on page 1 of M.R.P. 318.

1. Section (ii) should be reworded to read:-

(ii) Subsidiary charts. These should show the winds and streamlines at [illegible] if possible [illegible] should also be given the [illegible].

2. [illegible] in Section [illegible] should be reworded to read:-

3. Section IV should be reworded to read:-

(v) Forecast charts should be drawn where possible; they should include [illegible]

4. [illegible]

G.149266.

CONFIDENTIAL
contents must not be quoted.

AIR MINISTRY M.R.P. 319

METEOROLOGICAL RESEARCH COMMITTEE. 8th January, 1947

An Appreciation of Various Methods of Measuring the Light Scattered by the Upper Atmosphere from a Search-Light Beam.

by

F. E. Jones.

Introduction

The experiment proposed by Tuve, Johnson and Wulf (1) for exploring the meteorology of the upper atmosphere by measuring the molecular scattering using a modulated search-light beam has been developed by Johnson, Meyer, Hopkins and Mock (2).

The purpose of this paper is to consider the fundamental electrical problems involved in this experiment and to suggest other and possibly better methods of tackling the problem.

The Scattering Phenomenon

Quoting from the paper (2) by Johnson, Meyer, Hopkins and Mock, the amount of light scattered from a unit volume of air is given by :-

$$K = \frac{32\pi^3(\mu - 1)^2}{3N\lambda^4}$$

where

N = number of molecules per c.c.
λ = wavelength of radiation in cms.
μ = refractive index of scattering gas.

For air $\mu - 1 = \alpha N$ where $\alpha = 1.08 \times 10^{-23}$. The paper assumes α independent of wavelength giving

$$K = \frac{330\alpha^2 N}{\lambda^4}$$

If $E_{o\lambda}$ is the radiation incident on the scattering volume, then the scattered radiation:-

$$E_{s\lambda} = \frac{E_{o\lambda}\, 330\alpha^2 N}{\lambda^4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The Electrical Problem

From equation (1) it can be seen that for maximum range the electrical system must be capable of transmitting as much power, $E_{o\lambda}$, as possible and the receiver must be capable of detecting as small amount of power, $E_{s\lambda}$, as possible. As in all similar problems the fundamental determining factors are:-

(a) The amount of mean power transmitted per unit solid angle.
(b) The time required to make a reading - this fixes the bandwidth of the receiver and hence the limiting noise.

It is to be noted that the maximum range of a system is independent of the distribution of the transmitted power in time, i.e., one can use sinusoidal, frequency, pulse or any other form of modulation without affecting the maximum range as long as the mean power is constant for each case.

0426590 -1

Enclosure 12 to Report No. R.7452-47
Military Attache, London.

The Transmitter

For any given optical system the determining factor for long range is the mean brightness of the source.

Three transmitters have been considered:-

(a) The standard search-light.
(b) The high pressure gas-filled discharge lamp.
(c) The spark in air.

Discussions with G.E.C. Research Labs. and R.R.D.E., lead the author to believe that as far as brightness is concerned there is nothing to choose between the standard search-light and the gas-filled discharge lamp. However, as it is necessary to mechanically modulate the search-light, some 30 per cent of its power will be lost for sinusoidal modulation. Electrical modulation can be employed in the case of the gas-filled tube and in addition, high frequency modulation becomes possible. The limiting brightness is determined by the melting point of the electrodes. There is some divergence of opinion between the G.E.C. and R.R.D.E. on the possibility of pulse modulating these tubes, although the evidence given by the G.E.C. to show that it is possible seems convincing. R.R.D.E. have promised to contact Siemens on the possibility of pulse modulating the Siemens tubes.

No information has so far been found on the limits of mean brightness attainable with the spark discharge in air. Little effort seems to have been put into this method of producing a luminous source presumably because the efficiency is very much lower than that of the gas-filled lamp.

The Receiver

The receiver consists of a photo-emissive cell followed by an amplifier. The fundamental limit of sensitivity will be set by the time that can be taken in making an observation in that the noise input voltage fluctuations are given by the Johnson-Nyquist equation(3).

$$E^2 = 4kTRb$$

where

k = Boltzmann's constant.
T = absolute temperature.
R = resistance.
b = bandwidth.

As the constancy of the atmospheric density at any height is presumably not known, it would be as well to make the bandwidth of the receiver as flexible as possible. This would enable 'fine structure' of the density to be observed, if present, at a sacrifice of maximum range, by using a comparatively wide bandwidth, and by simple changes enable the extreme range to be achieved by using the narrowest possible bandwidth.

From equation (1) it can be seen that it will be preferable, apart from the selective absorption bands, to transmit on the shortest wavelengths. Furthermore, if the energy can be confined to a narrow band of frequencies it may be possible to use a filter on the receiver to minimise the noise due to the night-sky current in the photo-cell.

The Optical System

The efficiency of the optical system will be governed by the effective areas of the transmitter and receiver, and by the cost. The transmitter mirror might well be a normal 150 cm. search-light mirror but for the receiver a better mirror is warranted in view of the small size of the sensitive surface on a modern photo-cell.

Methods of Measuring the Scattered Light

In order to distinguish between the night-sky and scattered radiation it is necessary to modulate the transmitted beam and then arrange that the receiver only accepts this form of modulation.

Three forms of modulation have been considered:-

(a) sinusoidal,
(b) frequency,
(c) pulse.

The case of sinusoidal modulation has been dealt with by Johnson, Meyer, Hopkins and Hook(2), although because of the usual predominance of flicker noise in low frequency circuits it would appear to be preferable to work at a higher modulation frequency than 10 cycles per second.

The disadvantages of the sinusoidal system are:-

(a) In order to get range discrimination it is necessary to put the receiver and transmitter considerable distances apart. This may make the alignment of transmitter and receiver difficult especially when the receiver has a very narrow bandwidth, and in addition reduces the range.

(b) It is difficult to make a receiver with a bandwidth variable and sufficiently narrow for the purpose of receiving very weak signals.

The frequency modulation system would overcome the first disadvantage but does not seem worth pursuing because a mean frequency of 100,000 cycles per second, modulated to a depth of 20 per cent, is necessary to achieve a one mile range discrimination.

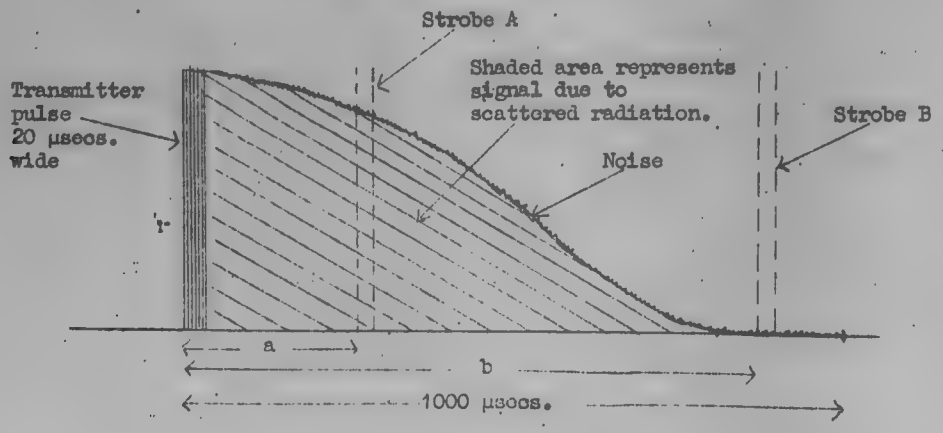
The pulse system overcomes both disadvantages of the sinusoidal system and in addition appears to offer other advantages as far as absolute measurements of scattered radiation are concerned. A possible pulse system is therefore described in some detail but its success depends on the possibility of pulsing the high-pressure gas-filled lamps.

Pulse System

In the pulse system the energy is transmitted in short, regular pulses instead of being evenly distributed in time.

Assuming a 10 KW high-pressure mercury vapour tube(4) is used as a source, it is suggested that the pulse should be 10 to 20 microseconds long repeated 500 times per second. In order to keep within the mean power limit of the tube the peak power through the tube would be 1000 to 2000 kilowatts.

On the receiver side, a cathode-ray tube display would present the information as indicated in the diagram:-



4/26 590-3

The trace is 1000 μsecs. (100 miles) long. The gates or strobes A and B (each half the pulse width in length) are adjustable in position from the ground ray. The energy in each of these pulses is integrated separately, and compared in an 'electrical balance' type of circuit. Thus the energy scattered by an illuminated layer of say 1 mile depth (10 μsecs.) at height 'a' can be compared with the energy scattered by a similar layer at height 'b'. For absolute signal to noise ratio measurements it will be necessary to move strobe B along to the end of the trace where no appreciable signal is received. The time of integration can best be determined by a resistance and capacity circuit and can therefore easily be made variable. Time constants from the order of microseconds to a few hours have been employed in this type of circuit.

In the pulse system the transmitter and receiver can be situated side by side. Furthermore, fluctuations in the light output of the source will not matter when comparisons are being made of light scattered from different heights.

Finally, it has been suggested (5) that increased range can be obtained by using more than one transmitter. This is indeed true, since two equal transmitters illuminating the same scattering area will double the apparent brightness of the area. Thus in the pulse case, the arrangement might well be a receiver situated in a ring of transmitters.

Recommendations

If the 'search-light experiment' is undertaken in this country it is recommended that the pulse system be used in conjunction with a gas-filled tube. This will first entail an investigation into the possibility of pulse modulating the G.E.C. or Siemen's high-pressure mercury vapour lamp, and if necessary, the development of a lamp specifically for the experiment. The pulse transmitter modulator and receiver, including the strobe and integrating technique, is well known and should present few difficulties. The transmitter and receiver mirrors might well be normal 150 centimetre search-light mirrors in the first instance although a better mirror for the receiver is worth while. Possible applications of filters to reduce the noise due to the night-sky should be borne in mind. Provision should be made for the installation of additional transmitters should it be found necessary to increase the range of the system beyond that possible with one transmitter.

Acknowledgments

The author is indebted to a number of his colleagues at T.R.E. (particularly Dr. F.C.Williams) for some useful discussions on the project and to G.E.C. Research Labs. and R.R.D.E. (Mr. D.H.Jones) for information on high pressure mercury vapour discharge lamps.

References

1. Tuve, Johnson and Wulf. Terr. Mag. 40. 452-454 (1935)
2. Johnson, Meyer, Hopkins and Hock. J. Opt. Soc. Am. 29. 512-517. (1939).
3. Physical Review 32. 97 and 110 (1928).
4. Paper 'High Pressure Mercury Vapour Lamps' by D.H.Jones, M.R.P. 315, December, 1946.
5. Letter from Prof. S. Chapman to the Author. (1946).

426590-4

9.1.47.

AIR MINISTRY

Acc. 47. 1924

METEOROLOGICAL RESEARCH COMMITTEE

THE DIURNAL VARIATION OF FOG AT SEA

by H. Jameson.

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

INTRODUCTION.

In reply to a query as to whether fog shows a diurnal variation at sea, the percentage frequency of fog was extracted from the Hollerith cards in the Marine Branch of the Meteorological Office, for certain 5° Marsden squares in the North Atlantic. As apparently little or nothing has been published on this subject, the results obtained may be of some interest.

These Hollerith cards contain weather observations taken by voluntary observers in British ships, since 1855, and entered up in logbooks, which are forwarded to the Marine Branch of the British Meteorological Office. The observations were taken at the end of each watch, that is, at 0000, 0400, 0800, 1200, 1600, 2000, ship's time.

The Marsden squares were selected from the foggy regions of the North Atlantic, as samples of conditions near the Banks and in the open ocean. These squares are 184 A, B, C, D; 183 A, B, C, D; 150 A, B, C, D; 149 B, C, D, (see fig. 1). Of these, 150 C and 150 D were slightly modified from the normal 5° square, to agree better with the outline of the coast, by taking away the 1° squares, 88, 89, 95 to 99 from 150 D, and adding 93 and 96 to 150 C. Observations were only extracted for those months, May to August, inclusive, when the liability to fog is greatest in the most foggy areas near Newfoundland.

The mean frequencies for the various squares, when taken separately for each month, in general varied so irregularly during the day that the figures for the four months were finally combined together for each square, while in the open ocean, 10° squares, 183 and 184, were used.

The final figures for percentage frequency are shown in Table I, together with the number of observations for each watch. These figures are plotted, roughly in their relative geographical positions, in fig. 2.

G.147,767.

TABLE I

Percentage Frequency of Fog, May to August inclusive

Local Time

Square		0000	0400	0800	1200	1600	2000	Mean	Range	Range/Mean
150B Lat. 40 to 45° N. Long. 55 to 60° W.	Percentage Frequency	8.6	10.8	9.1	7.6*	8.2	9.4	8.9	3.2	0.36
	Number of observations	592	612	626	628	597	594			
150A Lat. 40 to 45° N. Long. 50 to 55° W.	Percentage Frequency	10.2	13.5	10.6	9.1	8.8*	10.1	10.4	4.7	0.45
	Number of observations	507	495	489	503	534	512			
149B Lat. 40 to 45° N. Long. 45 to 50° W.	Percentage Frequency	12.0	13.3	14.5	12.8	8.7*	12.2	12.3	5.8	0.47
	Number of observations	543	541	530	516	514	540			
150D Lat. 45 to 50°N. Long. 55 to 60°W.	Percentage Frequency	28.1	29.8	32.3	27.5	29.4	25.9*	28.8	6.4	0.22
	Number of observations	353	382	403	381	374	356			
150C Lat. 45 to 50°N. Long. 50 to 55°W.	Percentage Frequency	34.8	36.1	36.6	31.8	30.5*	33.1	33.8	6.1	0.18
	Number of observations	615	609	587	605	607	660			
149D Lat. 45 to 50°N. Long. 45 to 50°W.	Percentage Frequency	32.5	34.5	36.3	32.6	31.5*	32.8	33.4	4.8	0.14
	Number of observations	547	549	547	538	537	545			
149C Lat. 45 to 50°N. Long. 40 to 45°W.	Percentage Frequency	11.0	12.2	11.6	10.5	9.9*	10.8	11.0	2.3	0.21
	Number of observations	408	416	440	448	436	433			
184 ABCD Lat. 50 to 60°N. Long. 30 to 40°W.	Percentage Frequency	5.9*	7.1	6.7	7.1	6.2	7.4	6.7	1.5	0.22
	Number of observations	1210	1216	1227	1218	1212	1209			
183 ABCD Lat. 50 to 60°N. Long. 20 to 30°W.	Percentage Frequency	4.0	4.3	4.5	3.7	3.1*	4.4	4.0	1.4	0.35
	Number of observations	1514	1479	1494	1549	1521	1533			

NATURE OF DIURNAL VARIATION.

The frequency of fog shows, in most of the squares, a diurnal variation, with maxima (underlined) in the early morning, and minima (marked with an asterisk) during the afternoon. Fig. 2 shows that this is most marked in the neighbourhood of Newfoundland, particularly in squares 149B and D, 150A and C. This diurnal variation is also quite appreciable in squares 150 B and 149 C, though less marked, but seems to be absent in 184, and is very slight in 183.

150 D, a modified quarter-square which includes the Banks south of Newfoundland, but excludes any part of the Gulf of St. Lawrence, seems irregular at 1600. The curves for the separate months were plotted, and June and August were examined for errors in computation, and for bias. The computations were correct, and the distributions in years seemed reasonable.

SIGNIFICANCE OF VARIATION.

It may be asked how far the values shown in Table I are accurate, and why statistical methods have not been applied to obtain an indication of the reliability of the results. If the observations were a perfectly random selection, the calculation of the standard deviation at each watch would enable the standard deviation of the difference between two successive watches to be calculated. The observations, however, are very far from random, since in any one square a ship is liable to make two or more consecutive observations in neighbouring positions, and nearly all the observations dealt with are grouped in such series of consecutive observations. The result must be that there is a definite correlation between the deviations from normal of any frequency and the one at the succeeding watch, so that these deviations are likely to be of the same sign and of somewhat the same order of magnitude.

The standard deviations of the frequencies were calculated and were found to be of the order of at least one or two per cent, for frequencies based on 400 to 600 observations. Such values would make ranges of two to six per cent. open to suspicion, were it not for the lack of random sampling. However, the consistency with which the maxima appear at 0400 or 0800 and the minima at 1600 make it clear that the diurnal effect is real, even with such large standard deviations for the frequencies at each watch.

The conclusion may be drawn that the manner in which marine data are grouped together, in sets of observations from series of consecutive watches, which is so disadvantageous when isopleths of frequencies or mean values are to be drawn, is of considerable advantage for the study of diurnal variations.

ACCURACY OF OBSERVATIONS.

There remains the question whether the diurnal variations obtained are due, wholly or in part, to observational bias, rather than to physical causes.

Because of the absence, in general, of distant points of reference at sea, fog observations there are largely a matter of the observer's judgment, and any variation between the frequency of observations by day and at night might be due to a tendency to faulty judgement in allowing for the effect of darkness. The opinion of experienced marine officers is that any variation due to this cause would be only slight. The frequencies for squares 183 and 184 certainly suggest that this is the case. Table II gives, for latitude 55°, and between May 1st and August 31st, the times of sunrise and sunset and of civil twilight (when the sun has a depression of 6° below the horizon). Daylight conditions of visibility extend for an appreciable interval before sunrise or after sunset. This interval will

depend, to some extent, on the weather, but civil twilight should give, at least approximately, the dividing times between day and night conditions of visibility.

TABLE II

Latitude 55°

Date	Morning civil twilight	Sunrise	Sunset	Evening civil twilight
May 1	3.37	4.21	19.34	20.16
16	3.02	3.51	20.02	20.50
June 1	2.35	3.29	20.27	21.21
16	2.22	3.20	20.41	21.39
July 1	2.27	3.25	20.42	21.40
16	2.49	3.41	20.30	21.23
Aug. 1	3.21	4.07	20.04	20.51
16	3.53	4.35	19.33	20.15
31	4.25	5.03	18.57	19.35

In latitude 55°, the only observations taken entirely in darkness were those at 2400. Observations at 0800, 1200, 1600, were entirely in daylight, while the majority of those at 0400 and 2000 were taken between sunrise and sunset, and of the remainder, all, except a few late in August, were taken between the times of morning and evening civil twilight. These frequencies may therefore be regarded as daylight frequencies. The frequencies for these two squares show very little diurnal variation among themselves, and the fact that in both squares the frequency of the night observation, at 2400, varies so little from the frequencies for the daylight observations suggests that the effect of darkness has been adequately allowed for.

It may therefore be accepted that the statistical evidence shows the existence of a definite though small diurnal variation in fogginess in the marine regions near Newfoundland. This diurnal variation should depend, wholly or in part, on the evaporation of the fog particles by the sun's heat. Cloudiness might be expected, therefore, to show some relation to fog clearance and fog persistence. The mean cloudiness during the season May to August is set out in Table III below.

TABLE III

Mean cloud amounts, May to August inclusive

Marsden Square	163	154	149C	149D	150C	150D	149B	150A	150B
Mean cloud amount	7.7	8.1	7.7	7.9	7.3	6.6	6.6	6.4	6.3

That it does not reveal any such close correlation may be partly because other factors may be involved, and partly because thick fog itself may be booked as cloud, so that the mean cloud amount is artificially raised, particularly in those areas where fog is most prevalent, and does not represent the likelihood of sunlight falling on the fog top.

All the figures in Table I, except those for 154, which show no trace of diurnal variation, were harmonically analysed for the 24-hour and 12-hour periods. The values obtained are shown in Table IV, the oscillation being expressed as:-

$$a + m \sin (x + M) + n \sin (2x + N)$$

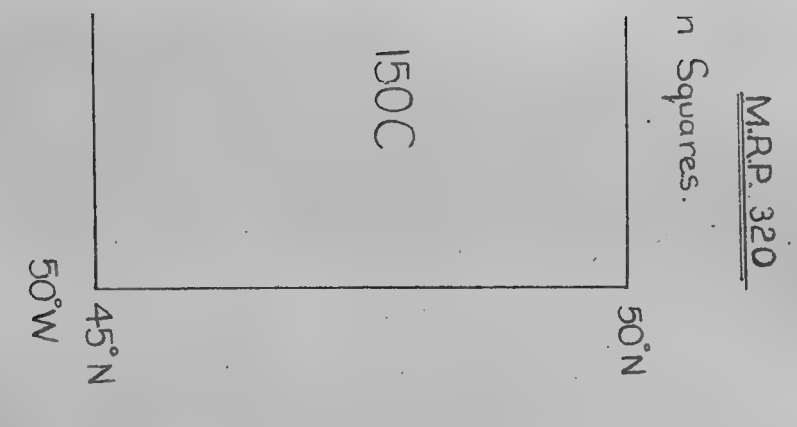
While the zero from which M and N are measured is local midnight.

/TABLE IV

TABLE IV

Square	a	m	M	n	N
150 B	8.9	1.1	51°	0.9	279°
150 A	10.4	1.9	35°	0.9	302°
149 B	12.3	2.0	3°	1.4	174°
150 D	28.8	2.1	338°	1.1	286°
150 C	33.8	3.0	27°	1.1	210°
149 D	33.4	1.9	356°	1.2	222°
149 C	11.0	1.0	25°	0.3	250°
183 ABCD	4.0	0.5	36°	0.5	199°

The phases M show a certain amount of agreement, ranging from 338° to 51°, while the values for the squares in which the diurnal variation seems most marked in fig. 2, 149 B and D, 150 A and C, range only from 356° to 35°. The phases N show a wider range, from 174° to 302°, the values for 149 B and D, 150 A and C, showing the same range.



M.R.P. 320

n Squares.

1/3872

depend,
least a
visibil

Date

May 1
16
June 1
16
July 1
16
Aug. 1
16
31

In
at 2400
the maj
and of
times o
regard
very li
the fre
frequen
has be

It
existen
marine
wholly
Cloudin
and fog
set out

Marsden

Mean cloud amount	7.7	8.4	7.7	7.9	7.3	6.6	6.6	6.4	6.3

That it does not reveal any such close correlation may be partly because other factors may be involved, and partly because thick fog itself may be booked as cloud, so that the mean cloud amount is artificially raised, particularly in those areas where fog is most prevalent, and does not represent the likelihood of sunlight falling on the fog top.

All the figures in Table I, except those for 104, which show no trace of diurnal variation, were harmonically analysed for the 24-hour and 12-hour periods. The values obtained are shown in Table IV, the oscillation being expressed as:-

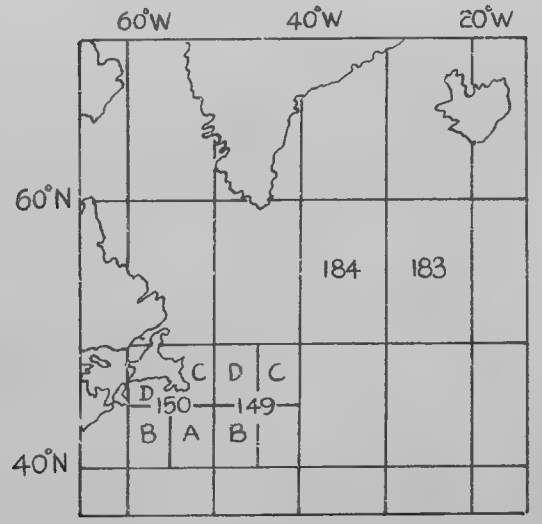
$$a + m \sin (x + M) + n \sin (2x + N)$$

While the zero from which M and N are measured is local midnight.

/TABLE IV

M.R.P. 320

Positions of Marsden Squares.



Shapes of modified Marsden Squares.

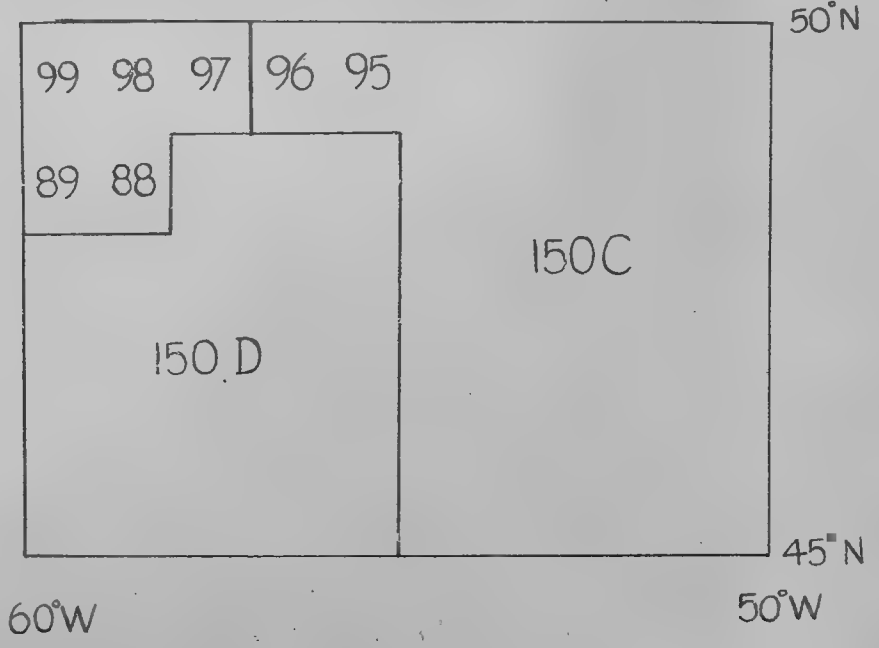
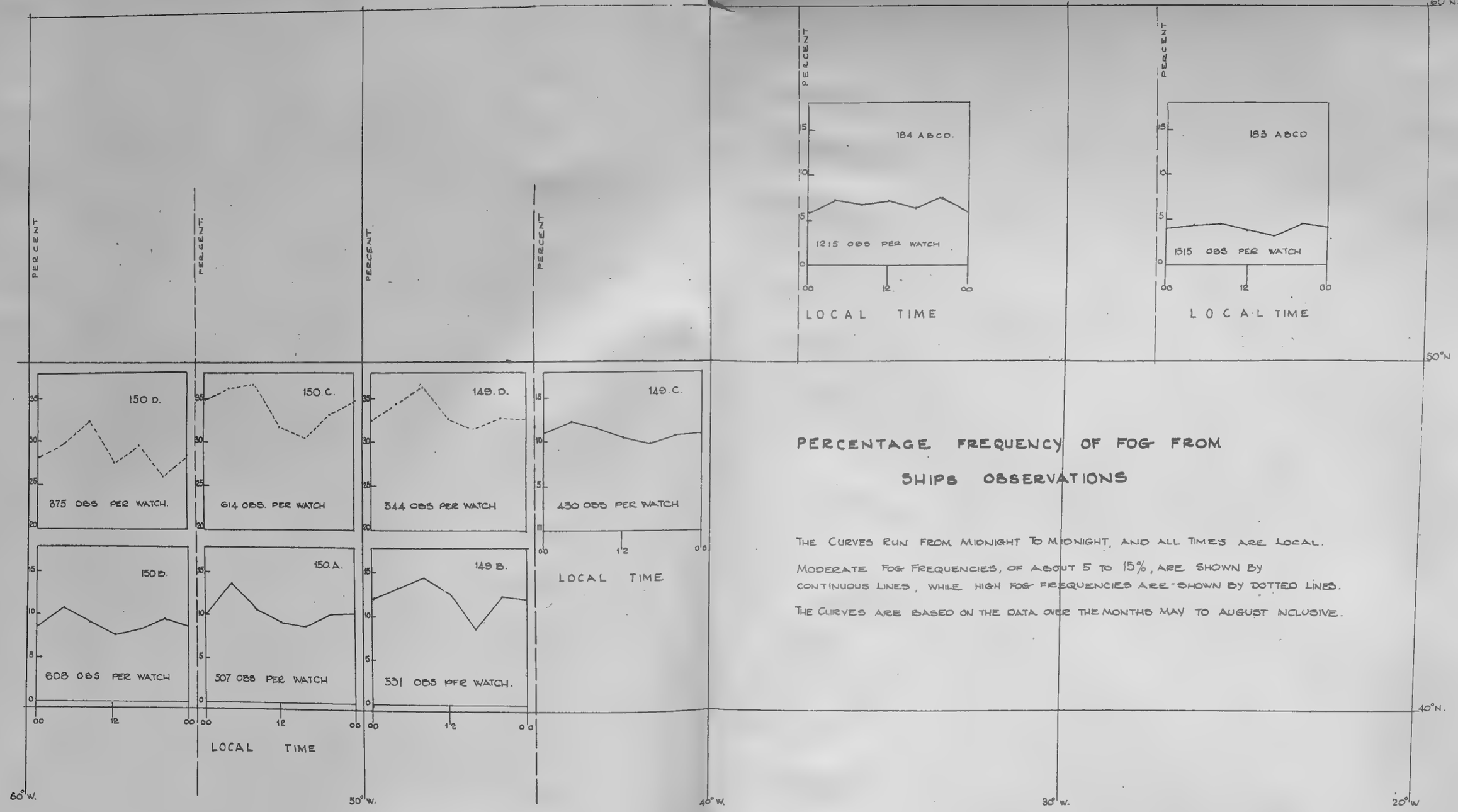


FIG 1

1/2872



PERCENTAGE FREQUENCY OF FOG FROM SHIPS OBSERVATIONS

THE CURVES RUN FROM MIDNIGHT TO MIDNIGHT, AND ALL TIMES ARE LOCAL.

MODERATE FOG FREQUENCIES, OF ABOUT 5 TO 15%, ARE SHOWN BY CONTINUOUS LINES, WHILE HIGH FOG FREQUENCIES ARE SHOWN BY DOTTED LINES.

THE CURVES ARE BASED ON THE DATA OVER THE MONTHS MAY TO AUGUST INCLUSIVE.

FIG. 2.

1/2872

Date	Time	Station	Top of Ascent if below 200 mb.
July 3	0000	LER	
"	0606	BCN	400
"	1148	ALD	400
"	1200	LIV	
"	1200	PEN	
"	1754	ALD	400
July 4	0000	LIV	
"	0000	PEN	
"	0600	LAR	
"	1200	BCN	400
"	1200	LAR	200
"	1200	PEN	
"	1800	ALD	400
July 5	0000	DMK	
"	0100	LIV	
"	0600	LAR	200
"	1200	BCN	400
"	1200	LAR	200
July 9	0600	ALD	400
"	1154	ALD	400
"	1200	LIV	
"	1800	LER	200
July 14	0000	LIV	
"	0000	PEN	360
"	0600	LAR	
July 16	0000	DMK	200
"	1200	PEN	
"	1748	ALD	400
July 18	0548	ALD	400
"	0600	BCN	400
"	1148	ALD	400
"	1200	BCN	400
"	1742	ALD	400
"	1800	LER	
"	1806	BCN	400
July 19	0000	DMK	
"	0600	BCN	400
"	1148	ALD	400
"	1800	LIV	
July 25	0000	PEN	
"	1748	ALD	400
July 26	0000	PEN	
"	0600	LAR	
"	1200	ALD	400
"	1200	LAR	
"	1740	ALD	400
"	1800	LER	
July 27	0548	ALD	400
"	0600	BCN	400
"	1148	ALD	400
"	1200	PEN	
"	1800	LER	
July 29	0000	PEN	
"	0600	ALD	400
"	0600	LIV	
"	1200	ALD	400
"	1200	BCN	400
"	1200	LIV	
"	1800	BCN	400

/Aug. 3

Date	Time	Station	Top of Ascen if below 200 mb.
Aug. 3	0000	DMK	350
"	0000	LAR	
"	0000	LIV	
"	0600	LAR	
"	0600	LIV	
"	1200	BCN	400
"	1200	LAR	
"	1200	LIV	
"	1200	PEN	
"	1748	ALD	400
"	1748	BCN	400
Aug. 4	0000	LIV	200
"	0000	PEN	
"	0100	DMK	200
"	0600	BCN	400
"	0600	LAR	
"	1154	BCN	400
"	1200	LAR	
"	1800	BCN	400
Aug. 5	0000	LAR	200
"	0600	LAR	
"	0600	LER	
"	1200	LAR	200
"	1800	LER	200
Aug. 8	0000	DMK	
"	0000	LAR	280
"	0000	PEN	
"	0600	LAR	
"	1200	ALD	400
"	1200	PEN	
"	1742	ALD	400
Aug. 9	0554	ALD	400
"	0600	BCN	400
"	0600	LAR	
"	1148	ALD	400
"	1200	LAR	
"	1800	ALD	400
"	1800	BCN	400
"	1800	LAR	
Aug. 10	0000	DMK	
"	0000	LIV	
"	0000	PEN	
"	0554	ALD	400
"	0600	LAR	
"	0600	LIV	
"	1200	LIV	
"	1200	PEN	
"	1300	LAR	
"	1800	BCN	400
"	1800	LAR	
"	2100	BCN	400
Aug. 11	0000	DMK	
"	0000	LAR	
"	0000	LIV	
"	1200	LAR	
"	1200	PEN	
"	1800	LAR	

AIR WEATHER SERVICE
[illegible] REFERENCE FILES

This report has been published ... Its CONFIDENTIAL ... contents must not be quoted.

TECHNICAL REFERENCE FILE

AIR MINISTRY

M.R.P. 328

METEOROLOGICAL RESEARCH COMMITTEE

17th February, 1947

A Review of Progress in the Development of some Meteorological Instruments

(by the Staff of the Instruments Branch)

AIR WEATHER SERVICE TECHNICAL REFERENCE FILES

1. The following notes summarise the progress, from the commencement, and the present position of the more important development work recently undertaken in or for the Meteorological Office. Development work is also in progress in the Instruments Branch on a number of other projects including sea temperature apparatus, equipment for the determination of the size distribution of raindrops from aircraft, intensity of rainfall recorders, light-weight batteries for radio-sondes, balloon psychrometer, precision aneroid barometer and improvements to the mercury barometer, the ventilated psychrometer and pens for recording instruments.

2. Frost Point Hygrometers

These instruments were developed by Dr. Dobson and Mr. Brewer for use in aircraft to measure the humidity of the upper atmosphere. They are the only instruments available for this purpose, as aircraft psychrometers cannot be used at temperatures much below the freezing point. Their use led to the discovery in 1943 of the extremely low humidity in the stratosphere. There have been four stages of development, as described below, but the same general principle has been employed throughout. Air from a scoop outside the aircraft is directed as a fine jet on to a polished black metal surface forming the top of an inverted thimble. A jet of cold liquid is directed from below into the interior of the thimble, so forming a deposit of dew or frost on the top furface. The temperature of the thimble is measured electrically and is adjusted by varying the jet of cooling liquid until the deposit neither increases nor decreases. This gives the dew or frost point of the air.

(a) Aircraft Frost Point Hygrometer Mk. I. This was described in MRP 126 (13.1.43), and several instruments of this type were made by Mr. Brewer at Boscombe Down. The thimble was cooled by a jet of petrol or alcohol, surrounded by solid CO_2 in a thermos flask, the jet being controlled by air pressure from a hand operated rubber bulb. Light from a small lamp was concentrated on the thimble by internal reflection in an elliptical piece of perspex sheet (a hole was cut at each focus of the ellipse, one containing the lamp and the other the top of the thimble). The temperature of the thimble was measured by means of several thermocouples embedded in it, connected to a microammeter.

(b) Aircraft Frost Point Hygrometer Mk. II. This was developed during 1944-45, and has now superseded the Mk. I type, from which it differs in a number of details as follows:-

(i) The thimble temperature is measured by measuring the resistance of a length of fine platinum wire wound round the skirt of the thimble. This is done by means of a Wheatstone Bridge indicator mounted on the same base as the hygrometer, its dial being calibrated directly in °F.

(ii) The same Wheatstone Bridge can be used to measure the temperature of the air by means of a platinum resistance element mounted outside the aircraft.

(iii) The rubber bulb is replaced by a lever operated plunger pump.

(iv) A small electric heater may be used to heat the thimble if required to remove a deposit too thick to use.

(v) The brightness of the lamp can be varied.

(vi) The ellipse is of glass instead of perspex, because perspex could hold and give out a certain amount of moisture.

/70

70 of these instruments have been made by the Pullin Optical Co. and are being fitted to Meteorological aircraft. They are illustrated in Figs. 1, 2 and 3.

(c) Aircraft Frost Point Hygrometer (Pressurised) Mk. IIA. This was designed during 1945 for use at high altitudes in pressure-cabin aircraft. It is essentially a simplified version of the Mk. II type, rearranged so as to fit inside an airtight metal cylinder through which pass the air inlet and outlet tubes, the electrical connections, and the handle for operating the pump. Since the very low frost points are found in the stratosphere the thimble is cooled by a jet of liquid oxygen from the pump. The deposit is observed through a lens system in the top of the cylinder, and the thimble temperature measured by a platinum resistance thermometer, as in the Mk. II, though the bridge has to be mounted separately. The thimble heater and lamp dimming switch have been omitted.

20 of these instruments have been made by the Pullin Optical Co.

(d) Aircraft Frost Point Hygrometer Photo-electric Mk. I. This was developed in 1944-45 and is described in MRP's 212 and 235. It is an advance on previous types in that the deposit of dew or frost controls the amount of light falling on a photocell, which is connected to a simple amplifier and a microammeter. The temperature of the thimble is adjusted to give a steady reading on the microammeter, so eliminating the uncertainty in judging by eye whether the deposit is increasing or decreasing.

The instrument may be remotely controlled, e.g. from inside a pressure cabin, and being designed for high altitude work uses liquid oxygen as coolant. The thimble is illuminated by a beam of light at an angle of about 45 degrees, and the directly reflected beam is absorbed in a light trap. The photocell, mounted just above the thimble, is illuminated by scattered light whenever there is a deposit on the thimble.

In the experimental models used hitherto (illustrated in Fig. 4) the thimble temperature has been measured by thermocouples connected to a microammeter. In the production models (six of which are now being made by the Baldwin Instrument Co.) a length of platinum wire wound round the thimble is connected to a Sangamo Weston Ratiometer, a direct reading resistance indicating device. It is expected that a first sample instrument will be received from Baldwins in the near future.

(e) Future Developments. It is hoped that the photo-electric model will be developed into an automatic instrument in which the density of the deposit controls the temperature of the thimble. By this means the thimble will be kept always at the frost point temperature, and the time taken for each observation will be much reduced. A further step forward would be to record the frost point by an autographic method. (J.R. Bibby and G.E.J. Hartley, in co-operation with Dr. Dobson).

4. Photo-electric Visibility Meter

(a) Requirements. The need arose in 1943 for a reliable means of measuring the visibility at night, using a photo-electric cell instead of the human eye. Hitherto the only instrument available for this purpose has been the Gold Visibility Meter, in which a distant light is viewed through a dark glass whose opacity can be varied until the light is judged to be just invisible. This involves errors due to the varying sensitivity of the human eye. To avoid such errors the method was adopted of measuring the apparant brightness of a distant light by means of a photo-electric cell.

If such a measurement is made on a very clear night, and then repeated on a night with poor visibility, the latter value will be less, and will enable the extinction coefficient of the atmosphere on the second night to be calculated.

The relevant formula is

$$I = I_0 \, d^{-2} \, e^{-\mu d}$$

/where

where I is the measured intensity of the night in question, I_o the intensity on a perfectly clear night at unit distance, d the distance from lamp to photocell, and μ the extinction coefficient.

From this measurement, the equivalent daylight visibility can be calculated from Koschmieder's formula

$$\mu V = 3.91$$

where V is the visibility. This formula can be deduced theoretically by making certain assumptions, all of which are usually fulfilled in practice to a sufficient degree of accuracy.

(b) Experimental work. The early experimental work by G.E.W. Atkins is described in M.R.P.'s 157 and 226, and it led him to recommend the following apparatus: A source of light consisting of a 12v and 36w lamp in a 19 cm parabolic reflector; a photocell of the EEL selenium rectifier type, 67 mms diameter, mounted with a 10 cms diameter lens to concentrate the light on it; the photocell to be connected to a Tinsley portable galvanometer of 100 ohms. resistance giving full scale deflection (10 cms) for about 4 microampers. The sensitivity of this arrangement is such that the light and photocell should be about 300 metres apart.

(c) Trials. Apparatus on these lines was tested at an operational station during 1944. By cutting down the amount of stray light reaching the photocell, it was possible to measure the extinction coefficient in weak daylight, and so to compare the measured visibility with that observed by eye in the usual manner. In addition, comparisons between the photo-electric and the Gold visibility meters were made during darkness. The trials are described by J.R. Bibby in M.R.P. 236, the chief conclusions being:

(i) Providing a light source of sufficient constancy can be obtained, this is a reliable method of measuring the visibility at night, or in weak daylight. It covers a range of visibilities from about 300 yards to 5 miles with errors little, if any, greater than those encountered in estimating visibility in daylight.

(ii) Koschmieder's formula is verified, to the degree of accuracy possible in estimating the visibility by eye.

(iii) The Gold visibility meter was in good agreement above about 2000 yards, but for lower visibilities appeared to give values about 30% too high. (This is partly due to the different relation between μ and V which Gold assumed.)

(d) Present Position. The present design is similar to that described above, with the following modifications:

(1) The lamp is fed from a constant voltage transformer, instead of the accumulators previously used.

(2) A small auxiliary photocell is attached in front of the lamp, in order to allow for any variations in its brightness. This allowance is made by adjusting the sensitivity of the galvanometer by a variable high resistance in parallel with it.

Twenty outfits of this type are in course of production but a small modification is being introduced so as to incorporate various accessories such as switches, resistances and terminal blocks with the galvanometer in a complete unit. (J.R. Bibby.)

4. Radar Wind Equipment

(a) Ground Equipment. The use of the GLIII radar set for the measurement of upper winds became well established during the war. It was, however, limited in range and in strong winds measurements were restricted to heights below those required for high altitude flying. The first improvement introduced was the fitting of a range doubler to the standard set. An experimental model

designed in the Instruments Branch gave very successful results and a production model was then developed by R.R.D.E. The range doubler, combined with the use of a high power magnetron, increases the slant range of the set from 36000 to 66000 yards, which with balloon-borne reflectors ascending at their normal rate of 1350 ft./min. allows winds to be determined up to 50,000 feet when the average wind does not exceed 60 m.p.h.

(b) Balloons. The next step was to increase the rate of ascent of the balloon reflectors. This could be done either by increasing the free lift of the balloon or by decreasing the drag of the reflector. Increasing the free lift is no use, however, unless a larger balloon is used, otherwise the height of burst is correspondingly reduced. Experiments are in hand to produce a much larger balloon than those at present available, but a more promising line has been found in the development of a special balloon of normal size with two necks at opposite ends of a diameter, which can be used in tandem with a second balloon of the same size. Preliminary trials with pairs of such balloons carrying radio sondes indicate that heights exceeding 70,000 feet can be readily obtained.

(c) Reflectors. To obtain a reflector with less drag than the existing type (fig.5) which consists of metallised paper on a wooden framework, the use of gauze instead of paper is being tried. The main essentials are that the gauze should be a good reflector of 10 cm. waves, which means that it should have a low electrical resistance and not too open a mesh and that it should be capable of being stretched into a flat plane without sagging appreciably under its own weight or under moderate wind pressure. It was decided to increase the size of the reflector from 3 feet to 4 feet 6 inches partly to compensate for the slight loss to reflecting power as compared with that of metallised paper and partly to increase the range of detection. It was also decided to use a collapsible duralumin tube framework instead of wood so as to enable the reflector to fold into a smaller package. Attempts to obtain a reflector made with knitted wire gauze have so far been unfruitful but an alternative design using nylon net plated by a patent process (Calendar-Suchy) has been evolved. Some experimental models have been flown recently and provided adequate tensioning of the gauze planes is ensured the design meets the requirements. It is proposed to arrange a production contract in the near future so that trials can be carried out on an operational basis. (R.J. Scrase).

5. Radiosonde Theodolite and Microwave Transmitter

(a) Requirements. Following the experience obtained in trials with G.L.III equipment used for radio wind measurement in the summer of 1943, it was decided to develop a 'radio theodolite' purely for meteorological wind-finding purposes. It was also planned to incorporate, at a later date, an improved type of meteorological unit to form a complete radiosonde equipment capable of a high degree of accuracy. The maximum permissible vector error in wind velocity was specified as 2.7 knots and the maximum range of the equipment as at least 100 miles.

It was agreed that the required order of accuracy could be achieved only by employing centimetre wavelength transmission and that in order to obtain a reasonable size ground aerial system the wavelength should not be greater than 10 cms and, if possible, even shorter.

(b) Early Experiments. A contract was therefore placed with Messrs. B.T.H. in the early part of 1944 for the development of this equipment and, in the first instance it was decided to carry out experiments on a wavelength of 3 centimetres. An experimental ground receiver was built and flight trials with a transmitter mounted on an aircraft were carried out in the summer of 1944. These trials showed that with our present knowledge of valve design, it was impracticable to employ a wavelength of 3 cms and consequently the next stage of development was planned for 10 centimetres.

A modified G.L.III trailer was used as the ground equipment and, due to the reduction in the D/F accuracy when using longer wavelengths, it was decided to incorporate some form of ranging device to reduce the vector error in wind velocity at greater ranges. The system finally chosen employed

/sinusoidal

sinusoidal modulation of the carrier frequency, the range being measured as a phase change; this system was developed at the N.P.L.

(c) Trials. Trials on this equipment were carried out at T.R.E. over the period 1945/6 and in the summer of 1946 complete details of the performance of the equipment were compiled. The results of these trials was, briefly, that whereas the accuracy requirements could be fulfilled, the maximum range of the equipment was less than 60 miles. Difficulty was also experienced in modulating the lightweight balloon transmitting valve and the design was thus unsuitable for production. The experimental model is shown in Fig. 6.

(d) Modified Design. The whole system was then reconsidered in the light of previous experience and the following design evolved at T.R.E. The balloon transmitter is quiescent normally and is triggered from a ground transmitter radiating 2 microsecond pulses 500 times per second on a frequency of 152 Mc/s. These pulses are picked up by a small receiver in the balloon equipment and after detection are used to set the centimetre valve oscillating. The ground receiving aerial consists of a parabolic mirror and direction finding will be obtained in the normal manner. Due to various mechanical difficulties with this type of aerial it may be advisable later to employ a 'lens' type, development of which has been started by the War Office for another purpose. Ranging will be done by the usual radar method of measuring the time interval between the radiated 152 M/cs and the received centimetre pulses. The various contacts of the chronometric meteorological unit will vary the widths of the transmitted pulse and the different width pulses will be channelized in the ground receiver and used to operate the meteorological recorder unit.

The experimental model of this latest design is nearing completion and tests on its performance will take place, it is hoped within the next few months (E.J. Merrer).

C. The Development of a Chronometric Radio-Sonde

(a) Requirements. At a meeting in December 1943, the requirements of the forecasting branches of the Meteorological Office for a precision radio-sonde were formulated as follows:-

Probable error in temperature	±	.5° F throughout the range + 100° F to - 120° F.
" " " "	±	2 mb from 1000 mb to 300 mb
" " " "	±	1 mb from 300 mb to 30 mb

The existing Kew pattern radio sonde fell far short of these accuracy limits, with probable errors of ± 2° F and 5 mb. No statement was made about the accuracy of humidity readings, as the limitations here are in the element itself (gold-beaters' skin) rather than in the telemetering system.

(b) Programme. The various types of radio-sondes were reviewed by Mr. Dymond, and it was concluded that the most hopeful line of attack would be a chronometric system on the lines of the Canadian instrument. The most serious objection to the Canadian radio-sonde was that any variation in the clock rate during a complete cycle of the helix (about 15 seconds) would introduce errors. It was decided to obviate this by generating synchronising pulses at a rate of about 33 per second which could be used to ensure absolute synchronism between the helix rotation and the ground recorder. A development contract was placed with Messrs. Kelvin, Bottomley and Baird in August 1944, calling for 6 chronometric units and a recorder. As the programme was regarded as part of the Radio-Sonde-Theodolite programme, the units were minus radio transmitter, the intention being to use them to modulate the microwave transmitter of the RST. The units and recorder were delivered in October 1946, and arrangements are now being made to test them as complete radio-sondes by attaching them to a simple 27 Mc/s transmitter.

(c) Chronometric Unit. The chronometric unit is illustrated in Figs. 7 and 8. The elements are a "Hiflex" bimetallic strip, a pair of phosphor-bronze aneroid capsules, and a strip of gold-beaters skin. Each element operates an arm at the end of which is a small spring contact, similar in principle to the

/contact

contact on the Canadian radio-sonde, its object being to make the effective duration of contact between the arm and the helix only about $\frac{1}{8}$th second. The helix is a raised ridge on a cylindrical former, one side being insulated to prevent any spurious contacts due to bounce of the spring contact as it is released by the helix. The helix is driven at a rate of about 4 revolutions a minutes by a small electrical motor, which also drives a device which produces two independent series of pulses, one of 2 and the other of 7 milliseconds duration, each at a recurrence rate of 33 a second. The electrical circuit is such that when no contact is made the 2 millisecond pulses are passed to the transmitter, whereas when a meteorological contact is made the 7 millisecond pulses are passed. There is also a fixed reference contact which indicates the start of each revolution of the helix.

(d) Recorder. The ground equipment consists of a suitable receiver (either 27 Mc/s or microwave, as the case may be), pulse cleaning filter and pre-amplifier, pulse width discriminator, recorder drive mechanism, and recorder. The actual recording mechanism is a set of four radial arms with a fine electrical contact at the end. The arms are driven by a motor synchronised to the 33 c/s input and sweep across the Teledeltos recording paper which is driven by another motor. The marker voltage for the Teledeltos paper varies according to the pulse width. Synchronising pulses are recorded as light spots and meteorological pulses as dark spots. The reference contacts are also recorded as dark spots and if the synchronism is good they will all lie in a straight line down the edge of the paper. The record is measured with the aid of a curved scale, the distance along the curve from the corresponding reference mark to the appropriate meteorological mark being a measure of the relevant meteorological parameter.

(e) Future Programme. It will be seen that the chronometric unit can be readily converted into a complete 27 Mc/s radio-sonde and 200 such complete instruments are on order. This is a possible replacement as it stands for the present Mk.II Kew radio-sonde. Recent changes in the main radio-sonde and Teledeltos programme, described above, make possible some changes in design in the chronometric unit when used as part of the microwave radio-sonde - in the new PST, pulses are transmitted from the ground and it will not be necessary to generate both the short and long pulses in the chronometric unit. The chronometric unit is then at the parting of the ways and may in future appear in two significantly different forms. (C.E. Ashford)

7. Daytime Cloud Height Meter

(a) Following reports received from America in the early part of 1943, it was decided to develop in this country an equipment for determining the height of a cloud base. This equipment was primarily to be used at airfields but would probably be of use in normal meteorological observation.

Two methods were considered viz (1) an equipment similar to the normal night cloud searchlight and alidade but using a sinusoidally modulated light beam as source and a photocell mounted at the focus of a telescope as the receiver in place of the observer's eye. (2) A radar method employing narrow pulses of light and an electronic means of measuring the time interval between the transmitted pulse and the received pulse after reflection from the cloud.

(b) Modulated Light Method. In this method a 1,000 watt high pressure mercury vapour lamp is used as the source and it was found that when energized from a 400 c/s supply the depth of modulation was approximately 78%. Several types of photocell circuits and associated amplifiers were designed but considerable difficulty was experienced in reducing the level of the noise due to the high background brilliance. Ranges of the order of 1000 feet were obtained from fixed objects against bright skies. Work on this design has almost ceased, however, since more promising results have been obtained from the second line of development.

/(c)

(c) Pulsed Light Method. In this method, which is being developed at T.R.E., narrow pulses of light are produced by the discharge of a condenser across a spark gap at the focus of a 20 inch mirror. Due to the very narrow pulse width (less than 1 microsecond) peak powers of the order of several megawatts may be obtained. The receiver consists of a similar mirror with a vacuum photocell mounted at its focus and the received pulse is then fed to electronic circuits similar to those employed in normal radar. Ranges of up to 20,000 feet on thin wisps of cloud have been obtained and operational trials are now awaiting completion of the second model of this equipment. At the present moment the minimum range at which the equipment will operate is 300 feet and further work is required to meet the meteorological requirement of 400 feet or less. (H.J. Ferror.)

G. 148459

PROTOTYPE PHOTO-ELECTRIC FROST POINT HYGROMETER
Mk I

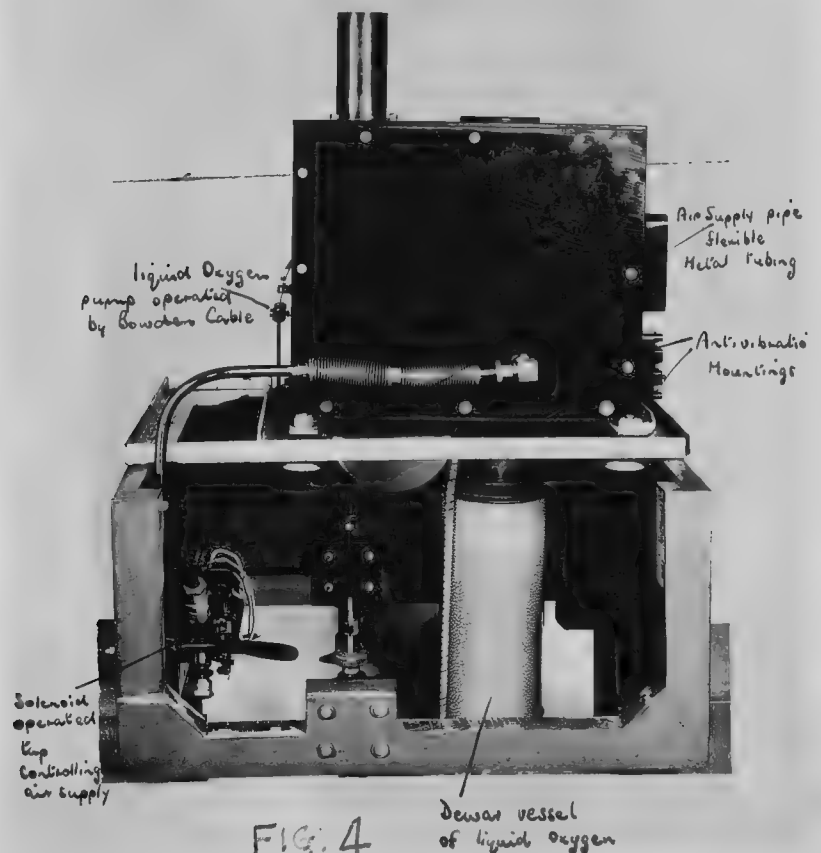


FIG. 4

[illegible] the M.R.II for radio-sonde. Recent changes in the Kelvin radio-sonde and theodolite programme, described above, make possible some changes in design in the chronometric unit when used as part of the microwave radio-sonde - in the new MSF, pulses are transmitted from the ground and it will not be necessary to generate both the short and long pulses in the chronometric unit. The chronometric unit is then at the parting of the ways and may in future appear in two significantly different forms. (C.L. Ashford)

7. Daytime Cloud Height Meter

(a) Following reports received from America in the early part of 1943, it was decided to develop in this country an equipment for determining the height of a cloud base. This equipment was primarily to be used at airfields but would probably be of use in normal meteorological observation.

Two methods were considered viz (1) an equipment similar to the normal night cloud searchlight and alidade but using a sinusoidally modulated light source as source and a photocell mounted at the focus of a telescope as the receiver in place of the observer's eye. (2) A radar method employing narrow pulses of light and an electronic means of measuring the time interval between the transmitted pulse and the received pulse after reflection from the cloud.

(b) Modulated Light Method. In this method a 1,000 watt high pressure mercury vapour lamp is used as the source and it was found that when energized from a 400 c/s supply the depth of modulation was approximately 70%. Several types of photocell circuits and associated amplifiers were designed but considerable difficulty was experienced in reducing the level of the noise due to the high background brilliance. Ranges of the order of 1000 feet were obtained from fixed objects against bright skies. Work on this design has almost ceased, however, since more promising results have been obtained from the second line of development.

/(c)

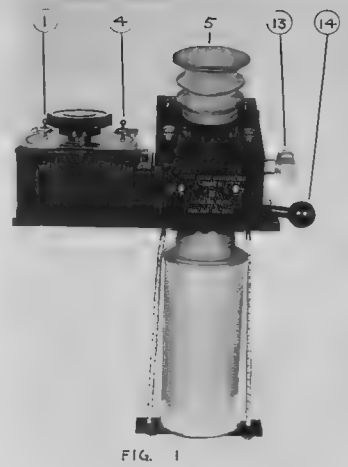


FIG. 1

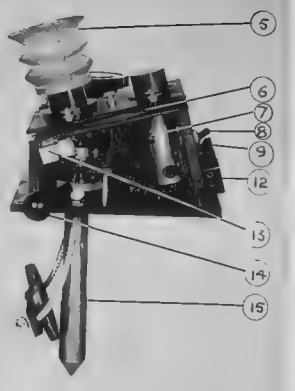


FIG. 2.

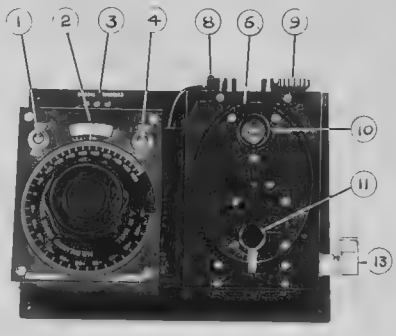


FIG. 3

PROTOTYPE PHOTO-ELECTRIC FROST POINT HYGROMETER
Mk I

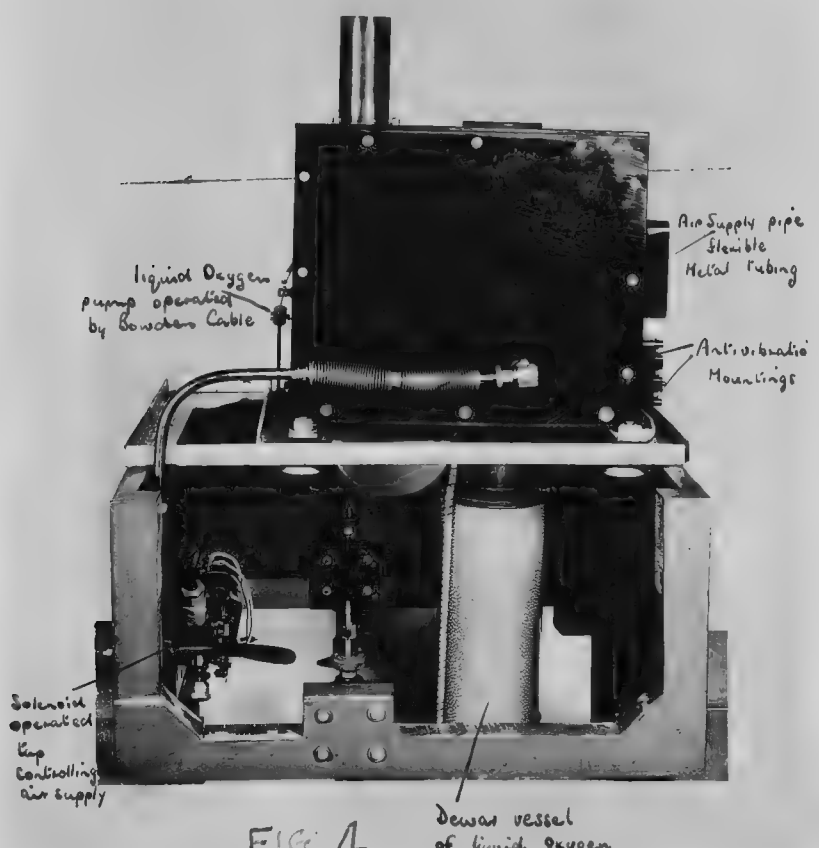


FIG. 4

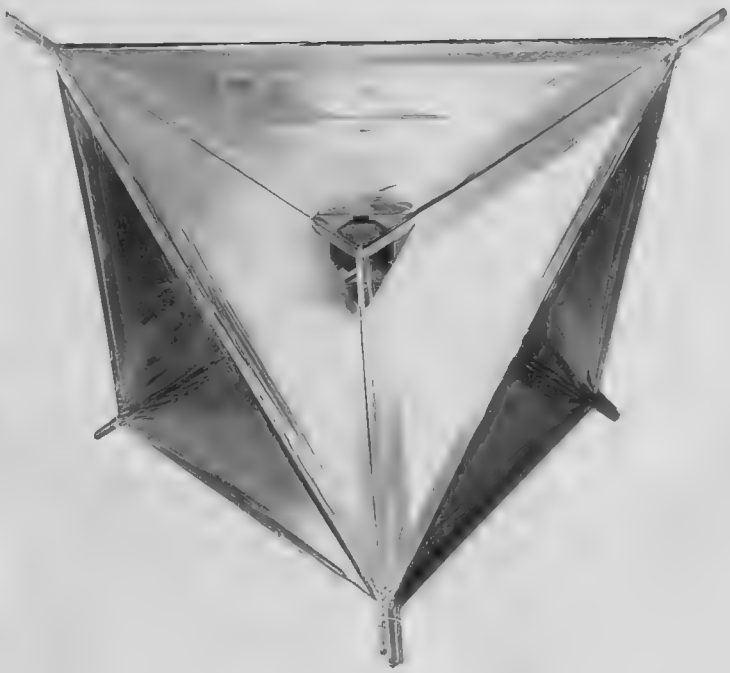


FIG.5. RADAR REFLECTOR.

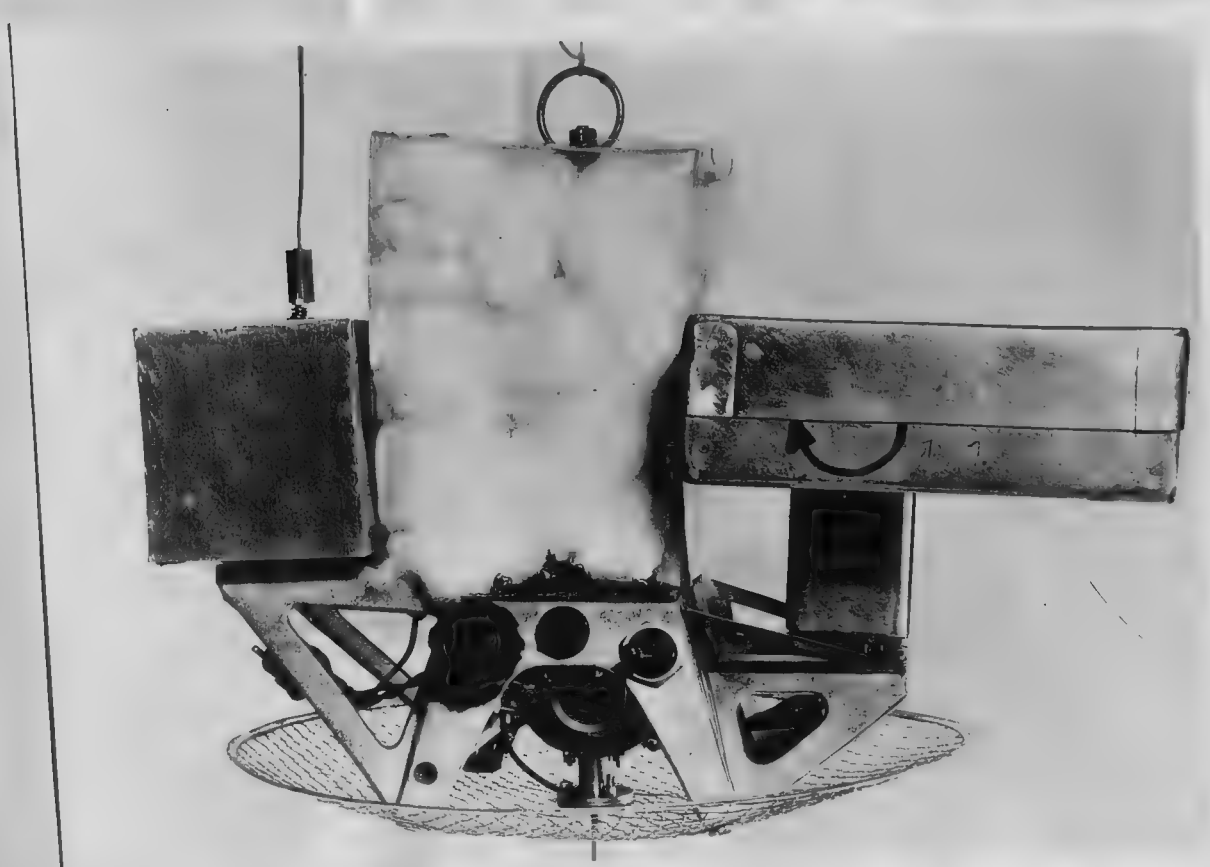


Fig.6. EXPERIMENTAL MICROWAVE RADIOSONDE.

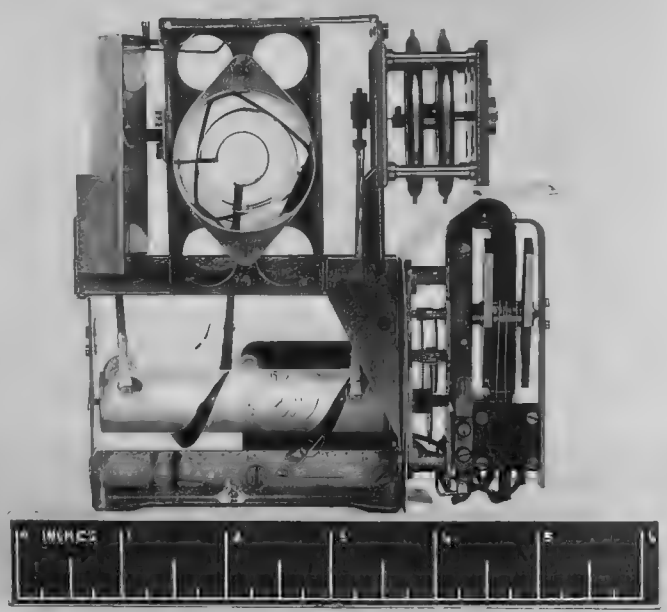


FIG.7. CHRONOMETRIC UNIT.

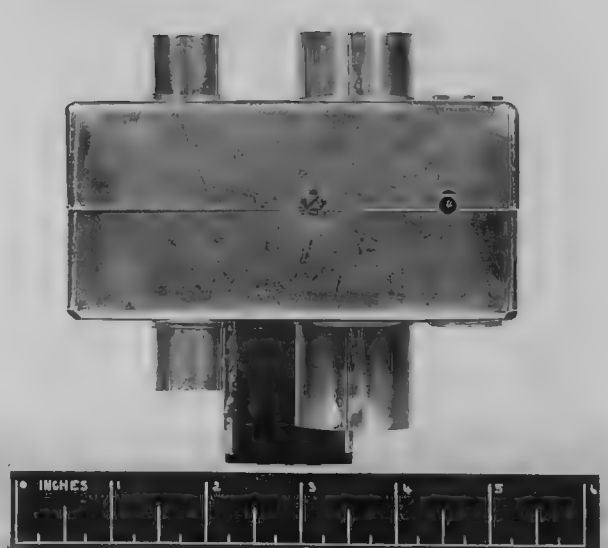


FIG.8. CHRONOMETRIC UNIT.

MRP 329

Prob. No.	Rcmdn. No.	Recommendation.	Rcmd. Priority.
B.13	9	That consideration of problem 13 to deferred until results from the Orlando investigation in the U.S.A. are available.	-
B.14	10	That consideration of nocturnal spreading of thunderstorms from hills to plains be deferred pending publication of results of a similar investigation in India.	-
	11	That Prof. Brunt be asked to supervise a critical review of the literature relevant to the problem of whether radiation from cloud tops affects the lapse rate sufficiently to affect stability and cause night precipitation.	P.1.
B.15	12	That Mr. Swinbank be asked to report on the problem of turbulent and radiative cooling of the lower layers of the atmosphere at night, and that Mr. John or some other officer with tropical experience be asked to investigate the forecasting of early morning fog and low stratus cloud in tropical regions.	P.2.
B.16	13	That further consideration of the effects of advection and divergence on the upper air temperature be deferred until problems 8 and 10 are settled.	-
C.17	14	That these calculations should be applied over wide areas but not to particular convergence zones until the number of observational stations is increased and the method of observation improved in accuracy. This work should be linked up with the work envisaged in recommendation 4.	P.1.
C.18	15	That it should be suggested to the Meteorological Department of the University of Chicago to prepare a comprehensive review and discussion of easterly waves and perturbations in tropical areas around the world in consultation with the other Meteorological Authorities directly concerned.	-
C.19	16	That the problem of the intrusion of the polar front into the tropics should be examined simultaneously with the problem of perturbations. (Recommendation 15)	-
	17	That the Director of the East Africa Meteorological Service be asked to prepare a memorandum formulating the problem of pressure surges as it affects East Africa. On receipt of this memorandum consideration can then be given to the question of whether the problem is suitable for investigation at a University.	
	18	That the causes and effects of large scale pressure surges, i.e. wide scale simultaneous time variations of pressure, should be examined in the Meteorological Office.	P.1.
C.20 and C.21	19	That the Weather Bureau, U.S.A. and the Meteorological Department, University of Chicago, be asked if they have any investigations in hand related to these two problems.	-

/3.

This report has not yet been published and is to be treated CONFIDENTIALLY. Its contents must not be quoted.

M.R.P. 329.

20th February, 1947.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

Report of the Tropical Meteorology Panel

1. Introduction. The Panel was constituted under minute 583 of the Meteorological Research Committee to recommend the best procedure to obtain solutions to the problems enumerated in M.R.P. 311. Two meetings of the Panel have been held, on 22nd January and 19th February 1947, at which these problems were considered.

A related paper, M.R.P. 326, was also considered.

The Panel's recommendations are submitted below.

The problem numbers quoted in the recommendations below refer to M.R.P. 311. This particular paper is an abstract from the larger paper M.R.P. 318.

2. Recommendations.

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

Prob. No.	Rcmdn. No.	Recommendation.	Rcmd. Priority.
A.1	1	That the M.R.C. should ask the various Meteorological Authorities who are directly interested in tropical areas whether they could undertake an investigation on the lines of Gold's "Barometric Gradient and Wind Force" for low latitudes.	-
	2	That the investigation described by Dr. H. Jameson in M.R.P. 279 should be extended in the Marine Branch of the Meteorological Office to include other areas and seasons.	P.2.
A.2 to A.7	3	That Sir Geoffrey Taylor be asked to supervise the investigation of problems 2 to 7 by a research worker at Cambridge University.	P.1.
A.8	4	That the methods of mapping the field of convergence and and divergence as set out in paragraph 18 of M.R.P. 318 be tried for a limited number of additional occasions for any tropical area for which observations are available.	P.1.
A.9	5	That consideration of problem 9 be deferred until problems 3, 4 and 8 have been explored.	-
B.10	6	That this is desirable in those areas where the number and accuracy of the observations is adequate.	-
B.11	7	That the effect of high ground, both with regard to the temperature of the upper air and also as a physical barrier to streamlines be investigated in East Africa and also by the research worker mentioned in recommendation 3.	P.2.
B.12	8	That Mr. Stinbank be asked to consider and report upon the effects of convective turbulence on the upper air Temperature distribution.	P.1.

/B.13

Acc. 4251925

Note in connection with M.R.P.329

When this paper was discussed by the Meteorological Research Committee stress was laid upon the need for the direct measurement of vertical velocity in connection with the measurement of convergence. The measurement of vertical velocity is included in the research programme of the committee but such measurements are also highly desirable in other parts of the world.

A.C. BEST
Secretary M.R.C.

G.149265

(A-1)

1.
Re
pr
22

Th

2.

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

Prob. No.	Rcmdn. No.	Recommendation.	Rcmd. Priority.
A.1	1	That the M.R.C. should ask the various Meteorological Authorities who are directly interested in tropical areas whether they could undertake an investigation on the lines of Cold's "Barometric Gradient and Wind Force" for low latitudes.	-
	2	That the investigation described by Dr. H. Jameson in M.R.P. 279 should be extended in the Marine Branch of the Meteorological Office to include other areas and seasons.	P.2.
A.2 to A.7	3	That Sir Geoffrey Taylor be asked to supervise the investigation of problems 2 to 7 by a research worker at Cambridge University.	P.1.
A.8	4	That the methods of mapping the field of convergence and and divergence as set out in paragraph 18 of M.R.P. 318 be tried for a limited number of additional occasions for any tropical area for which observations are available.	P.1.
A.9	5	That consideration of problem 9 be deferred until problems 3, 4 and 8 have been explored.	-
B.10	6	That this is desirable in those areas where the number and accuracy of the observations is adequate.	-
B.11	7	That the effect of high ground, both with regard to the temperature of the upper air and also as a physical barrier to streamlines be investigated in East Africa and also by the research worker mentioned in recommendation 3.	P.2.
B.12	8	That Mr. Swinbank be asked to consider and report upon the effects of convective turbulence on the upper air Temperature distribution.	P.1.

/B.13

A, R P's 330, 331, 334, 338, 341

21st February, 1947.

7885

This report has not yet been published and is to be treated CONFIDENTIALLY. Its contents must not be quoted.

M.R.P. 330.
21st February, 1947.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE.

The Shape of Raindrops and Mode of Disintegration of Large Drops.

M.R. 330
21 Feb 47

by

A.C. Best.

1. Introduction. The shape of raindrops is important in some applications of radar technique. In M.R.P.277 the present writer put forward some speculations regarding the shape of raindrops while falling through air with their terminal velocity and regarding the manner of disintegration of large drops. Since that paper was written some experimental evidence relating to these matters has become available which leads to a modification of the conclusions reached in M.R.P. 277.

2. Data available. In response to a request from the writer some photographs of falling water drops were taken by the Physics Section of the Chemical Defence Experimental Section, Porton. The water drops were formed artificially from tips and were photographed from the side after falling a distance of 35 feet. No attempt was made in this work to determine whether the drops had attained their terminal velocities but previous measurements by C.N. Davies (reported by O.G. Sutton in M.R.P. 40) have shown that the acceleration of water drops after falling a similar distance is very small except in the case of large drops falling through air at reduced pressure.

Porton have emphasised that in the present work the photographs of the largest drops show considerable variations in shape, due presumably to the persistence of oscillations from the time of release. Equilibrium conditions could not, therefore, have been reached and it is possible that the mean of the observed dimensions may depart significantly from the dimensions of a non-oscillating drop. This is not wholly a disadvantage in applying the results to natural raindrops as it is clear that large raindrops are associated with considerable variations in the vertical velocity of the air and it may well be that the natural state of a large raindrop is one of oscillation about a mean shape.

The nominal diameter of the equivalent sphere was obtained by Porton by weighing ten drops.

Enlarged photographs of the falling drops with a magnification of X5, were supplied by Porton and dimensions have been obtained by making measurements of these photographs. The photographs are reproduced in fig. I.

In the following paragraphs the terminal velocities of the drops have been used. These terminal velocities were obtained from M.R.P. 40.

3. The Measured Dimensions. The photographs showed that the drops are not always completely symmetrical about a vertical through the centre. Nevertheless there was little difficulty in selecting a maximum horizontal diameter and a maximum vertical diameter. In the following paragraphs it is assumed that the cross section of the drop above the maximum diameter is half an ellipse and the cross section below the maximum diameter is half of another ellipse.

It will be recalled that in MRP 277 it was assumed that these two ellipses are similar. It was appreciated then that the eccentricity of the lower ellipse is probably greater than that of the upper ellipse but the technique employed in that paper did not permit a distinction being made between the two halves.

With the assumption of a cross section consisting of two semi-ellipses measurements were made from the photographs in accordance with the sketch below.

/Fig. 2.

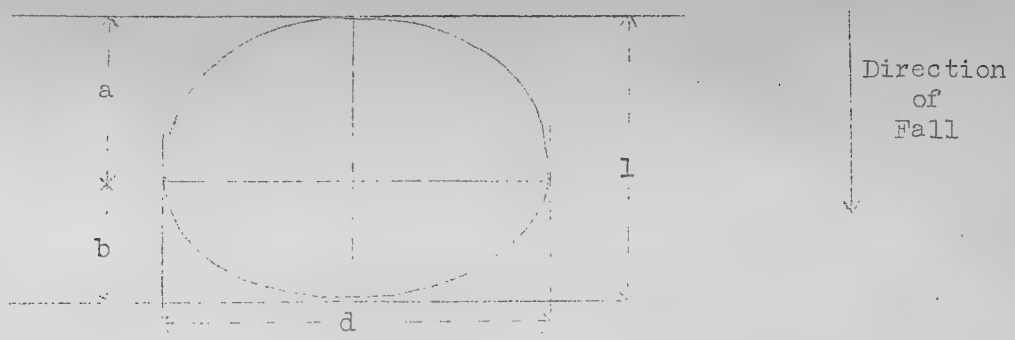


Fig. 2. Notation used for dimensions of drop.

The values of a, b, d & l reduced to "life size" from each individual photograph are tabulated in Table 1. The values of l were not obtained by adding the values of a & b but were obtained from separate measurements. Comparison of the values of l with the values of (a + b) indicates an uncertainty in the measured dimensions of about 0.05 mm. This is in good agreement with the value found by comparing a few measurements made independently by a colleague.

In Table 1 the quantity $D_1 = (l\, d^2)^{\frac{1}{3}}$ has been tabulated. If the shape of the drops is exactly as assumed this should be equal to D, the diameter of the equivalent sphere. Comparison of the values of D & D_1 in Table 1 shows that the discrepancy is never large. The differences may be due to any or all of three factors viz:-

(i) the departure of the shape of the drop from elliptical

(ii) variations in the true size of the drop

(iii) inaccuracy either in the nominal magnification of the photographs (this would be constant for all drops and seems improbable from the values in Table 1) or in making the measurements.

On the whole it seems more consistent to use the parameter D_1 to represent the size of the drop and this will be adopted in the succeeding sections. It is clear, however, that this does not affect the results much.

Table 1.
Dimensions of Drops

Drop	D	a	b	d	l	D_1
	mm	mm	mm	mm	mm	mm
A	2.8	1.42	1.13	2.86	2.55	2.75
B	"	1.40	1.17	2.86	2.55	2.75
C	3.35	1.65	1.32	3.44	2.94	3.26
D	"	1.76	1.23	3.44	2.94	3.26
E	4.0	2.02	1.41	4.34	3.43	4.01
F	"	2.11	1.38	4.34	3.44	4.02
G	4.4	2.18	1.40	4.85	3.59	4.39
H	"	2.23	1.46	4.87	3.68	4.44
I	4.8	2.42	1.75	5.25	4.11	4.84
J	"	2.39	1.40	5.43	3.84	4.84
K	"	2.31	1.37	5.54	3.73	4.87
L	"	2.64	1.52	5.17	4.17	4.81
M	5.2	2.90	1.73	5.45	4.61	5.16
N	"	2.41	1.36	6.39	3.78	5.36
O	"	2.66	1.37	6.35	4.05	5.47
P	"	2.83	1.31	6.03	4.19	5.34
Q	6.4	2.79	1.51	7.38	4.31	6.17
R	"	2.92	1.59	6.91	4.46	5.97
S	"	2.98	1.42	7.24	4.41	6.14
T	"	3.20	1.80	6.98	4.98	6.24

Note. D = nominal diameter of equivalent sphere
$D_1 = (l\, d^2)^{\frac{1}{3}}$

4. The Shape of a Falling Drop

In Table 2 the mean values of D_1, a, b, d/2 and of e_1 and e_2, the eccentricities of the upper and lower semi ellipses respectively are tabulated for each group of drops. Figure 3 shows a, b and d/2 plotted against D_1.

The closeness of the curves for a and d/2 shows that the top half of the drop departs but little from the spherical shape until D_1 reaches a value of about 4 mm. The curve for b emphasises the flattening of the lower half. Beyond D_1 = 4 mm b increases very little.

In M.R.P. 40 O.G. Sutton states that a drop of liquid falls as a rigid sphere provided $\rho d^2 g/T < 0.4$ where ρ is the drop density, d the diameter, g represents gravity and T is the surface tension. Putting $\rho = 1$, g = 981 and T = 75 this condition becomes d < 0.175 cm. It can be seen that the three curves in fig. 3, if extrapolated towards the origin, become indistinquishable in the neighbourhood of D_1 = 1.75 mm.

An interesting feature of Table 2 is the rather remarkable agreement between the values of D_1 / 2 and a. It will be noticed that "a" is very slightly greater than D_1 /2 except for the fourth and the largest drops. The difference in the case of the fourth drop is well within the experimental error and is probably without significance. The relative decrease in "a" for the largest drop is only slightly greater than the experimental error but it is thought that it may be significant. The reasons for this will be given later.

Table 2
Mean Dimensions of Drops

Drops	D_1	a	b	d/2	e_1	e_2
	mm	mm	mm	mm		
A. B	2.75	1.41	1.15	1.43	0.167	0.593
C. D	3.26	1.70	1.27	1.72	0.155	0.675
E. F	4.01	2.06	1.40	2.17	0.310	0.764
G. H	4.42	2.20	1.43	2.43	0.429	0.809
I.J.K.L.	4.84	2.44	1.51	2.67	0.410	0.826
M.N.O.P.	5.33	2.70	1.44	3.02	0.452	0.880
Q.R.S.T.	6.13	2.97	1.58	3.56	0.555	0.907

5. The Drag Co-efficient of a Falling Drop. Since the speculations contained in M.R.P. 277 were largely based upon assumptions regarding the drag co-efficient it is of interest to compute the drag co-efficient from the shape of the drops and the values of terminal velocity given in M.R.P.40. Using D_1 to compute the mass of the drop and assuming

$$\text{Air density} = 1.189 \cdot 10^{-3}$$
$$\text{drop density} = 1$$
$$g = 981$$

we easily get from the usual formula

$$\text{Drag co-efficient} = C_E = 1.1 \cdot 10^6 \cdot \frac{1}{v^2}$$

where v is the terminal velocity. The values of v corresponding to various values of D_1 can be obtained from M.R.P. 40. Assuming a value of 0.15 for the co-efficient of kinematic viscosity of the air Reynolds number R is given by

$$R = 6.67\ vd$$

and the drag co-efficient of the rigid sphere C_s can then be obtained from Table 1 of M.R.P. 277.

Using the sum of "a" and "b" in Table 2 to give 1 the drag co-efficients have been computed in this way and are given in Table 3.

/Comparison

Comparison of these figures with fig. 1 of MRP.277 indicates that inadequate allowance was made for the increased resistance of the deformed drop as compared with the equivalent sphere when constructing that figure.

Table 3.

Drag Co-efficients of a Raindrop.

D_1	l/d	C_E	R	C_s	$\frac{C_E}{C_s}$
mm					
2.75	.895	.487	1470	.386	1.26
3.26	.864	.493	1900	.368	1.34
4.01	.798	.500	2560	.354	1.41
4.42	.746	.501	2940	.348	1.44
4.84	.740	.534	3260	.344	1.55
5.33	.685	.549	3720	.341	1.61
6.13	.639	.598	4410	.337	1.78

6. The Mode of Disintegration of Large Drops. The question of the break up of large rain drops was examined in MRP 277. The considerations employed there depended upon the assumed shape of the deformed drop and, in view of the modification to these assumptions now found necessary, the mode of disintegration needs re-examination. Also it now seems possible to describe in more detail the actual process at the moment of break up.

It has been suggested by Ahlborn (Phys. Zeit. Vol. 32, 1931) that the flow of air past a falling raindrop sets up internal circulation and causes the drop to take the shape of a vortex ring. The essential action in such a process can be easily demonstrated by the following simple experiment. If a drop of coloured water is allowed to fall into a glass of clear water from a height of two or three millimetres it will be seen that the coloured drop almost immediately takes the form of a horizontal vortex ring with well marked circulation. The ring sinks slowly to the bottom of the glass, the radius of the ring increasing slowly as it falls. This experiment does not correspond in detail with the case of a raindrop falling through air of course. The frictional forces acting on the surface of the drop may, however, well be of the same order since, although the co-efficient of viscosity of water is approximity 100 times as great as that of air the terminal velocity of a large water drop in air is much greater than the velocity of sinking of the vortex ring in the glass of water. A similar process takes place when a chimney (e.g. of a locomotive) emits a smoke vortex ring.

With these considerations in mind it is suggested that large drops disintegrate for the following reason. The frictional forces acting on the surface of the drop set up a circulation, the internal motion being downwards in the centre of the drop. On drops which are just large enough for circulation to be set up, the surface tension forces are strong enough to prevent an opening forming in the centre of the drop. As larger and larger drops are considered, however, the tendency to form a vortex ring with a free passage for the air through the centre increases. Finally the air does break through the centre and a vortex ring is formed. The rupture of the surface of the drop must, however, be accompanied by violent oscillations of the vortex ring so formed and probably the ring disintegrates immediately into a number of component parts. In this connection the following quotation from M.R.P. 40 is of interest. Having spoken of the effect of circulation on the shape and velocity of drops Sutton writes -

> "Finally, very large drops will accelerate to a critical speed and burst. This phenomenon is very beautiful to watch from above - the drop suddenly flies apart into a circlet of smaller drops possessing quite a remarkable symmetry".

7. Energy of Disintegration. The photographs of the falling drops provide no direct evidence of the momentary existence of the drop in the form of a

/vortex

vortex ring. Several of the photographs of the drops of the two largest sizes showed a flattening of the lower ellipse such as one would expect if a dimple were forming in the lower surface, but the dimple itself could not be seen, even if there, owing to the camera being aimed in a horizontal direction.

It is obvious that the surface tension forces will resist the tendency to form a vortex ring and we may expect that such a ring will not form and therefore the drop will not disintegrate, unless there is a supply of energy available to overcome the surface tension forces. This energy may be supplied by the oscillation of the drop.

In Table 4 the quantities S_E and S_V are tabulated. These are respectively the surface area of the ellipsoidal drop (in square millimetres) computed by the application of the formula in para. 7 of M.R.P. 277 to each half of the drop separately (the notation of M.R.P.277 differs in the use of "a" and "b" from that of the present paper). The quantity S_V is the surface area of a vortex ring having the same volume as the ellipsoidal drop and having the radius of the ring equal to the radius of the cross section, i.e. having no central opening. The difference between the two represents the energy required to form the ring. Now, as pointed out in para. 8 of M.R.P. 277, Rayleigh's formula for the energy of vibration reduces to $E = 5A^2 T$ where A is the amplitude and T the surface tension. Hence by taking the square root of one fifth of the difference $(S_V - S_E)$ we get approximately, the amplitude of the vibration necessary to provide the requisite energy for disintegration. This amplitude (in mm) is shown in the penultimate column of Table 4.

It is probable that the decrease in amplitude for the largest drop is not significant.

The last column of Table 4 shows the difference between the smallest and largest values of d for the drops of each particular group. To provide the energy of break up these figures should be twice as large as the figures in the previous column.

It is to be noted that $(d_1 - d_2)$ does not necessarily give the actual amplitude of oscillation as there is no reason to suppose that photographs were taken at the instant of maximum and minimum displacement. This accounts for the decrease in $(d_1 - d_2)$ for the largest drop. It is, however, clear that the amplitude of vibration increases rapidly as the drop gets bigger and that for drops not much bigger than 6.13 mm diameter the energy of oscillation would be adequate to cause the formation of a vortex ring and hence disintegration of the drop.

Table 4
Surface Area of drop

D_1	S_E	S_V	$(S_V - S_E)/5]^{\frac{1}{2}}$	$(d_1 - d_2)$
mm	mm^2	mm^2	mm	mm
2.75	24.2	26.6	0.69	0.0
3.26	33.9	37.3	0.82	0.0
4.01	51.5	56.4	0.99	0.0
4.42	62.1	68.5	1.13	0.02
4.84	75.0	82.3	1.21	0.37
5.33	92.4	99.9	1.22	0.94
6.13	124.9	132.0	1.19	0.47

8. Summary. Photographs of drops of water of various sizes falling through air have been obtained and from these photographs the dimensions and shape of the drops have been obtained. It is shown that the upper half of the drop retains the hemispherical shape to a close approximation for quite large drops but that the lower half experiences considerable flattening.

Drag co-efficients for drops of various sizes are computed.

It is suggested that when a large raindrop disintegrates it does so because it first forms a vortex ring and almost immediately breaks up.

Enlarged Photographs of Drops of Water Falling Vertically Downwards with their Terminal Velocity

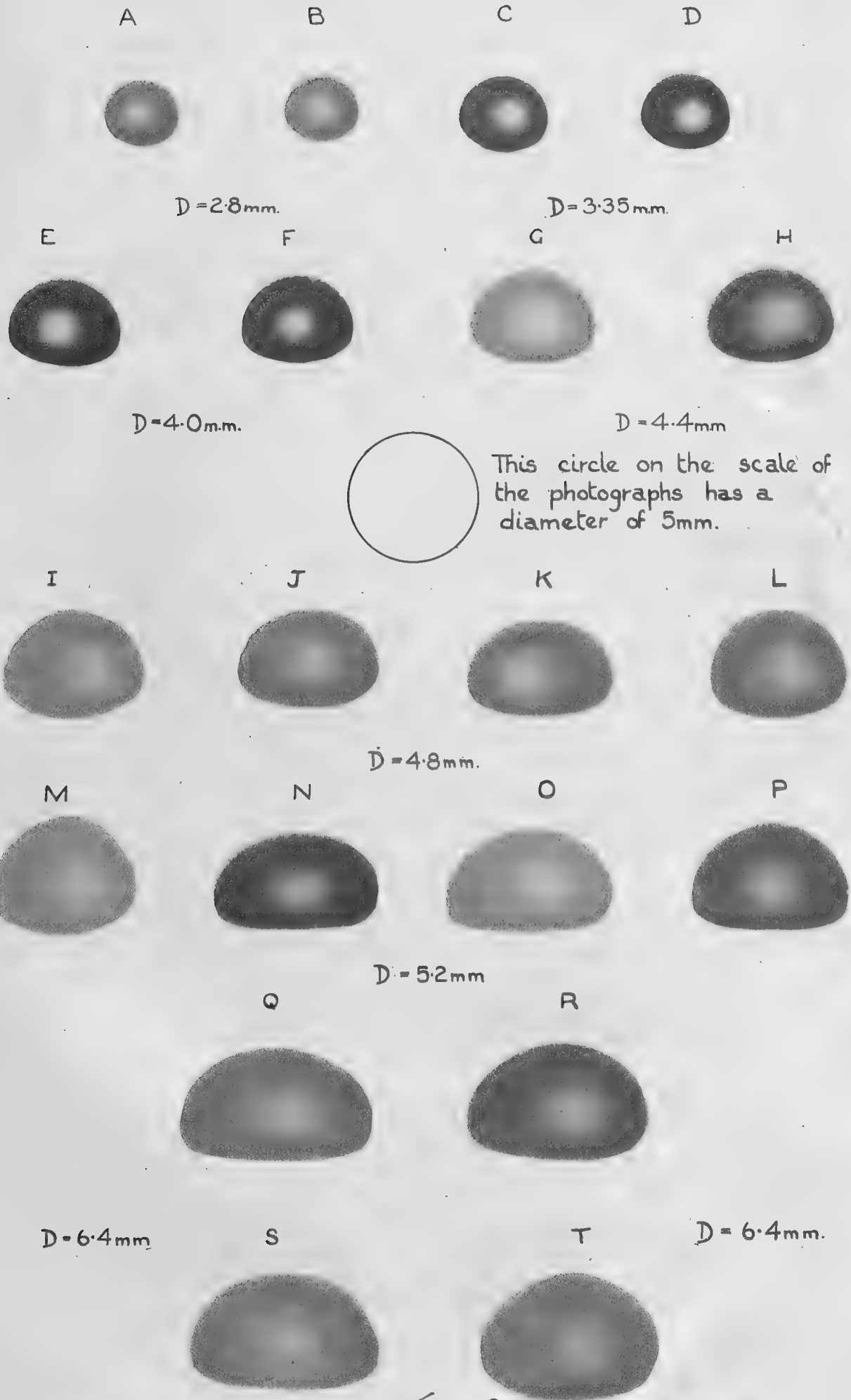


FIG. I.

Printed by Photographic & Reproductions Branch, Air Ministry, London, S.W.1. 1/2682

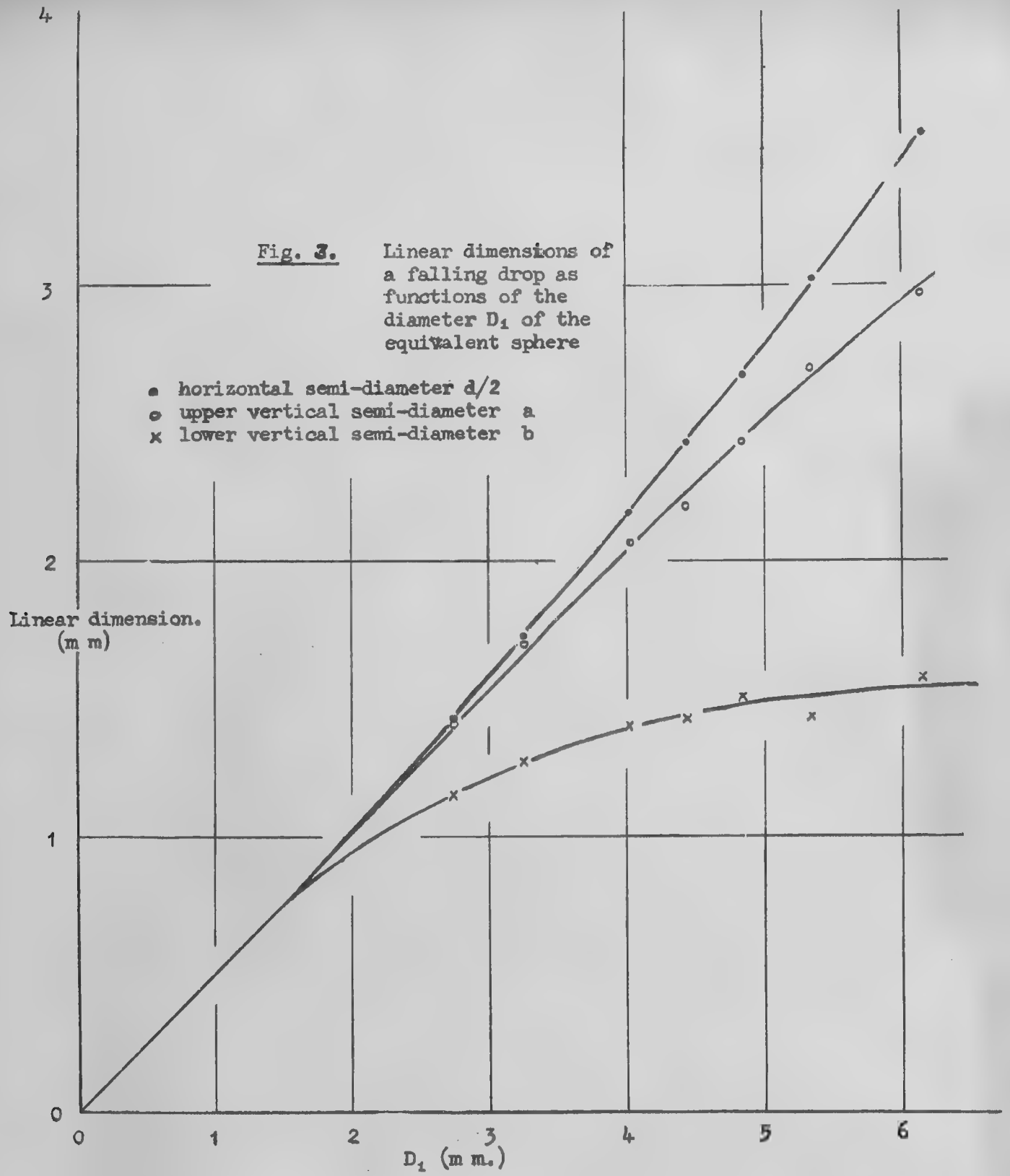


Fig. 3. Linear dimensions of a falling drop as functions of the diameter D_1 of the equivalent sphere

This report has not been
published and is to be treated
as confidential. Its
contents

...th which gives the minimum time of flight between two ...

...ary if an accurate track is to be obtained. A more ...

... and ignores several ...

... of the more laborious ...
... various shows ... may be accepted ...
... some of the more gross sources of error.

... which the method is immediately required is ... the ...
... saving in time which may be made by flying a "pressure ...
... r this purpose, the order of magnitude only is required ...
... saving is to minutes, 30 minutes, 1 hour or 2 hours), and ...
... hod in its present form is considered justified.

... is likely to be some, initially at least, on the ...

... w the track which ... minimum flight time ...
... New York route ... basis of the 500 mb.

CONFIDENTIAL
contents must not be quoted.

19th February, 1947

AIR MINISTRY

METEORLOGICAL RESEARCH COMMITTEE

An approximate method of estimating the pressure pattern flight path for minimum flight time in a stationary system of geostrophic winds

by

J.S. SAWYER

I. Introduction

One of the fundamental problems of "pressure pattern flying" is to determine the flight path which gives the minimum time of flight between two points. This is a peculiarly complex problem, and although graphical methods can be devised which obtain the desired result, they are laborious, and great care is necessary if an accurate track is to be obtained. A more simple and rapid method of obtaining the flight path for minimum time of transit is thus urgently needed, and the following is an attempt to provide it.

The method now proposed involves many approximations, and ignores several sources of error. However, no attempt has been made to estimate its accuracy on theoretical grounds, but comparison with the results of the more laborious graphical method on selected occasions shows good agreement and may be accepted as justifying the use of the simpler method. No doubt, refinements will be found in use to overcome some of the more gross sources of error.

The use for which the method is immediately required is to estimate the magnitude of the saving in time which may be made by flying a "pressure pattern route". For this purpose, the order of magnitude only is required (that is whether the saving is 10 minutes, 30 minutes, 1 hour or 2 hours), and the use of the new method in its present form is considered justified.

In future such a method may be required in planning a "pressure pattern flight". Such planning is likely to be done, initially at least, on the assumption of geostrophic winds on a forecast chart. The errors in constructing such a chart are likely to be considerable, and the use of a refined method of computing the flight path for minimum time would not be justified. For this purpose, also, the method now proposed may prove useful.

In order to investigate how the track which gives a minimum flight time varies with time in a changing pressure distribution, a few such tracks have been computed for the London - New York route on the basis of the 500 mb. contour charts at 12 hour intervals. These are discussed in the final sections of the note.

II. Basis of the Method

It can be shown that on a plane surface, and with a system of winds in which there is no change in wind with displacement at right angles to the route, a close approximation to the least time of flight is obtained when the aircraft is flown on a constant heading throughout. The resulting track will be referred to as a "single heading track", and minimises the effect of the component of the wind at right angles to the route to be flown. It is indicated in Figure 1 by the continuous curve A Z_2 Z_4 Z_6 B.

However the "single heading track" does not give the minimum time of flight in the practical problem in which wind varies both along and across the route, and does not solve the problem now considered which is to determine the flight path which results in the minimum time of flight between A and B with any given pressure distribution.

/For

For convenience, this latter will be referred to subsequently as "the pressure pattern track". As it is defined by the property of making the time of flight a minimum, small displacements of the track from the "pressure pattern track" will have very little effect on the flight time. (This is a property of all minimum points and curves). A close approximation to the pressure pattern track is, therefore, not needed to obtain a close approximation to the minimum flight time.

Essentially the method now proposed depends on the replacement of the "single heading path" (A Z_4 B in Figure 1) by a track in which the midpoint Z_4 is displaced at right-angles to the route by various distances, x, and the intermediate points of the track are displaced in proportion to their distance from the ends of the route A B (that is the points Z_2 and Z_6 midway between the terminals and the centre of the route are displaced by a distance x/2). The time taken by a flight along this new track is estimated for various values of x and the minimum found. The corresponding position of the mid-point C_4 is taken as the mid-point of the "pressure pattern route".

The procedure is then repeated for each of the half routes, AC_4 and C_4B and revised mid-points D_2 and D_6 which give the minimum flight time are found for these sections. The three points D_2, C_4, D_6, are regarded as determining the "pressure pattern route" with sufficient accuracy.

In devising a simple method of computing the time of flight along the tracks under trial the following approximations are made.

(a) The earth is treated as flat with the great circle AB as a straight line.

(b) The displacement of the "single heading track" and "pressure pattern track" from the great circle are regarded as small compared with the length of the track.

The following notation will be used

a = length of great circle AB

x = displacement of mid-point of route from "single heading track"

V = air speed of aircraft

α = angle between heading of aircraft and great circle when flying the single heading track.

v = mean component of wind in direction AB

α and x/a will be regarded as small quantities and powers greater than two will be neglected.

In order that the aircraft may reach the point C_4 instead of Z_4 from A its heading will have to be altered so as to make and angle β with the great circle, so that approximately

$$\frac{a}{2}\tan\beta - \frac{a}{2}\tan\alpha = x$$

The component of the aircraft speed in the direction AB when flying the new track is then V cos β

where $V\cos\beta = V/(1+\tan^2\beta)^{\frac{1}{2}}$

$= V/(1+\tan^2\alpha + \frac{4x}{a}\tan\alpha + \frac{4x^2}{a^2})^{\frac{1}{2}}$

$= V(1 - \frac{1}{2}(\tan^2\alpha + \frac{4x}{a}\tan\alpha + \frac{4x^2}{a^2}))$ approx.

Suppose the mean wind component in the direction AB is v_1 over the section of route AC_4, the mean component of the ground speed of the aircraft in this section resolved along AB is then, approximately,

$$v_1 + V(1 - \frac{1}{2}(\tan^2 \alpha + \frac{4x}{a}\tan \alpha + \frac{4x^2}{a^2}))$$

Similarly the component along AB of the aircraft ground speed over the section C_4B is

$$v_2 + V(1 - \frac{1}{2}(\tan^2 \alpha - \frac{4x}{a}\tan \alpha + \frac{4x^2}{a^2}))$$

where v_2 is the mean component of the wind in the direction AB on the second part of the route.

Thus, approximately, the mean component of the aircraft ground speed along AB over the whole route is

$$V + v - \frac{V}{2}(\tan^2 \alpha + \frac{4x^2}{a^2})$$

where $v = (v_1 + v_2)/2$

Thus the time of flight t along the route AC_4B is given by

$$t = \frac{a}{V(1 - \frac{1}{2}\tan^2 \alpha - \frac{2x^2}{a^2}) + v}$$

$$= \frac{a}{V(1 - \frac{2x^2}{a^2} + \frac{v}{V})}(1 + \frac{1}{2}\tan^2 \alpha)$$

Thus for a given pressure pattern and airspeed the minimum time of flight is obtained when $\frac{v}{V} - \frac{2x^2}{a^2}$ is a maximum.

In application of the method v is estimated for various values of x. It is convenient, therefore to have a diagram which gives $\frac{a}{V(1 - \frac{2x^2}{a^2} + \frac{v}{V})}$ as a function of x and v. A separate diagram is required for each length of route, a, and airspeed V.

Such a diagram is illustrated for the London-New York route ($a = 50^\circ 6'$) and for an airspeed of 200 knots (V) (see Figure 2).

We enter the diagram with the displacement x of the centre point of the route (x) taken along the horizontal scale. Degrees (60 nautical miles) are used for convenience. We proceed upward until we meet the appropriate curve indicating the mean tail wind v (knots). The time of flight is then given by the ordinate (on the vertical scale).

If the representative points are plotted for several values of x and corresponding values of v, a smooth curve can be drawn through them. The minimum flight time is immediately apparent from the lowest point of the curve. The corresponding value of x determines the mid-point of the route C_4.

/The

The same diagram can be applied to determining the minimum flight time over the separate halves of the route, but in this case the scale for x must be doubled and the scale for the flight time halved, as indicated by the subsidiary scales of Figure 2.

A subsidiary diagram is included on the right hand side to correct the time of flight for values of α other than zero. This does not affect the position of the route as determined for minimum flight time. The curves indicate the time scale to be used for any value of α as given on the horizontal scale at the base. It will be seen later that it is necessary to calculate the total drift of the aircraft at right-angles to its track (d) but not essential to calculate for other stages of work. For this reason a scale has been added by which the latter can be used in place of α to obtain a corrected time of flight.

III. Practical Application of the Method

The application of the method can be divided into three stages

(i) The plotting of a "single heading track"

(ii) The estimation of mean head winds over modified routes.

(iii) The determination of the route for minimum time.

The operations involved under (ii) and (iii) are then repeated for the two halves of the route as indicated above.

The three stages of the work are described below and illustrated by means of a worked example for the route London-New York. Although the procedure appears complicated, it can usually be completed in an hour or less.

It is convenient to work on a chart the projection of which causes the great circle route to appear as an approximately straight line - a conical projection is usually suitable. The contours of the isobaric surface (or isobars at a constant level) from which it is intended to work are drawn in on this chart.

(i) Plotting the "single heading track"

(a) Draw the great circle track on the chart on which the pressure contours (or isobars) are already drawn (see Figure 3).

(b) Divide the route into eight equal sections by points X_1, X_2, ... X_6 and X_7. Draw perpendiculars to the route at these points and mark off equal intervals, each 60 nautical miles (1 degree of arc) along them.

(c) Determine the difference in contour height between the terminals, (H = height at B - height at A) and calculate the drift of the aircraft at right-angles to its course for the whole route when flown on a single heading. This is given in degrees of arc by the equation $d = \frac{-21.4}{60 \sin \phi} \frac{H}{V}$ where d is measured to the right of the route. It is convenient to have a table to give d directly from H for a given route and air speed V. (The latitude ϕ is taken to be the mean value over the route when this is in a more or less east-west direction.)

Such a table is given as Table I for the London - New York route. In the example (Figure 3), H = 17600 - 18500 = -900 feet.

so that d = 2.1 degrees of arc.

/(d)

(d) The course the aircraft would take in still air if flying on the heading required for single heading flight, is marked off by indicating on the perpendiculars through X_1 X_2 etc, points Y_1 Y_2 etc. such that $X_1 Y_1 = \frac{-d}{8}$,

$X_2 Y_2 = \frac{-2d}{8}$, $X_3 Y_3 = \frac{-3d}{8}$ etc.

This has been done in Figure 3.

(e) The displacement of the aircraft from its still air course, when making a "single heading flight" depends only on the pressure contour height at the point reached. The next step is, therefore, to label the contours which pass near the route with the values of the displacement as obtained from the formula $d = \frac{-21.4}{60 \sin \phi} \frac{\Delta h}{V}$ or from a table such as Table I. (Δ h here respresents the contour height of the chosen contour minus the contour height at the starting point.)

(f) The intersections (Z_1, Z_2, ... Z_7) of the "single heading" flight path with the perpendiculars are now obtained directly by noting the points on these perpendiculars where the displacement as given by the pressure contours is the same as the actual displacement from the still air track ($Y_1 Z_1$, $Y_2 Z_2$, etc.)

This can be done by inspection using the scale of degrees already marked on the perpendiculars.

(g) The "single heading" track is then drawn through Z_1 Z_2 Z_3 etc.

(ii) Computing the mean tail wind

(a) The tailwind component is measured at selected points along each of the perpendiculars, by placing the geostrophic scale along the perpendicular and noting the distance between the contours in terms of geostrophic wind. Graphs are then drawn (Figure 4) showing the variation of the tail-wind component with distance from the great circle route (in degrees of arc). These graphs should extend to about 14 degrees from the single heading track at the centre of the route, although they need not be so extensive along perpendiculars nearer to the ends of the route. Four or five points are usually sufficient to draw each curve satisfactorily. It will also often be possible to estimate the region in which the "pressure pattern route" will lie and to restrict the work to the areas likely to be crossed, or adjacent to them.

(b) The tail-wind component of the geostrophic wind is also measured at both terminals and the mean taken.

In the example in Figure 3:-

Tail-wind component of geostrophic wind at London	=	-10 knots
" " " " " " New York	=	-75 knots
Mean tail-wind at terminals	=	-42 knots

(c) The mean tail-wind over the routes of the type described in section II is next calculated for various displacements of the mid-point from the single heading path. This is best done in tabular form, the tail-wind components being taken from the graphs drawn as indicated in sub-paragraph (a) above.

A suitable form for the table is indicated by Table II. The values inserted are those for the example given in Figure 3 and 4 etc.

/(iii)

(iii) First approximation to pressure pattern route

The values of the mean tail-wind component obtained as indicated in paragraph (ii) (c) are plotted against the displacement of the mid-point of the route on Figure 1. The curved lines give the scale for the mean tail-wind and are used during plotting.

The curve derived from the example given in Table II etc., is indicated by R S in Figure 1. This has a well defined minimum for a displacement of the mid-point of the route of about 6 degrees from the "constant heading" path. The approximate time of flight on this path is 15.5 hours compared with 18.4 hours on the "single heading" flight plan, a saving of 2.9 hours.

(iv) Second approximation to pressure pattern route.

(a) The position of the first approximation to the "pressure pattern route" is now marked on the graphs of the tail-wind components (Figure 4). The displacement obtained for the mid-point, as obtained in paragraph (iii) is marked off from the single heading track on the graph for the perpendicular through X_4; $\frac{3}{4}$ of this on the graphs for perpendiculars through X_3 and X_5; $\frac{1}{2}$ on those through X_2 and X_6 and $\frac{1}{4}$ of the displacement on the graphs for the perpendicular through X_1 and X_7.

(b) The mean tail-wind (parallel to the great circle) is now computed for modifications of each half of the route for which the mid-points are displaced from the positions of these mid-points in the first approximation. This is done for each half of the route in a manner exactly analogous to that used for the whole route.

The calculations for this example are reproduced in Table III.

(c) The mean tail-wind components are now plotted against the displacement of the mid-point of the half sections of the route, as was done for the whole route (paragraph (iii) above). The subsidiary scales at the bottom and side of the diagram must, however, be used. The curves obtained in the present example are indicated by R_1 S_1 and R_2 S_2 for the first and second half routes respectively. In the present example, the minima of these curves occur for values of the displacement of $1\frac{1}{2}$ degrees and 2 degrees respectively.

(d) The intersections of the pressure pattern route with the perpendiculars through X_2 X_4 and X_6 can now be marked on Figure 3. The intersection with the central perpendicular through X_4 was already obtained in the first approximation (paragraph (iii)) and its distance from the great circle route is already marked in Figure 4. This can be transferred to Figure 3.

The additional displacements of the second approximation to the pressure pattern route obtained in sub-paragraph (c) above for the intersections with the perpendiculars through X_2 and X_6 can now be added to the displacement from the great circle of the first approximation indicated on Figure 4 and transferred to Figure 3.

(e) If it is desired to complete the pressure pattern route, this is done by inserting "single heading" tracks between the points now determined (and the terminals). The technique outlined in paragraph (i) above is used for this purpose.

(f) An approcimation to the time of flight was obtained in paragraph (iii) above for the first approximation to the pressure pattern path. This can be corrected by subtracting the saving produced by the revised path on each half section of the route, obtained by noting the amount (on the scale at the extreme left) by which the minima of the curves R_1 S_1 and R_2 S_2 are below their intersection with the axis.

In the example:-

Time over first approximation route	=	15.5 hours
Time saved by modifying 1st part of route	=	0.1 hours
Time saved by modifying 2nd part of route	=	0.2 hours
Revised time for pressure pattern route	=	15.2 hours

The time for a "single heading flight" is given by the intersection of RS with the axis (namely 18.4 Hours).

IV. Comparison with other methods of computation

The pressure pattern route on the synoptic situation reproduced in Figure 3 has also been calculated by a direct graphical method. The resulting track is given in Figure 5 for comparison. There is close agreement with that computed by the present method and given in Figure 3.

The minimum flight time as computed by the graphical method is 15.5 hours, a difference of 18 minutes from that computed by the approximate method. Errors are involved in both processes and it cannot be stated which is the more nearly correct.

The time taken to fly a great circle track on the same synoptic situation has been computed as 21.5 hours; this is to be compared with 18.4 hours for a "single heading" flight and 15.2 hours for a "pressure pattern" track.

It may be remarked that the saving of time in the example considered is greater than normal, although not exceptional. The case was, however, selected in order to give a clear-cut distinction between the various possible flight plans.

Table IV gives the results of a similar comparison of the "pressure pattern" track as determined by the present approximate method and the direct graphical method, when both were applied to 9 other synoptic situations. Assuming the graphical method to give a true result for the "pressure pattern track", the probable error of the minimum time of flight as determined by the present method is 0.3 hours and the probable error is the position of the "pressure pattern track" is less than 1.2 degrees of latitude for all points on it.

The occasions selected for the comparison were chosen in order to test the system on a variety of pressure distributions. They cannot, therefore, be regarded as representing a random selection of occasion, and the saving of time as given by them is probably somewhat greater than the average for the route.

V. Remarks on the Applicability of the Method particularly in Pressure Situations changing with time.

The present method of determining the pressure "pattern flight path" was devised mainly with a view to estimating the ultimate limit to the saving of time by the use of "pressure pattern flying" on various routes, and for this purpose it is considered the application of the method to individual synoptic charts will give results of the right order of magnitude without the need for allowing for changes in the pressure situation with time. If the possible saving of time was found to justify the adoption of "pressure pattern methods", it was intended that the modification of the

method for application to moving pressure systems should be studied.

However, it is interesting to note that the position of the "pressure pattern track" is dependent on the main features of the pressure distribution, rather than on the detailed configuration of the isobars. As a result when the method is applied to the 500 mb. pressure contour charts the pressure pattern track does not normally change very rapidly with time. In order to illustrate this the "pressure pattern tracks" have been calculated from the pressure chart 12 hours and 24 hours later than that in the example worked in the text. The results are included in figure 5 for comparison.

The effect of a rapidly moving depression on the "pressure pattern track" is illustrated by figures 6 and 7 which show the contours of the 500 mb. surface at times separated by a 12-hour interval. The corresponding "single heading" and "pressure pattern tracks" are included.

It is interesting to note that if an aircraft flew on the "pressure pattern track" calculated on the basis of the pressure distribution in Fig. 6, but the actual pressure distribution actually encountered was that shown in Fig. 7 (i.e. 12 hours later) then the duration of the flight would be 19.1 hrs. compared with 20.3 hrs. on the single heading track and 18.5 hrs. on the "pressure pattern track" computed from the true pressure system. If, on the other hand, the aircraft were to adopt the headings necessary to follow the "pressure pattern track" on the assumption that the pressure field was as in Fig. 6 whereas it was actually that in Fig. 7, then it would arrive very near to New York in 18.6 hours. The actual track which would be followed under these circumstances is also included in Fig. 7.

These few results suggest that the "pressure pattern track" at the 500 mb. level is not very sensitive to changes in the pressure field and that in the planning of a flight the method could be used with advantage even though the assumed pressure field differed in some degree from that actually experienced. The results so far obtained are, however, by no means adequate to establish such a claim

TABLE I

London - New York Route ($\phi = 50^{\circ}$) - Air Speed 200 knots

Drift caused by stated difference of pressure contour height between terminals

Difference of contour Height H (or Δ H)	100	200	300	400	500	600	700	800	900	1,000	Feet
Drift d	0.2	0.5	0.7	0.9	1.2	1.4	1.6.	1.9	2.1	2.3	Degrees arc

Difference of contour Height H (or Δ H)	1,100	1,200	1,300	1,400	1,500	1,600	1,700	1,800	1,900	2,000	Feet
Drift d	2.6	2.8	3.0	3.3	3.5	3.7	4.0	4.2	4.4	4.7	Degrees of arc

Difference of contour Height H (or Δ H)	2,100	2,200	2,300	2,400	2,500	2,600	2,700	2,800	2,900	3,000	Feet
Drift d	4.9	5.1	5.4	5.6	5.8	6.0	6.3	6.5	6.8	7.0	Degrees of arc

TABLE II

Calculation of Mean Tail-Wind on Route London – New York

	Route for which mid-point is displaced n degrees from "single heading track"							
	n = -12	n = -8	n = -4	n = 0	n = 4	n = 8	n = 12	
			(knots)					Insert in this row values of tailwind
	- 11	- 12	- 13	- 15	- 17	- 18	- 17	on perpendicular through X_1 for points -3,-2,-1,0,1,2,3 degrees from single heading tr
	- 19	- 16	- 12	- 9	- 6	- 2	+ 3	" " " " X_2 " " -6,-4,-2,0,2,4,6 " "
	- 27	- 25	- 22	- 15	0	+ 17	+ 27	" " " " X_3 " " -9,-6,-3,0,3,6,9 " "
	- 32	- 45	- 45	- 25	+ 17	+ 27	+ 25	" " " " X_4 " " -12,-8,-4,0,4,8,12 " "
	- 82	- 84	- 74	- 32	+ 42	+ 50	+ 45	" " " " X_5 " " -9,-6,-3,0,3,6,9 " "
	- 90	- 90	- 80	- 55	+ 13	+ 30	+ 37	" " " " X_6 " " -6,-4,-2,0,2,4,6 " "
	- 90	- 88	- 87	- 78	- 67	- 50	- 38	" " " " X_7 " " -3,-2,-1,0,1,2,3 " "
	- 42	- 42	- 42	- 42	- 42	- 42	- 42	Mean tail-wind component at terminals
Total	-393	-402	-375	-271	- 60	+ 12	+ 40	
Mean	- 49	- 50	- 47	- 34	- 7	+ 1	+ 5	

TABLE III

Calculation of Mean Tail-Wind on 1st half of route London - New York (second approximation)

	Route for which mid-point displaced n degrees from 1st approximation to pressure pattern track							
	n = -6	n = -4	n = -2	n = 0	n = 2	n = 4	n = 6	
	(knots)							
	- 13	- 15	- 16	- 17	- 18	- 17	- 15	Insert in this row values of tailwind on perpendicular at X_1 for points -3,-2,-1,0,1,2,3 degrees from 1st approx. to pressure pattern track.
	- 14	- 11	- 7	- 4	0	+ 3	+ 7	" " X_2 " " -6,-4,-2,0,2,4,6 " " " "
	- 8	- 3	+ 2	+ 8	+ 15	+ 20	+ 23	" " X_3 " " -3,-2,-1,0,1,2,3 " " " "
	+ 7	+ 7	+ 7	+ 7	+ 7	+ 7	+ 7	Insert mean value of tailwind at A and at X_4 on 1st approximation to pressure pattern track.
Total	- 28	- 22	- 14	- 6	+ 4	+ 13	+ 22	
Mean	- 7	- 5	- 3	- 1	+ 1	+ 3	+ 5	

Calculation of Mean Tail-Wind on 2nd half of route London - New York (second approximation)

	Route for which mid-point displaced n degrees from 1st approximation to pressure pattern track							
	n = -6	n = -4	n = -2	n = 0	n = 2	n = 4	n = 6	
	(knots)							
	- 5	+ 20	+ 44	+ 50	+ 51	+ 50	+ 49	Insert in this row values of tailwind on perpendicular at X_5 for points -3,-2,-1,0,1,2,3 degrees from 1st approx. to pressure pattern track
	- 88	- 72	- 25	+ 23	+ 33	+ 38	+ 40	" " X_6 " " -6,-4,-2,0,2,4,6 " " " "
	- 88	- 84	- 72	- 59	- 45	- 34	- 31	" " X_7 " " -3,-2,-1,0,1,2,3 " " " "
	- 25	- 25	- 25	- 25	- 25	- 25	- 25	Insert mean value of tailwind at B and at X_4 on 1st approximation to pressure pattern track.
Total	-206	-161	- 88	- 11	+ 14	+ 29	+ 33	
Mean	- 51	- 40	- 22	- 3	+ 3	+ 7	+ 8	

TABLE IV

Comparison of methods of determining the "pressure pattern track".

Synoptic Chart utilised		Route	Minimum time of flight (over "pressure pattern track")		Difference between results of two methods		Maximum deviation of pressure pattern track from great circle	Saving of time Single heading minus pressure pattern	Saving of time Great Circle minus pressure pattern
Date & Time	Pressure Level		As determined by graphical method	As determined by method described in present note	In minimum time of flight	Maximum distance between tracks determined by two methods			
	mbs		hrs.	hrs.	hrs.	(degs. of latitude)	(degs. of lat.)	hrs.	hrs.
1 Jan 1946 04-06 GMT	500	London - New York	16.3	16.3	0.0	1.5	9.0	2.7	3.9
2 Jan 1946 04-06 GMT	500	London - New York	17.2	17.2	0.0	1.0	8.0	1.6	4.4
4 Jan 1946 04-06 GMT	500	London - New York	20.7	21.4	+0.7	4.0	16.0	1.0	2.3
23 Jan 1946 04-06 GMT	500	London - New York	16.9	17.0	+0.1	1.5	10.5	2.8	4.7
19 April 1946 16-18 GMT	500	London - New York	16.3	15.4	-0.9	1.2	6.0	2.4	2.7
2 Dec 1946 16-18 GMT	500	London - New York	17.1	16.8	-0.3	0.5	13.5	3.4	3.6
14 Dec 1946 16-18 GMT	500	London - New York	15.5	15.4	-0.1	1.0	8.5	2.9	6.1
21 Jan 1946 16-18 GMT	700	London - New York	16.7	17.2	+0.5	1.5	12.0	1.3	2.8
10 Jan 1946 04-06 GMT	500	New York - London	11.9	11.5	-0.4	1.4	12.0	1.1	1.8

FIGURE 1.

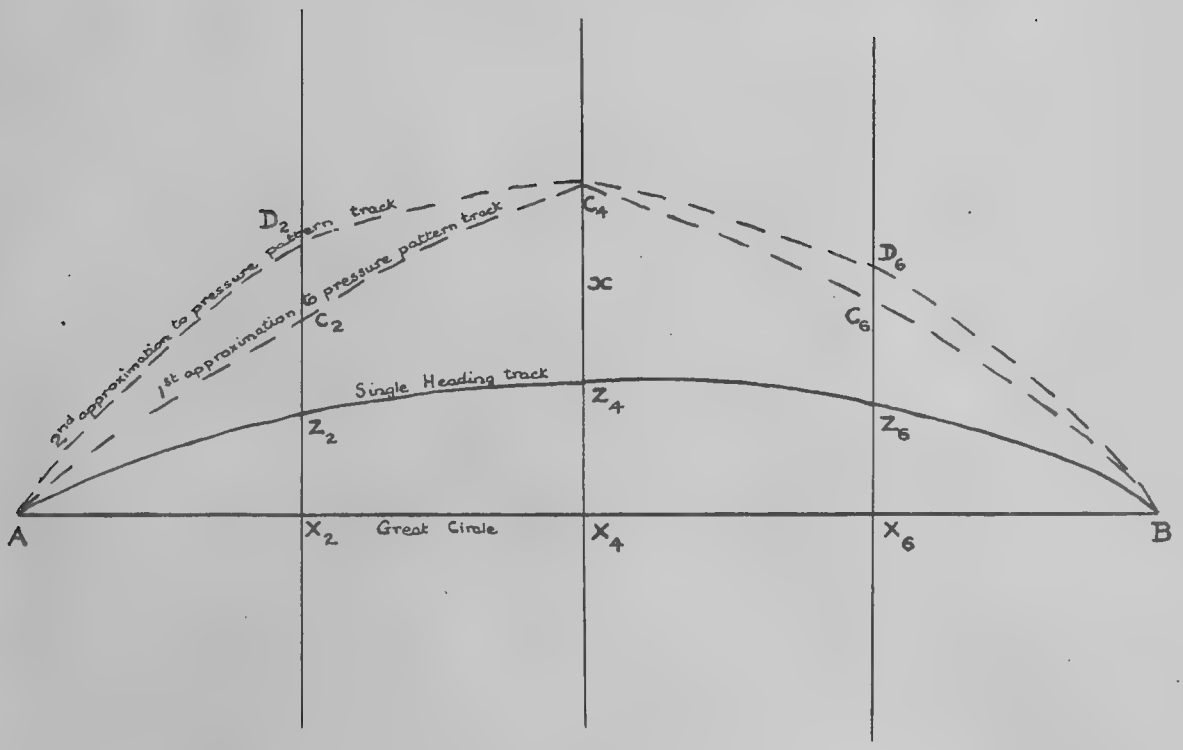

D2
C4
D6
2nd approximation to pressure pattern track
1st approximation to pressure pattern track
x
C2
C6
Single Heading track
Z4
Z2
Z6
A
X2
Great Circle
X4
X6
B

FIGURE 2 – FOR DETERMINING MINIMUM TIME OF FLIGHT ON ROUTE LONDON – NEW YORK (AIRCRAFT AIRSPEED 200 KNOTS)

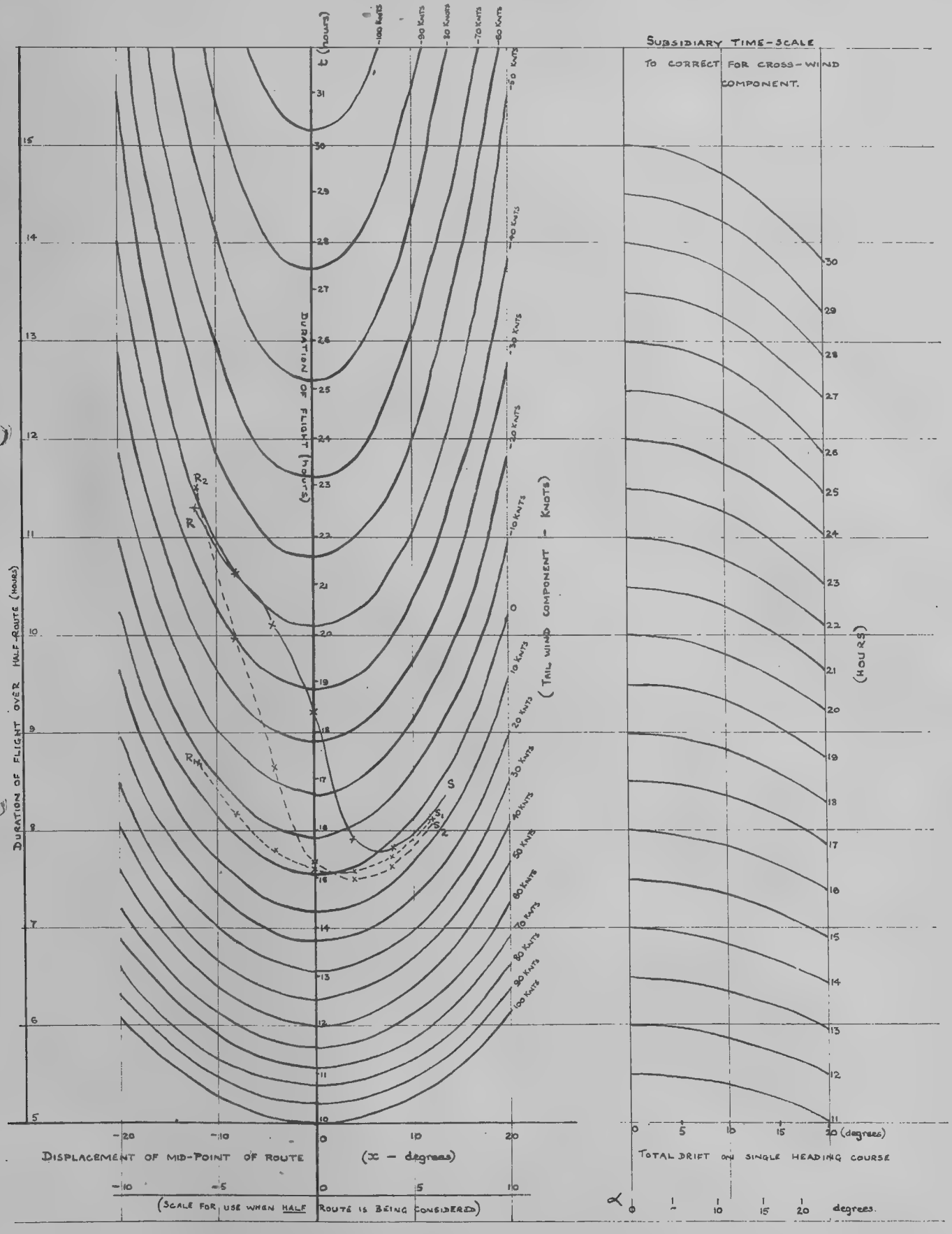


FIGURE 3. – EXAMPLE OF CALCULATION OF PRESSURE PATTERN TRACK BASED ON 500 MB CONTOURS FOR 1600 G.M.T. – 14th DECEMBER 1946.

Route LONDON – NEW YORK

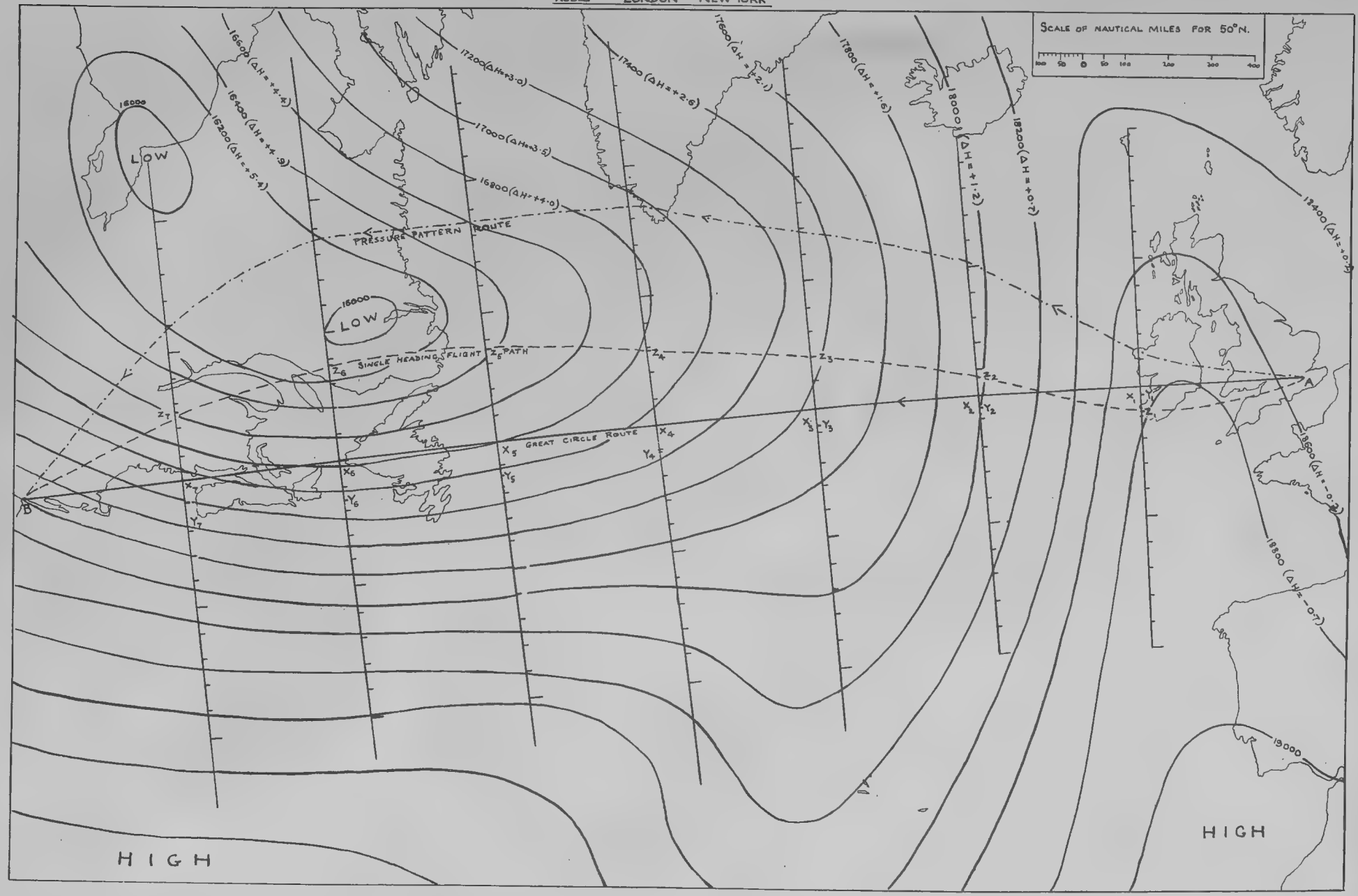


FIGURE 4 - GRAPHS OF TAILWIND COMPONENTS AT POSITIONS ALONG PERPENDICULARS TO LONDON — NEW YORK AIR-ROUTE 1600 G.M.T. 14 DECEMBER 1946.

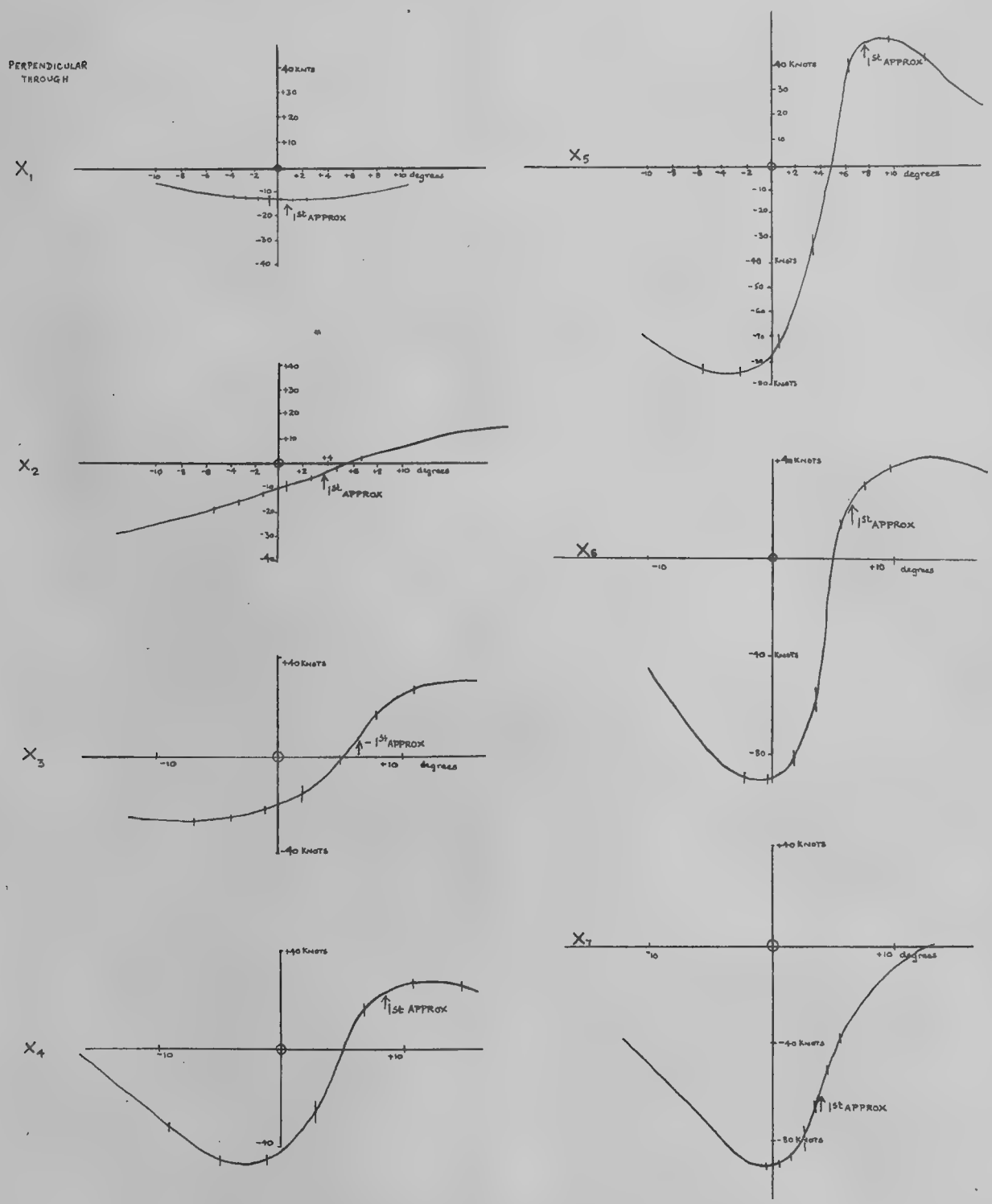


FIGURE 5. PRESSURE PATTERN TRACK BASED ON SYNOPTIC CHART GIVEN IN FIG. 3. AND ON CHARTS 12 AND 24 HOURS LATER.
(The continuous curve has been computed by a direct graphical method for comparison with result of approximate method given in Fig. 3.)

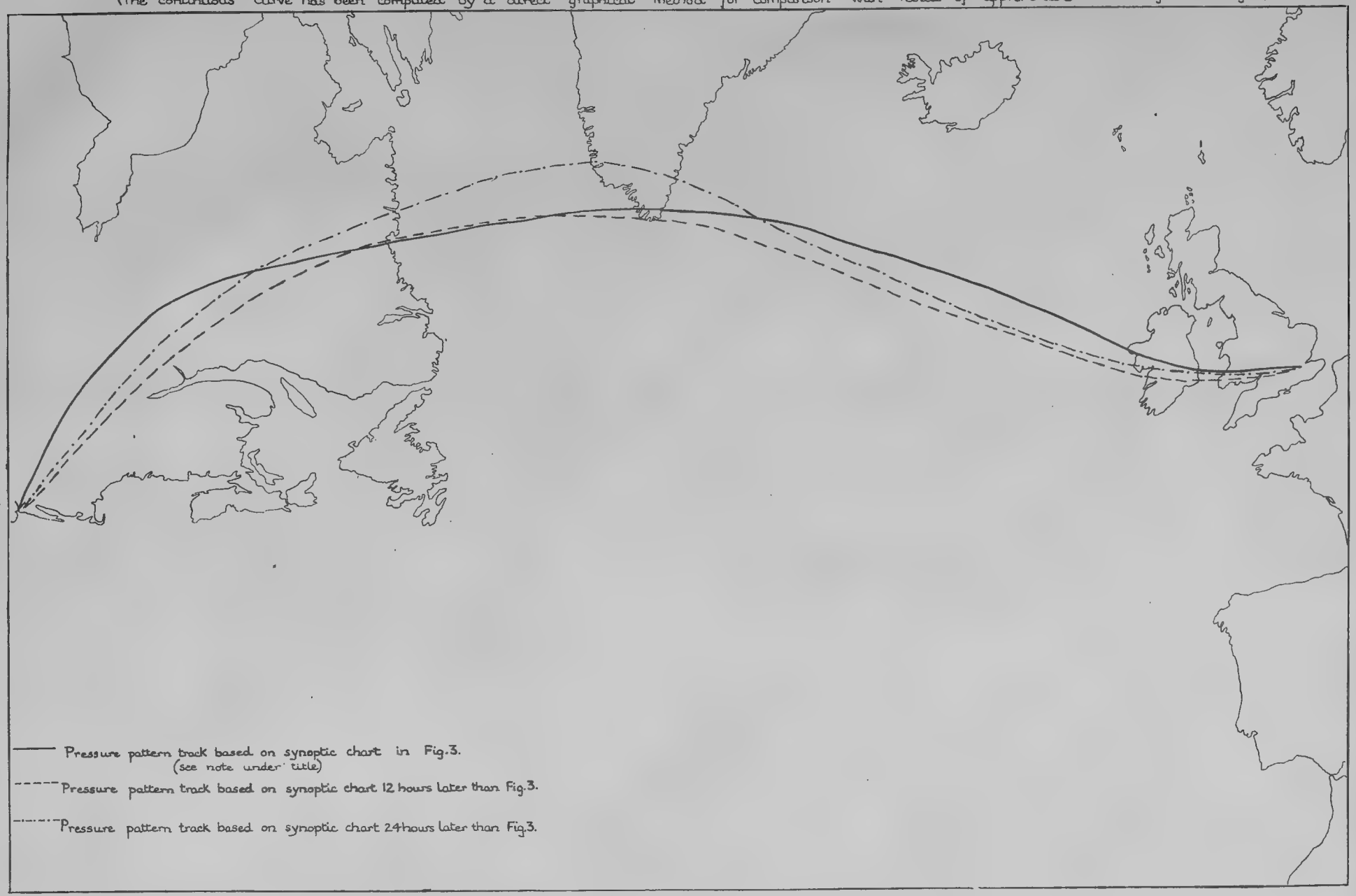
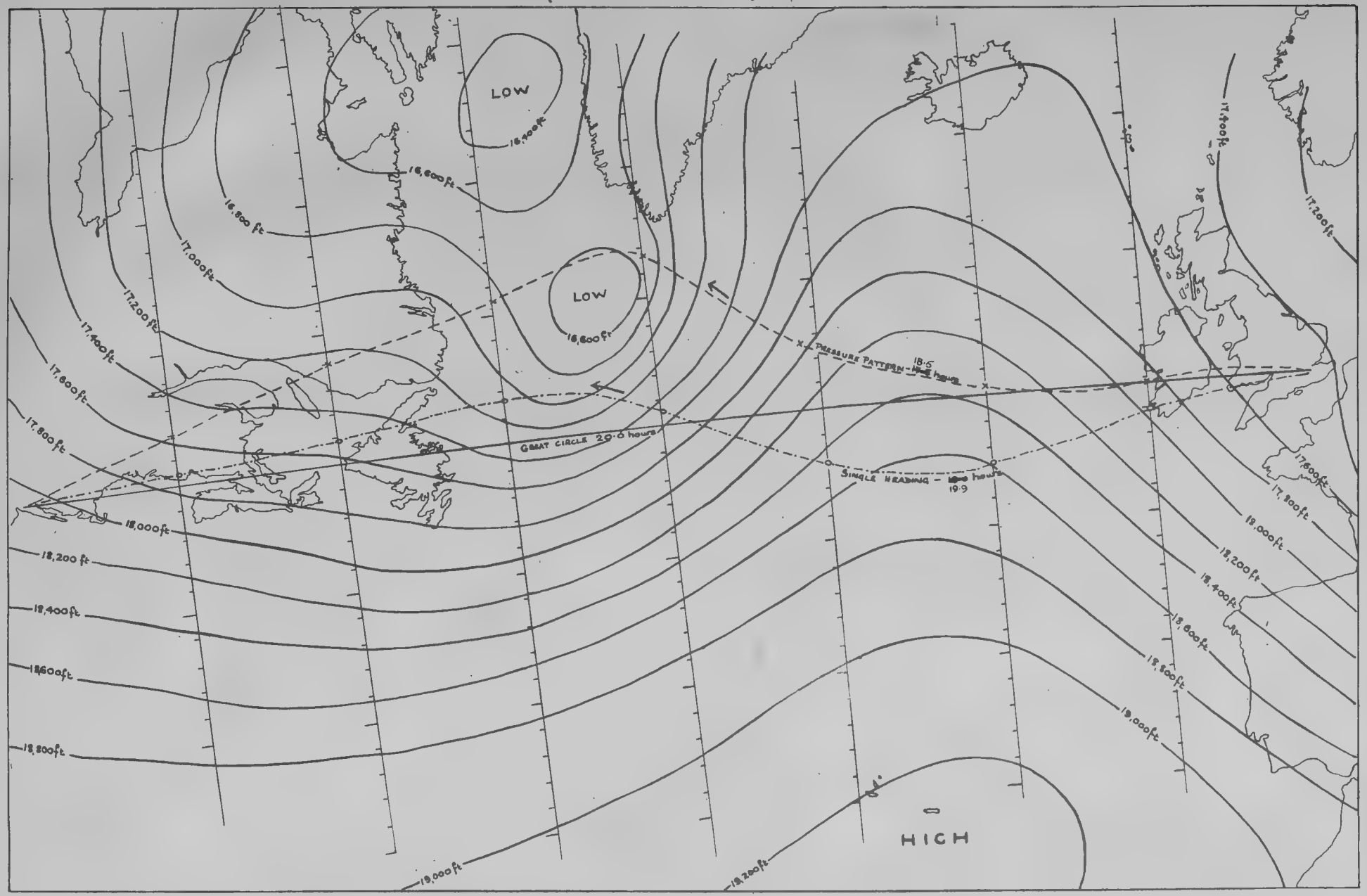


FIGURE 6. CONTOURS OF 500 MB SURFACE 0400 - 0600 G.M.T. 4th DECEMBER 1948 AND ASSOCIATED PRESSURE PATTERN AND SINGLE HEADING TRACKS.
(AIRCRAFT SPEED 200 KNOTS)



AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

Seasonal Charts of the Mean Height of the Tropopause

by

A. DEWAR AND E.S. SAWYER

NOV 20 1951

The preparation of these charts was undertaken primarily to provide information as to the distribution of mean tropopause height required in connection with the investigation of "Upper Winds over the World" at present being carried out in the Climatology and Investigation Branches of the Meteorological Office. The charts will probably prove of value also in answering other questions, particularly in connection with aviation, where an approximate height of the base of the stratosphere is required.

The only co-ordination of local data on tropopause height which has hitherto been published appears to be diagrams showing the variation of mean tropopause height with latitude, first drawn by Shaw (9) and modified by Ramanathan (10). The geographical variation of mean tropopause height is indeed mainly dependent on latitude, but sufficient observational data is now available to permit the mapping with considerable accuracy of the longitudinal as well as latitudinal variation over N. America and Europe. The contours can also be interpolated over the N. Atlantic with reasonable confidence.

An estimate of the effect of the continental land masses in distorting the otherwise latitudinal variation of mean tropopause height can be made from theoretical considerations as indicated in later paragraphs, and the soundness of these ideas is confirmed by comparison with the observed heights over America, Europe and the Atlantic. It has therefore been considered possible to extend the contours of mean tropopause height from the Atlantic region to the major part of the globe by means of analogy and physical considerations, and by utilising the scanty data which are available for the other parts of the world. The results are given in the accompanying seasonal charts of mean tropopause heights. It must however be emphasised that outside Europe and N. America the charts are of a tentative nature.

Observational material

The data available for the mean height of the tropopause were divided into three main classes :-

(a) Heights obtained from irregular sounding balloon ascents covering a comparatively long period.

(b) Heights obtained from regular and frequent radio sonde ascents over a comparatively short period.

(c) Other values, often based on a few or even single ascents.

In drawing the contours of mean tropopause the reliability of the observations was regarded as in the order indicated.

The data included under (a) were obtained from various publications (1). Where necessary, they were converted to seasonal mean values in thousands of feet.

The data included under (b) were obtained from both British and American radio sonde observations. The British observations had been co-ordinated by the Central Forecast Section into published daily charts of the contours of the tropopause. These charts were used directly, the mean tropopause heights being evaluated for about a dozen points over the area 45° - 65° N, 15°E - 15°W. The charts used were for the period June 1943 - May 1945 and the work was carried out in the Investigation Branch.

/Mean

Mean monthly tropopause heights for a number of radio sonde stations in the U.S.A. have been published (2) and also curves showing the variation of mean temperature with height for the mid-months of the different seasons. The former data are given for a period of only one or two years (1940-41), whereas the latter generally cover five years (1939-43). It was decided to use the mean temperature data as they were available for a longer period and a larger number of stations. However, it was realised that the height obtained by applying to the mean temperature-height curve, the normal rule for assessing the tropopause, would not necessarily be the true mean tropopause height. A comparison was therefore made for those stations for which material was available in both forms for the same period. A systematic difference in height was found for the summer and autumn periods, but in winter and spring the differences were negligible. To eliminate this, the heights obtained from the mean temperature curves over the U.S.A. have been reduced by 3,000ft. in summer and by 5,000ft. in autumn before the contours were drawn.

The data under headings (a) and (b) were practically limited to North America and Europe and the data classified under (c) above were collected to serve as a guide in extending the contour lines. The data were taken from several sources (4).

For Greenland a value was obtained from a graph of wind velocity against height. An annual value for Melbourne, Australia, was adjusted by comparison with values for England, to give seasonal values. Some annual mean contours of the tropopause over Brazil (5) were plotted but appeared much too low. As it was not clear on what observations they had been based it was considered best to disregard them. A few radio sonde ascents over Burma, many of which failed to reach the tropopause, were analysed statistically by Miss N. Carruthers and although the true mean is subject to a rather wide range of uncertainty, the observations indicate that the maps as drawn give heights of the correct order over Burma.

For the Gulf of Mexico monthly means of temperature (6) for the levels 15 - 20 km from December 1942 to November 1945 were combined into seasonal values, and mean tropopause heights estimated from plotted temperature height curves. In many cases there were only a few values throughout the period at the higher levels (17 - 20 kms). For Siberia, tropopause heights occasionally published in the German Upper Air CLIMAT's between 1937 and 1940 were used. These were scanty and erratic, but on the whole suggested a comparatively low tropopause over the Siberian anticyclone in winter.

Other scattered observations of tropopause height exist and could have been extracted and considered if time and staff had been available.

Physical Considerations

Two main factors determine the local mean tropopause height.

(i) the balance which would be attained between radiation and turbulence in an atmosphere without latitudinal movement.

(ii) the advection of low tropopause from higher latitudes or of high tropopause from lower latitudes by Southward or Northward air movements.

The balance between radiation and turbulence is such that it would be expected that at a fixed latitude high mean temperature in the troposphere will be accompanied by a high tropopause and low mean temperature by a low tropopause. Thus the tropopause in winter is expected to be higher over the oceans than the continents latitude for latitude, and the reverse in summer. Flohn (8) also holds this view. The association of low tropopause with low troposphere temperature is also supported by the relatively low values of tropopause height

/obtained

obtained from the few soundings over the southern hemisphere when compared with those of the northern hemisphere.

It is also known that the main air masses of the globe carry with them tropopause heights typical of their source region, which are only modified slowly as they pass into new latitudes or areas. Thus regions of the world frequently affected by "warm" air masses, that is with a predominant air movement towards the pole in the middle troposphere will be regions of high tropopause for their latitude and that regions predominantly covered by "cold" air masses, that is with a predominant movement towards the equator will be regions of low tropopause for their latitude.

General

The values obtained from observational material were plotted on mercator projection charts of the world. The mean contours were drawn with due regard to the reliability of the data as assessed above, and utilising the physical principles underlined where data was absent or unreliable. For the southern hemisphere the values at Batavia and Melbourne were used as starting points and the remainder completed from the physical considerations and analogy.

The problem of defining the tropopause has been ignored in the above, but very real difficulties are introduced both in high latitudes where the change in lapse rate is often ill defined and in sub tropical latitudes where a multiple tropopause is very common. It is hoped that at a later date it may be possible to re-examine the data in order to ensure, so far as possible, that a uniform definition of the tropopause has been employed.

References

(1) Wegener, K. Die Tropopause in Europa. Ann. der Hyd., 49. 1941 pp. 113-117.

Koppen und Geiger - Handbuch der Klimatologie. Bd. 1 Teil.F/2/2. 12,29,43,53.

Gutermann, I.G. - Vertical temperature gradients above Franz Joseph Land, Moscow. (Met. Hydrol. 1938 No. 6 pp. 56 - 65 (Russian)) Summarised translation in Milton, Bull. Amer. Met. Soc. 19., 1938. pp. 433-436.

(a) Toronto, Dept. Transport, Air Services Branch, Met. Division. - Meteorology of the Canadian Artic. Toronto 1944

(b) Paris, Office National météorologique. - Étude climatologique sur le bassin mediterranean. Paris, 1940 p. 70.

Moltchanov, P. Results of application of radio-sounding methods in the exploration of the upper air in the Artic regions over a period of 6 years (1930-36) Leningr. Bull. Artic Inst. 6. 1936 p. 238.

K.R. Ramanathan - Discussion of results of sounding balloon ascents at Agra during the period July 1925 to March 1928 and some allied questions. - Memoirs of the I. Met. D. Vol XXV Part V 1930.

K.R. Ramanathan and Ramakrishnan - Discussion of results of sounding balloon ascents at Poona and Hyderabad during the period October 1928 to December 1931 - Memoirs of the India Met. Depot Vol XXVI Part IV.

N.K. Ssur and K.P. Ramakrishnan - Discussion of results of sounding balloon ascents at Madras in the months of June to November during 1934 - 1935 - Memoirs of the India Met. Dept. Vol. XVII Part II.

(2) U.S. Mon. Weath. Rev. - Jan. 1939 - Jan. 1941. Table 4.

(3) Ratner, B. Upper air average values of temperature, pressure, and relative humidity over the United States, and Alaska - Washington D.C. July 1945.

(4) Palmer, E. Über die Temperatur - verteilung in der Stratosphare Met. Z. 51. 1934 p.21.

Bureau, R., Douguet, ., et Wehrle. Ph. Radiosondages dans les mers Australes. Paris C.R. 208, 193[illegible], p. 1419.

Carmichael H. and Dymond E.G. Upper air investigations in North-West Greenland. London. Proc. Roy. Soc. 171, 1939, pp. 345-359.

Loewe, F. Discussion of eight years of aerological observations obtained by means of aeroplanes near Melbourne. Melbourne. Commonwealth Bureau of Meteorology. Bull. No. 27.1940 p. 6.

(5) Serra, A. [illegible]orna, L. [illegible]s, Ensaio de ar de America do Sul. Translation in Report No.403 Washington, U.S.A.A.F. Weather Information Branch. Fig. 1.

(6) U.S. Mon. Weath. Rev. - Dec. 1942 to Dec. 1945. Table 1.

(7) Bureau R. Douguet, M., et Wehrle Ph. Radiosondages dans les mers Australes. Paris. C.R. 208. 1939.pp. 1419 - 1420.

(8) Flohn, H. Kontinentalitat und Ozeanitat in der freien atmosphare. Met. Z., 60. 1943., p.350.

(9) Shaw Sir Napier - Manual of Meteorology 2nd edition p. 116 (First published 1927).

(10) K.R. Ramanathan - Discussion of results of sounding balloon ascents at Agra during the period July 1945 to March 1928 and some allied questions - Memoirs of the I. Met. D. Vol. [illegible] Pt. V. 1930

9185

its
contents must

24.5.41.
Jnl. R.A.S. Q.J.R. Met. Soc.
74 (321-322): 349-360, July-Oct '48
AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

The Fine Structure of Wind in the Free Air

by C.S. Durst, B.A.

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

Summary

The Fine Structure of Wind at heights of about 5,000 feet and 20,000 feet is examined by means of smoke puffs released at the appropriate height and followed in a camera obscura. These puffs are released over the camera at short intervals during a period of an hour.

It is found that there are fluctuations in the wind of the free air, the periods of which are 5 to 15 minutes. In the cases considered the vector standard deviations of the fluctuations range from 0.5 knots to 3.5 knots. On making similar calculations from 5 minute mean winds measured from surface anemograms fluctuations of the same order of magnitude are found.

The observations are of necessity made in clear air. It is believed the fluctuations would be greater in some types of cloud.

No obvious connection has at present been found between the magnitudes of the fluctuations and the wind speed or the source of the air or the temperature structure.

1. Historical

The research given below is the result of a discussion on 1st August, 1946 between Mr. J.W. Frame, Orfordness Research Station, and Mr. C.S. Durst of the Meteorological Office. The latter expressed the view that there were comparatively small scale fluctuations in wind during short periods even in the free air and suggested that an aircraft circling over the Orfordness Camera Obscura and emitting smoke puffs at short intervals would settle this question. Mr. Frame developed this suggestion with the results given below, the field work being mainly done by Mr. Hicks and Mr. York of Orfordness Research Station. The subsequent handling of the data was done by Mr. G.H. Gilbert under the supervision of Mr. Durst.

2. The Method of Measurement of Upper Winds

In order to determine wind velocities above ground level, smoke puffs are fired by an aircraft at the appropriate height above a 30" camera obscura. The track of the puff across the table of the camera obscura is plotted manually by an observer, who marks the position of the puff by pencil every 10 seconds. The co-ordinates of each point referred to a 6" grid ruled on the table are read off at the conclusion of the exercise.

To reduce the readings of the positions on the table into actual velocities, it is necessary to know the height of the puff accurately. This is assumed to be the same as the height of the aircraft at the time of firing, and is determined by the pressure altimeter.

As the effective area of the sky covered by the camera obscura depends upon the height at which the aircraft is flying, trials at low heights would provide little data, if the puff is fired when over the centre of the table. The pilot of the aircraft is therefore instructed by radio to fly over the upwind boundary of the area and to fire the puff when crossing a diameter of the table. This gives the maximum length of trace. By circling under guidance from the ground, the aircraft is brought back to the same point approximately every two to five minutes during a period of an hour. Thus the wind is determined about 15 to 25 times in succession.

/In

In order to determine the variation of wind velocity with place at the same time, two or three puffs were fired in quick succession on the same runs giving simultaneous winds 400 feet to 2,000 feet apart.

3. The Data.

The present series consists of 12 sets of observations of up to an hours duration, and at heights of 2,000 to 4,000 feet; in 7 of these observations the two puff method was used. These 12 sets were made between 19th September and 18th October, 1946; the arrangement being that a months trial should be given. The heights at which the observations were made ranged from about 2,000 to 5,000 feet. In the case of a thirteenth observation the aircraft was at a height of about 20,000 feet. It is proposed to make further observations at the rate of about one a week throughout a year and at heights up to 25,000 feet, and possibly to 40,000 feet.

The data available from the present series are set out in table I. In these each "observation" consists of a series of "runs" (i.e. occasions when a smoke puff was followed across the table of the camera obscura and plotted at 10 second intervals); the total duration of an "observation" might be up to an hour. On those observations when two puffs were simultaneously followed across the table the plots of simultaneous runs are numbered a and b.

4. The reduction of the data.

Each run was treated as follows:-

(a) Components of the position of each plotting were measured in inches off the six inch grid marked on the tables, thereby giving co-ordinates on the table which are converted at once into space co-ordinates in feet by multiplying by the height of the aircraft in feet and dividing by 29 (the focal length of the lens being 29 inches). Mean velocities were then calculated over each 20 second intervals, the velocities being given in components referred to the axis of the plotting table which bore 159° true.

(b) Mean values of the wind velocity during each run were determined (i.e. from the observation of each smoke puff) and from these the average wind speed and direction during the observation; these are given in col.8 of table I. Then the standard deviations of the runs were determined from the average wind speed and these are set out in columns 9, 10 and 11.

(c) Diagrams were also drawn of the departure of the components, along and across wind of the mean wind velocities during each run, as is shown in fig.1, the ordinate being the time of the first plot of the particular smoke puff on the plotting table. Fig.1 is for the observations on 4th October, 1946. To illustrate all the observations on this scale would involve too much space but Fig. 2 gives a rather more condensed picture.

In addition in columns 13 to 22 there are set out certain particulars of the winds temperatures and weather of the air over East Anglia at the times of observation.

Figures I and II show graphically the results of the investigation. For II consists of a series of diagrams each of which shows the measurements made during one observation of anything up to two hours duration. Time is measured along the abscissae, velocity along the ordinates. Each point represents the departure (along or across wind) from the mean velocity (of the whole observation) of a run starting at the time indicated, so the graphs represent the fluctuations of wind measured at short intervals over a fixed point of the earth's surface. Temperature-pressure height, diagrams are also given.

5. Check on the Accuracy of Observations.

It is very important to find out from any internal evidence what accuracy the observations may be expected to have.

In the first place when two puffs are followed across the table simultaneously it is seen from figs. I and II that the resulting curves of the departures of the winds deduced from simultaneous runs when plotted are broadly similar.

/It

It might, however, be objected that any unrecognised alteration in height of the aircraft between one run and another would have the effect of making the apparent winds from both puffs in error by the same amount. Such an error in height of the smoke puff would make the wind speed incorrect but not the direction. Hence provided there is no big general swing of the wind direction, agreement between the standard deviations of the downwind components and those for the cross wind components is a criterion that the errors due to incorrect height are comparatively small. In Table I accordingly the standard deviations are given for the cross wind components as well as down wind, and the diagrams in fig. 2 are shown resolved along and across wind. It is seen that the fluctuations are as large across as down wind, indeed the mean value of all the observations combined of the standard deviation across wind is 1.4 kts. and that along wind is 1.3kts. The diagrams too show that the agreement between the fluctuations in any two series of runs is as great in both directions. For each case of observations with double puffs the vector correlation coefficient* was calculated between the simultaneous runs and these are set out in column 12 of Table I.

Secondly the observations on 4th October were examined in greater detail to endeavour to obtain a degree of accuracy for the 20 second mean values of wind determined from the smoke puffs. It was found that the standard deviation of a 20 second determination of wind from the mean value of its run was 1.1 knots. In making this computation no attempt was made to reject large values, but it is probable that some of the 1.1 knots was due to inevitable inaccuracies in plotting and measurement as can be seen to be the case by the trend of the grouping of dots in Fig. 3 and particularly Fig. 3(b). A further calculation was made, excluding by a mechanical procedure excessive values. (The standard deviation was then reduced to 0.5kts.) However, let us suppose that the apparent fluctuations are due to pure chance and that there were on the average 7 plottings in each run. The mean values of the runs would have a standard deviation of about 0.4 knots, (and if we exclude excessive values this standard deviation becomes 0.2 knots).

In addition in a number of cases where simultaneous observations on two smoke puffs were available correlation coefficients were calculated between the simultaneous departures of 20 second means from the mean of the run. These coefficients varied between 0.4 and -0.3.

To sum up the conclusions of this section, the accuracy of a determination of wind by camera and obscura a smoke puff which was observed for one or two minutes, is high and is probably of the order of ± 0.4 or 0.5 knots, but the accuracy of the 20 second means is more doubtful.

6. Magnitude of Deviations due to Real Wind Changes.

The standard deviations of vector departures of the individual runs from the mean speed of the observation are shown in column 9 of Table I. They range from $3\frac{1}{2}$ knots down to just over $\frac{1}{2}$ knot, and the mean is about 2 knots. This result must not be taken too generally for it has to be remembered that all these observations were made in clear air; in cloud the fluctuations would be greater.

/The

---

* The vector correlation coefficient is $\Sigma\, |V_1||V_2| \cos.\theta / (\Sigma V_1^2 \Sigma V_2^2)^{1/2}$ where V_1, V_2 are vector velocities and θ the angle between them.

The fact which stands out most strikingly in Figure 2 is the reality of the eddies which can be seen affecting the successive smoke puffs at about 4,000 feet (but even at a height of 20,000 feet similar fluctuations may be seen in the observations made on 20th December, 1946). The sizes of these eddies are greater than the distance apart of the successive smoke puffs and so are of the dimensions of 2 to 10 miles diameter.

From Figure 2 and the calculations above one has the impression that a small parcel of air moving in wind at 4,000 feet retains its own velocity for some minutes but that a neighbouring parcel some mile or two away may have a very different velocity. To put it in mathematical terms the velocity following the path of the fluid has not much change but the velocity measured at a particular stationary point has greater fluctuations. These fluctuations seem from figure 1 to be fairly slow in coming about and to occupy about 5 minutes to a quarter of an hour in passing across the camera.

It was remembered that the same sort of longer period fluctuations in wind were found near the surface at Cardington; they are shown in Geophysical Memoir 54 Fig. 32 p.37 as well as elsewhere. To make a comparison between the fluctuations near the surface and at 4,000 feet, the anemograms for Felixstowe were obtained for the days on which wind measurements were made at Orfordness. In Fig. 2 the measurements made from the Felixstowe anemograms on 23rd September and 11th October 1946 are plotted from $3\frac{1}{2}$ minute means in a similar manner to the upper air observations at Orfordness. It is seen that the fluctuations are of the same type though the measurements cannot be considered to be of great accuracy. The standard deviations on the two occasions are set out in Table II below.

Table II Standard deviation of Wind at about 2,000 feet over Orfordness and at the surface at Felixstowe.

Date	Time	Orfordness		Felixstowe	
		Wind speed and direction	Standard deviation	Wind speed and direction	Standard deviation
23rd Sept.	1550 to 1615	257° 15 kts.	3.5 kts.	265° $12\frac{1}{2}$ kts.	2.6 kts.
11th Oct.	10.15 to 1100	074° 24 kts.	0.7 kts.	073° 20 kts.	1.3 kts.

From this analysis there seems to be some evidence that the larger eddies, which are pictured in Geophysical Memoir 54, extend up to or above 2,000 to 4,000 feet but it requires more data of a similar character to those already collected to determine the precise form which they take and to what are due the large scale fluctuation.

Altogether there were 66 cases in which two smoke puffs were observed simultaneously. These were analysed according to distance apart; the magnitude of the vector difference in departures of wind during each run from the mean wind of the observation was plotted against distance apart of the two puffs. There was no obvious correlation from this plotted figure, though the distances apart varied from just over 100 feet up to 1700 feet (and in one case to 2100 feet). The great majority were 400 to 700 feet apart but the deviations were not consistently smaller than those at 1100 to 1600 feet apart.

A comparison was also made between the wind speed and the standard deviation of the velocity during runs from that of the whole observation. While 12 observations are too few for a final conclusion, there appeared to be no correlation.

7. Deviation of wind at 20,000 feet.

While these notes were still being written, observation was received of wind at about 20,000 feet made on 20th December 1946 vide obs. 13 of Table I. The fluctuations revealed by this observation are of the same type (apparently) as those seen at 2,000 to 4,000 feet and of the same order of magnitude the standard deviation being about 1.3 knots. Again there is no evidence that height errors are serious.

/8.

8. The Bearing of these results on the accuracy attributed to pilot balloon winds.

In Report No. M/Res 76 issued by A and AEE in February, 1936 comparison is made of the accuracy of pilot balloon observations by comparison with smoke puff observations. These experiments were made at Orfordness and the conclusion drawn was that the pilot balloon observations were often in error by as much as 3 knots; the accuracy of the smoke puffs was assessed at about $\frac{1}{2}$ a knot. The importance of the variation of wind with place, however, does not seem to have been fully appreciated when M/Res 76 was written; comparison was made between the wind measured by smoke cloud over the camera obscura and the simultaneous determination of wind by a balloon which was perhaps 5 miles distant and thus from the finding of the present investigation might well be in quite a different wind regime.

stribution

ituation

gh pressure

h pressure

bars

made in
cold front

Pressure

8 b							5700)	Continental	Wedge of High Pressure
9 a 9 b	10.10.46	10.41	12.18	8	43	- 6	2000	Modified Polar Continental	Anticyclone centred to north of Scotland
10 a 10 b	11.10.46	10.17	10.59	12	78	-	-	Modified Polar Continental	Anticyclone centred to north of Scotland.
11 a 11 b	14.10.46	10.28	11.24	13	69	10	4000	Polar Maritime	Wedge of high pressure
12 a 12 b	18.10.46	11.16	11.53	8	41	- 8	3000	Polar Maritime	Anticyclone centred over North Sea.
13 a 13 b	20.12.46	14.26	15.33	8	80	- 6	2200	Modified Polar Continental	Wedge of high pressure

Th
eddies
feet (l
observa
than th
of 2 to

Fr
parcel
minutes
differe
path of
statior
to be f
hour in

It
were fo
54 Fig.
fluctua
were ob
In Fig.
and 11t
the upp
are of
accurac
below.

Table I
the sur

Date

23rd Se

11th Oc

Fr
which are pictured in Geophysical Memoir 54, extend up to or above 2,000 to 4,000 feet but it requires more data of a similar character to those already collected to determine the precise form which they take and to what are due the large scale fluctuation.

Altogether there were 66 cases in which two smoke puffs were observed simultaneously. These were analysed according to distance apart; the magnitude of the vector difference in departures of wind during each run from the mean wind of the observation was plotted against distance apart of the two puffs. There was no obvious correlation from this plotted figure, though the distances apart varied from just over 100 feet up to 1700 feet (and in one case to 2100 feet). The great majority were 400 to 700 feet apart but the deviations were not consistently smaller than those at 1100 to 1600 feet apart.

A comparison was also made between the wind speed and the standard deviation of the velocity during runs from that of the whole observation. While 12 observations are too few for a final conclusion, there appeared to be no correlation.

7. Deviation of wind at 20,000 feet.

While these notes were still being written, observation was received of wind at about 20,000 feet made on 20th December 1946 vide obs. 13 of Table I. The fluctuations revealed by this observation are of the same type (apparently) as those seen at 2,000 to 4,000 feet and of the same order of magnitude the standard deviation being about 1.3 knots. Again there is no evidence that height errors are serious.

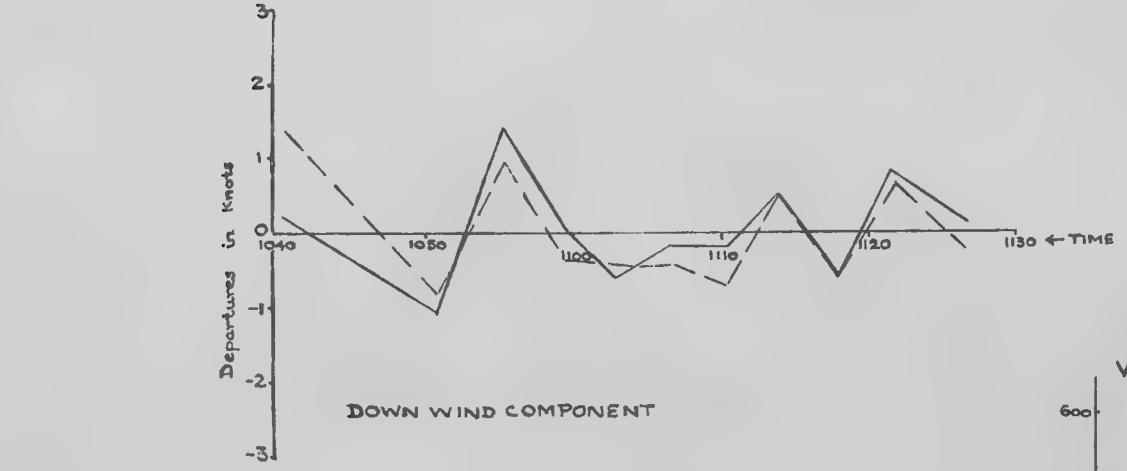
/8.

TABLE I. Data of Wind Structure at Orfordness September - December 1946.

Number of observation	Date	Time of start	Time of end	Number of runs	Average duration of run	Average height	Average wind speed and direction	Standard deviation of vector departures	Standard deviation of departures down wind	Standard deviation of departures cross wind	Vector correlation co-efficient of departures of double puffs	Radio winds at Downham Market preceding and following smoke puff observations		Average lapse rate of temperature from surface to level of observations	12.00 hr. observations at Felixstowe					Type of Air Mass	Pressure distribution or synoptic situation
															Weather	Type of low cloud	Type of medium cloud	Total cloud amount	Height of low cloud		
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
		G.M.T.	G.M.T.		Seconds	Feet	Knots	Knots	Knots	Knots		Knots	Knots	Degrees F per 1,000ft.				tenths	feet		
1	19.9.46	14.43	15.40	16	79	3420	264°17	2.0	1.2	1.7		280°25	250°19	4.1	partly cloudy	Cu.	Ac.	4 - 6	3000	Polar Maritime	Wedge of high pressure
2	21.9.46	10.01	11.03	14	86	4150	308°22	2.7	2.3	1.3		300°38	300°33	3.8	cloudy	Large Cu.	Ac.	9	2500	Polar Maritime	Wedge of high pressure
3	23.9.46	15.51	16.17	11	45	2560	257°15	3.5	2.8	2.1		260°20	260°18	4.3	showery	Sc.	-	10	1000) 4000)	Polar Maritime	Depression
4	24.9.46	15.14	16.10	20	60	2800	250°15	1.6	1.3	1.1		270°27	200°11	4.6	hazy	Large Cu.	Ac.	9	2000	Polar Maritime	Parallel isobars
5 a	4.10.46	10.40	11.27	11	66	4000	286°20	1.5	0.7	1.3)	.93	290°23	270°24	4.0)	cloudy	Sc.& Large Cu.	-	9+	2500) 5700)	Polar Maritime	Observations made in advance of a cold front occlusion.
5 b								1.6	0.7	1.5)											
6	7.10.46	11.42	12.29	9	46	1980	343°8	2.1	1.2	1.7		040°8	030°4	4.3)					2500)	Polar	Wedge of High Pressure
7 a	7.10.46	15.16	15.38	8	81	3800	005°5	0.7	0.5	0.4)	.77	030°4	100°11	4.6)	cloudy	Cu. & Sc.	-	9+	4000)	Continental	
7 b								0.5	0.3	0.4)											
8 a	8.10.46	15.27	16.12	9	59	3890	048°16	2.6	1.3	2.2)	.88	090°18	100°16	4.9)	cloudy to partly cloudy	Sc.& Large Cu.	Ac.	7 - 8	2000) 5700)	Polar Continental	Wedge of High Pressure
8 b								3.0	1.7	2.5)											
9 a	10.10.46	10.41	12.18	8	43	5000	081°31	2.1	1.0	1.8)	.81	090°32	080°34	2.8)	partly cloudy	Cu.	Ac.	4 - 6	2000	Modified Polar Continental	Anticyclone centred to north of Scotland
9 b								2.2	1.0	1.9)											
10 a	11.10.46	10.17	10.59	12	78	5070	074°24	0.8	0.7	0.4)	.57	090°35	070°21	2.8)	cloud-less	-	-	-	-	Modified Polar Continental	Anticyclone centred to north of Scotland.
10 b								0.6	0.4	0.3)											
11 a	14.10.46	10.28	11.24	13	69	2910	024°9	0.9	0.5	0.8)	.83	010°10	300°8	4.1)	cloudy	Sc.	-	10	4000	Polar Maritime	Wedge of high pressure
11 b								1.0	0.5	0.9)											
12 a	18.10.46	11.16	11.53	8	41	2040	109°18	1.3	1.1	0.8)	.69	120°12	130°15	4.9)	cloudy to partly cloudy	Sc.	Ac.	7 - 8	3000	Polar Maritime	Anticyclone centred over North Sea.
12 b								2.2	1.8	1.2)											
13 a	20.12.46	14.26	15.33	8	80	19400	001°8	1.3	0.8	0.9)	.68	050°7	040°12	2.9)	partly cloudy	Sc.	-	4 - 6	2200	Modified Polar Continental	Wedge of high pressure
13 b								1.4	1.2	0.5)											

FIG. 1. DEPARTURES OF WIND VELOCITY FROM THE MEAN VALUE OF ALL RUNS M.R.P. 338

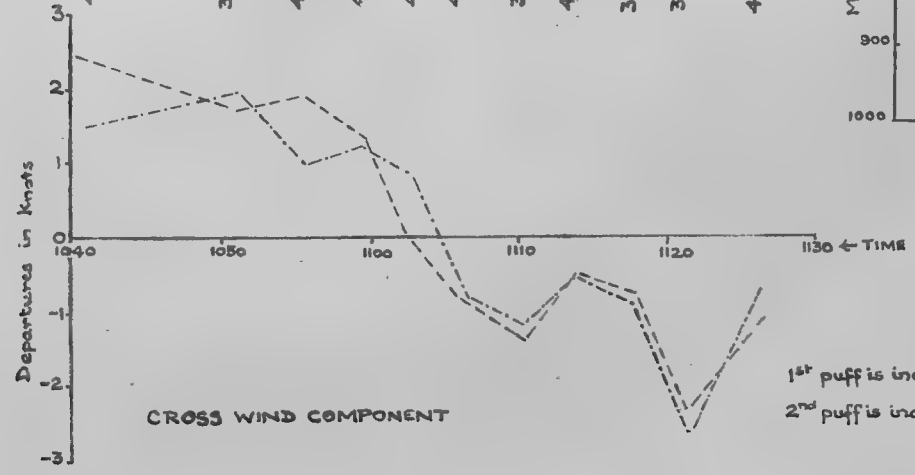
4th OCTOBER 1946

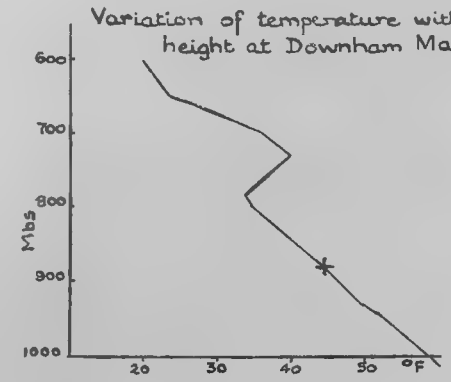


Distance apart of puffs in feet.

Height of puffs in feet.

4000 (370) 3730 (560) 4150 (420) 4000 (600) 4023 (620) 4023 (490) 3900 (600) 4120 (700) 3890 (540) 3990 (520) 4120 (720)





DOWN WIND CROSS WIND

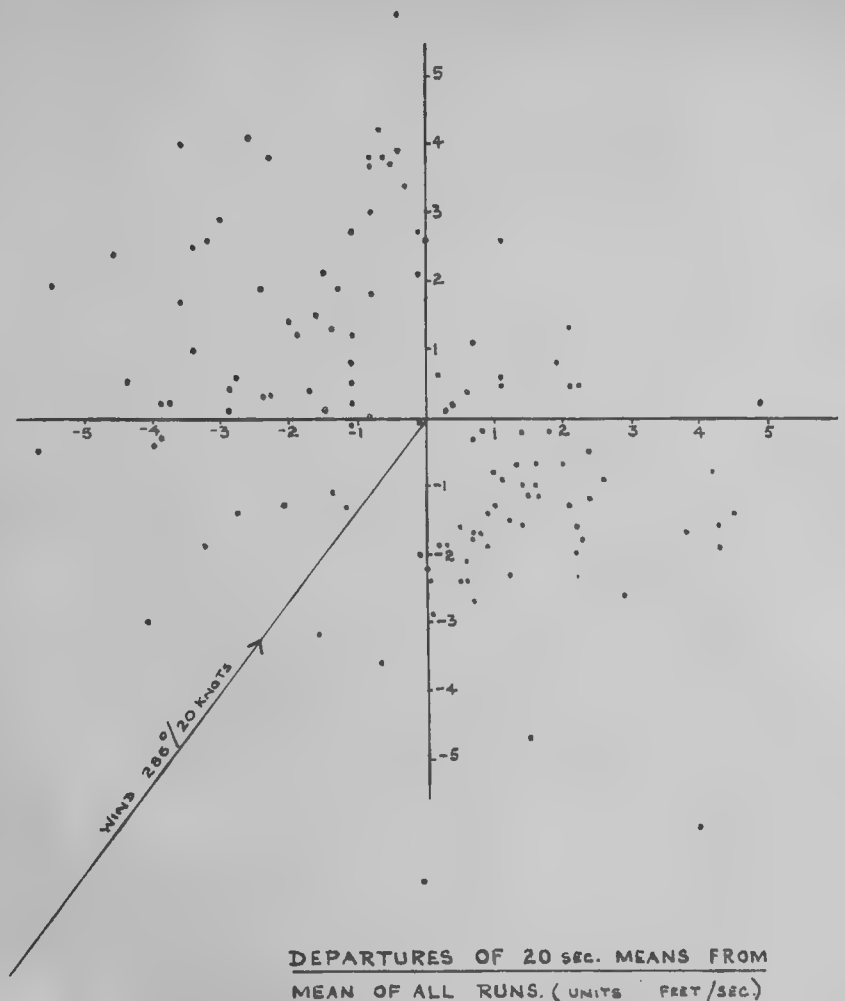
1st puff is indicated: ——— - - - - -

2nd puff is indicated: — — -·-·-·-

1/2770

FIG. 3(a)

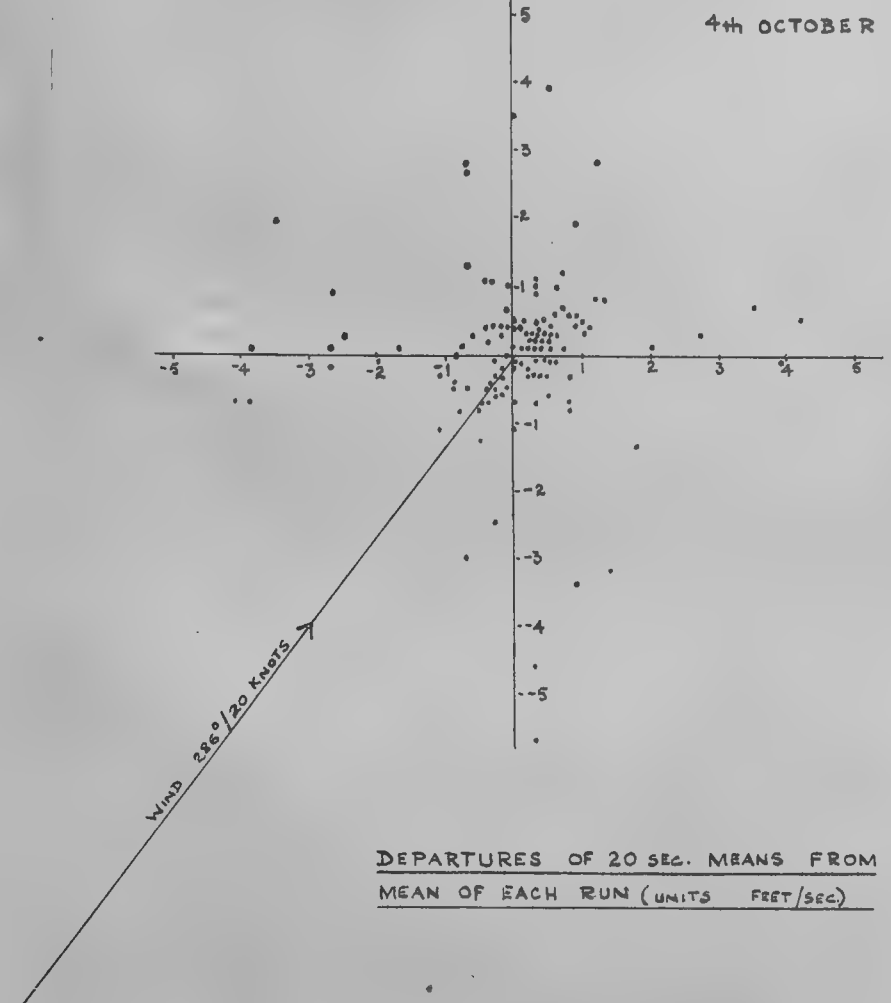
4th OCTOBER 1946



DEPARTURES OF 20 sec. MEANS FROM MEAN OF ALL RUNS. (UNITS FEET/SEC.)

FIG. 3(b)

4th OCTOBER 1946



DEPARTURES OF 20 SEC. MEANS FROM MEAN OF EACH RUN (UNITS FEET/SEC.)

12780

This re... publi... Co... contents must not be quoted

AIR MINISTRY
METEOROLOGICAL RESEARCH COMMITTEE

M.R.P. 341
28th April 1947

High Altitude Sounding Balloons

by
O.M. ASHFORD

1. Introduction

In September 1945, the Instruments Branch of the Meteorological Office was asked to consider the design of a balloon capable of carrying a useful load of about 1 kilogram to a height of 100,000. The object of this note is to report progress to date.

2. Performance of Existing Balloons

Sounding balloons are normally made in three sizes 350 gm., 500 gm. and 700 gm. The average bursting height with a load of 1500 gm. is in the region of 60,000 ft., but occasional ascents have been made to 75,000 ft. The main factors limiting the bursting height are the decrease in extensibility as the temperature falls, chemical action of oxygen and ozone and mechanical defects. Improvements might therefore be made (a) by using larger balloons or more than one balloon so that the elastic limit for a given load would not be reached until a greater height (b) by using better anti-oxidants and/or dyes in the rubber mix (c) by using materials other than natural rubber or (d) by using a valving technique to allow some hydrogen to escape when the internal pressure becomes too great.

3. Experiments with Large Balloons

It was estimated that a balloon weighing 10 kilograms, capable of expanding to 40 ft. diameter, would be necessary to carry a load of 1 kilogram to 100,000 ft. As a step in this direction, Messrs Guide Bridge produced six balloons weighing 4,500 gm., four of which were flown at Larkhill with the results shown in Table I. The average bursting height of 71,000 ft. is appreciably greater than for smaller balloons, and the relationships between bursting height and $\frac{W_2}{L}$ and free lift may be significant. Further experiments are called for. The heights were measured by radar.

4. Experiments with Balloons in Tandem

To obviate the difficulty in tying two ordinary balloons together without the risk of the string chafing the rubber, Messrs. Guide Bridge produced some special 700 gm. balloons with two diametrically opposite necks. Twenty four of these balloons were flown at Larkhill, with the results shown in Table II. Unfortunately, radar observations could not be made, and the bursting heights had to be estimated from radio-sonde readings. It was found that at extreme heights the windmill operated switch of the M.O. radio-sonde does not function satisfactorily. The results were somewhat erratic but appreciably greater heights than usual were obtained on 11 of the 15 useful ascents. With the tandem balloons there are many possible variations in technique, e.g. upper balloon with greater free lift than lower, pressure equalised between balloons by connecting tube, etc., and it is considered that further experiments would lead to profitable results.

5. Experiments with Dyed Balloons

R. & D.E. had found earlier that balloons coloured with a special orange dye withstood exposure to daylight much longer than other balloons. A batch of seventy orange balloons was tested at Downham Market, and the bursting heights were if anything slightly lower than those obtained with white balloons, both by day and by night.

/6. Alternative

6. Alternative Materials

The obvious alternatives to natural rubber are artificial rubber and non-extensible materials such as that used by Dr. Regener. Of the artificial rubbers, neoprene is best for meteorological balloons, and the Americans claim to have made some capable of reaching 100,000 ft. (Spilhaus 1946). Neoprene latex is not made in this country and there are practical technical difficulties in transporting it from America. No immediate experiments can therefore be made over here, other than with American balloons.

The non-extensible materials used by Regener (Regener 1943) are being examined by R. & D.E. and it is possible that some may be made over here. The non-extensible balloons suffer from the disadvantage that they have to be very much larger at the start than ordinary balloons.

References.

Regener E. Deutsche Akad der Luftfahrtforschung
Schrifton Heft 37. 1943

Spilhaus A. Bull. Amer. Met. Soc. Vol. 27. p.405. Sept. 1946

G. 149565

No.	Date 1947	G.M.T. Approx.	Balloon				Weight carried Kg.	Free lift Kg.	Height burst ft.	Rate of ascent to 40,000 ft. ft/mins	Remarks
			No.	Wt. Kg. (W)	Length ins. (L)	W/L^2					
1.	Mar 27	0930	40	4.65	140	.238	1.9	5.5	80,100	1420	Radio-sonde carried in addition to radar reflector.
2.	" 31	1400	22	4.55	154	.192	1.2	5.0	45,000	1400	
3.	Apr 1	1400	26	4.39	144	.212	1.2	3.5	58,500	1310	
4.	" 2	1345	28	4.61	144	.223	1.2	2.5	(100,000)	800	Max. height measured 83,000 ft. when range too great for G.L. III. Balloon found at Reigate and height of 100,000 ft. deduced from data.

Table I. Ascents with 4,500 gm. balloons.

Ascent No.	Bursting Height ft.	Remarks
1.	49,000	Lower balloon burst first.
2.	51,000	" " " "
3.	64,000	" " " "
4.	> 66,000	" " " "
5.	74,000	" " " "
6.	53,000	Switch stuck on Temp. or Humidity and little subsequent change.
7.	56,000	Probable that upper balloon burst first and lower balloon remained floating.
8.	69,000	
9.	76,000	
10.	83,000	
11.	> 66,000	Switch stuck. Windmill started turning again after burst. Reading of 78,000 ft. on descent.
12.	> 73,000	" " " " " " " " " 93,000 " " "
13.	> 67,000	Burst not observed. Modulation failure.
14.	64,000	Two double-necked balloons used.
15.	71,000	Balloons joined by tube to equalise pressure.

Note:- Except in ascent No. 14, the upper balloon was a normal 500 gm. type.
Free lift of upper balloon normally 2200 gm., and of lower balloon 2000 gm.

Table II. Ascents with balloon in tandem.

MRP's 344, 346, 350, 352, 353

his reno ... be treated
... CONFIDENTIAL ... Its
contents must not be quoted.

M.R.P. 344 (Revised)
19.5.47.

C. E. Brunt. AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE.

A Note on the Adiabatic Lapse Rate for Saturated Air at Temperatures below 0°C

by

J.K. Bannon.

1. Introduction and Summary.

It is generally acknowledged that super-cooled water drops can exist at temperatures far below freezing and are of common occurrence in cloud.

In many cases, therefore, it is to be expected that as saturated air rises, as for instance in a large cumulus cloud, its surplus water vapour over and above its saturation water vapour content, will condense into super-cooled water drops and not into ice particles.

But a mass of air, at temperatures below freezing, ascending and condensing its surplus water vapour into super-cooled water drops, will cool at a different rate from that if the surplus water vapour condenses into ice. This is because the latent heat of vapourisation of water is less than the latent heat of sublimation of ice, and also because the pressure of saturated water vapour is less over ice than over water.

Thus, since less heat is released on condensation into water than on condensation into ice, the adiabatic lapse rate of saturated air condensing its surplus water vapour into super-cooled water drops, would be expected to be greater than that for air releasing its surplus water vapour to ice. This appears to be so, at any rate to temperatures as low as about 245 to 250°A; below 245°A, the different behaviour of the saturated vapour pressures over water and ice respectively, probably reverses the magnitudes, though the differences in the two lapse rates is very small.

Tables are given below for the computation of the two saturated adiabatic lapse rates, assuming that the heat content of the solid or liquid water is negligible compared with that of the air.

It is also shown how to make the necessary, (small), corrections to these saturated lapse rates to allow for the heat content of the condensed water vapour.

The cumulative effect of the difference between the two lapse rates is shown to result in a temperature difference of about 1°C between the tops of super-cooled water and ice clouds respectively, after convecting through 10,000 ft. above freezing level.

2. Lapse Rate of Saturated Air - process with respect to Ice.

Professor Brunt has given details of the calculation of the saturated adiabatic lapse rate in a well known paper. (Q.J.R.Meteor. Soc., 59, 1933, page 351). There he assumes that at temperatures below freezing the air condenses its surplus water vapour directly into ice. He gives a table (page 355) from which the lapse rate may be quickly calculated for any given temperature and pressure by means of the formula:-

$$\frac{\text{saturated lapse rate}}{\text{dry adiabatic lapse rate}} = 1 - \frac{Z - X}{Z + p} \qquad (1)$$

where p is the pressure in millibars and $Z - X$ and Z are given in the table.

/Unfortunately

45.5.334

Unfortunately the calculations there have several errors. The column (5) of this table is based on a value of $dL/dT = 0.5$, whereas it should be -0.5, and there are several other arithmetical mistakes. The table has, therefore, been recomputed over the range 305° to 230°A and is given below as Table I.

This calculation is based on assumptions implying that the heat content of any free water (or ice) held by the air is small compared with that of the air.

Table I

The Functions Z, X, and Z - X. --- Ice.

1	2	3	4	5	6	7	8	9	10
T	L	e	$\frac{de}{dT}$	$e.(c+\frac{dL}{dT})$	$L\frac{de}{dT}$	(5)+(6)	$Z = (7).\frac{\epsilon}{\sigma_p}$	$X = \frac{\epsilon Le}{A RT}$	Z - X
305	580	47.60	2.69	21.3	1561	1582	4108	821	3287
300	582	35.69	2.10	16.0	1222	1238	3215	628	2587
295	585	26.46	1.61	11.8	942	954	2478	476	2002
290	588	19.38	1.23	8.7	723	732	1901	356	1545
285	591	14.03	.92	6.3	544	550	1428	264	1164
280	593	10.02	.69	4.5	409	414	1075	192	883
275	596	7.05	.51	3.2	304	307	797	139	658
273	597	6.11	.44	2.7	263	266	691	122	569
273	677	6.11	.50	3.05	338	341	885	137.6	747
270	677	4.77	.39	2.39	270	272	706	111.5	595
265	677	3.12	.27	1.56	183	185	480	72.2	408
260	677	2.00	.18	1.00	122	123	319	47.3	272
255	677	1.26	.12	.63	81.2	81.8	212	30.3	182
250	677	.780	.077	.390	52.1	52.5	136.3	19.1	117
245	677	.472	.049	.236	33.2	33.4	86.7	11.9	74.8
240	677	.279	.030	.139	20.3	20.5	53.2	7.2	46.0
235	677	.161	.019	.080	12.9	13.0	33.7	4.2	29.5
230	677	.091	.010	.045	6.8	6.8	17.7	2.4	15.3

T = temperature in °Absolute.

e = saturation vapour pressure over water (above 0°C), or over ice, (below 0°C), in millibars.

L = latent heat of water (above freezing point), or the latent heat of sublimation of ice, (below freezing) in g-cals/gm

c = specific heat of water, (above 0°C), or ice (below 0°C).

σ_p, A and R have their usual meanings.

ϵ = the ratio of the specific gravities of water vapour and dry air.

3. Lapse Rate of Saturated Air - process with respect to Super-cooled Water.

The same calculations have been made assuming that the air is saturated relative to water, even below freezing point, and that the surplus water vapour condenses into super-cooled water drops. The results are given in Table II overleaf and cover the range 273° to 235°A.

/Table II.

This report has not yet been published and is to be treated CONFIDENTIALLY. Its

H.J. Margetson

contents must not be quoted.

23rd May, 1947

AIR MINISTRY

Aec. 48.5486

Con Brit Meteorological Research Committee

Anomalous Sound Reception Experiments

by

R.J. Murgatroyd

Summary.

This report describes the results of 16 experiments made in Central and Southern England between April 1944 - April 1945 in which recordings of sound received by anomalous paths from large explosions were utilised in an attempt to obtain data on winds and temperatures at great heights. The methods used and their limitations are outlined and the results of the recordings tabulated. Within the limitations imposed by the method the following main results were obtained:-

1. In these latitudes the wind at heights of 30-45 Km. above the earth's surface appears to exhibit a monsoon effect. In winter the direction is generally between South West and North West with speeds of 40 to 80 metres per second. In summer the direction is between North East and South East with speeds less than 20 metres per second.

2. The temperature at 40-45 Kms. appears to increase in summer to values approaching surface values. This increase is not evident below 45 Kms. in winter and a 20-40°C variation of temperature between summer and winter is likely at these heights.

Introduction.

The phenomenon of Anomalous Sound Propagation is well known. The zones of audibility around a large explosion are generally:-

1. An "inner zone" of audibility extending outwards from the explosion point with size and shape determined by the magnitude of the explosion and the wind and temperature distribution in the troposphere.

2. A "silent zone" at greater ranges from the explosion point.

3. One or more "outer zones" of audibility at greater ranges still from the explosion point. On some occasions further zones of silence followed by other "outer zones" have been observed. A sketch of a possible audibility distribution around a large explosion is given in Fig.1A.

The characteristic features of the sound propagation to the outer zones the so-called "anomalous propagation"in temperate latitudes may be summarised:-

1. The outer zones have a minimum range from the explosion point usually between 100 and 200 Km. and less in winter than in summer.

2. The time taken by the sound to reach the outer zone is usually one or two minutes longer than it would take theoretically if it travelled at its surface speed along the surface from explosion point to outer zone.

3. The outer zone may exist all around the explosion point but there is evidence that it is most frequently to the west of it only in summer and to the east in winter.

4. It is very common in the outer zone of audibility to hear two or more sounds from a single explosion (multiple sound arrivals).

The fact that sound travelling to the outer zone has this minimum range before it reaches the surface and takes the comparatively long time to obtain this range suggests that it must follow a path through the Upper Air before returning to the surface. It is unlikely that this will include a reflection at the highest point of its trajectory because a discontinuity sufficient to produce the reflection is improbable in the atmosphere. Hence it appears that the sound must be continuously refracted in its path. Such a path would imply that the maximum horizontal speed of the sound relative to the earth will occur at the top of its trajectory in order for it to be bent down to earth again. If we apply the usual laws for the speed of sound in a gas it is seen that the speed relative to the earth depends on the composition and temperature of the atmosphere and the wind at any level. Moreover since it can be shown generally that neither a sufficiently great sound speed nor a sufficiently large time delay in sound travelling to a given range in the Outer Zone is given by paths considered up to the present limit of direct measurement of the wind, temperature and composition of the atmosphere (circa 20 Km), the phenomena of Anomalous Sound Propagation are likely to provide data about the atmosphere at levels exceeding this. A bibliography of some of the works on Anomalous Sound Propagation is appended. The main results which they describe are summarised:-

1. The maximum trajectory heights attained by the sound "rays" are usually about 40-50 Km. above the earth's surface.

2. The most probable cause of the high sound speeds at these levels is that the temperature there approaches surface values i.e. the general picture of the temperature structure of the atmosphere up to a height of say 50 Km. in temperate latitudes consists of a layer from the surface to about 10 Km. (the troposphere) with temperature decreasing with height, an isothermal layer from 10 Km. to about 35 Km. and finally an inversion layer from 35 Km. to 50 Km. or so. Structures given by prominent workers in ^{o}K for summer are tabulated:-

Heights in Km.	20	25	30	35	40	45	50
Whipple.	219	220	223	245	282	310	336
Duckert.	219	220	223	267	316	335	344
Gutenberg.	219	220	223	256	306	330	349

3. The Outer Zones of audibility are not usually symmetrical about the explosion point and in fact appear most usually to the west of it in summer and the east in winter although sometimes they are observed on all sides. This observation suggests that the wind at these levels is likely to have predominantly easterly components in summer and westerly in winter - a monsoon effect.

During the period April 1944 - April 1945 the opportunity arose of performing set experiments to use the phenomena of anomalous sound propagation to obtain further information about the wind and temperature at high levels and the methods utilised are described in detail below.

Measurements Available.

1. The positions of the explosion points and the recording microphones can be surveyed accurately. Microphones are used in conjunction with an Einthoven String Galvanometer Recording Instrument which produces a trace of the sound waves received (which are of frequency 0-10 c.p.s. usually). The range X and azimuth α from explosion point to microphone can then be computed, preferably in terms of spherical co-ordinates although it was found that it was sufficiently accurate to compute them on the Cassini grid in these experiments.

This simplification introduced errors of less than 4 minutes of angle and 10 yards of range which are very small compared to other errors introduced in this work.

2. The total time of sound travel T of the sound from explosion to microphone is measured by marking the explosion time on the recording trace by means of an operator shouting over the telephone communication from explosion to recorder point. The error introduced in T by this method is the reaction time of an operator between hearing the sound over the telephone and pressing a switch on the recorder and should be between 0.5 and 1 second.

T is usually 10-15 minutes and was conveniently measured in "sound metres". A sound metre is defined as a unit of time equal to 1/337.6 seconds - the speed of sound being 337.6 m/sec. at 10°C and 50% Relative Humidity.

3. If instead of using a single microphone, a lattice of a minimum of 3 microphones is suitably deployed (say 5-10 Km. apart in triangles, or squares etc.), the relative arrival times of the sound at the various microphones can be measured. A knowledge of the surveyed positions of these microphones and the arrival times at each of them allows one to deduce the apparent azimuth of travel β of the sound waves at the microphones and the "characteristic velocity" A of the sound wave as it travels over the microphone lattice. The method of calculating β, and A from a set of microphone readings is given below. These calculations have to take into account the fact that the sound wave is propagated spherically and also corrections for the relative heights of the microphones above a horizontal plane should be determined.

The "characteristic velocity" A is a basic parameter of the refracted path taken by the sound and the accuracy of its determination must be high to get reasonable results in the calculations of the sound trajectories. For this reason it is advantageous to have a lattice of several more microphones - say about a dozen - and extract the most probable value of A and β for the lattice by statistical methods. The exact time of sound inception on a record for a given microphone is often difficult to determine due to the fact that the intensity of the received sound may not be much greater than that due to local sources of interference.

The value A is equal to the horizontal sound speed at the top of the trajectory of the sound path. The value of $\phi = \alpha - \beta$ is a measure of the sideways deflection of the sound in its path due to cross components of wind.

4. Routine wind and temperature measurements by radio-sonde methods were available to a height of about 18 Km. during each of these experiments.

Applications of the Measurements and Review of the way in which they can be used.

During this work three main assumptions had to be made.

1. That the travel of the sound from explosion to microphones could be treated in terms of refraction only.

2. That the general law for the speed of sound in a gas was valid throughout the path.

3. That the atmosphere could be considered to be horizontally stratified i.e. that at any given level the wind and temperature would be the same all over the area of sound travel.

When the characteristic velocity A is known the path of sound within the region of known wind and temperature measurements (0 - 18 Km.) can be calculated uniquely to give the time T^1, range X^1, and sideways deflection Y^1 of the sound path below 18 Km.

Since the total time T, range X and deflection Y of the sound in its entire path are also measured the values of the T-T', range X-X' and deflection Y-Y' to be ascribed to the portion of the sound path above 18 Km. can be found.

Hence the problem becomes one of finding a sound path above 18 Km. for a reception along a given azimuth to satisfy -

(a) A remaining time T - T'
(b) A remaining range X - X'
(c) A remaining deflection Y - Y'
(d) A horizontal speed of A at the top of the trajectory.
(e) Known values of wind and temperature at the base of this "remaining layer" i.e. at a height of 18 Km.

It should be noted that there is no measurement of the maximum height attained by the sound and in fact this height will be defined only by the fact that it is the height at which the horizontal speed of sound equals the characteristic velocity. Moreover the number of measurements available is comparatively few so that they will be satisfied by a large number of meteorological structures. The portion of the sound path above 18 Km. will not then be determined uniquely unless some simple assumption is made about the way in which temperature and wind vary with height e.g. linearly, parabolically etc., each type of assumption producing a different meteorological structure and a corresponding sound path. This is illustrated in Fig. 1 B. For this reason all that can be hoped for from these experiments is a broad idea of the way in which temperature and wind vary above the limit of the radio-sonde measurements. It might also be added that there is likely to be obtained more and better information about winds than there is about temperature from this work because:-

(a) There is only one method available of obtaining temperature information. This is to calculate the way sound speed varies with height along at least three azimuths of reception from a consideration of the time and range parameters of these sound paths and then solve for the two wind components and the temperature at each level to satisfy the sound speeds along the three azimuths at that level.

(b) The calculation of (a) also provides the wind at each level but in addition there are two other sets of data which will give information about the wind.

1. Assymetry in the audibility diagram will be a wind effect because the effect of temperature is the same in all directions.

2. The sideways deflection Y - Y' is due to cross-wind effects at right angles to the azimuth of sound reception considered. A method of utilising this information as well as a full description of the method outlined in (a) above will be given below.

(c) (a) above depends on one of the basic assumptions made i.e. that the sound propagation follows the standard law for the speed of sound in a gas (Laplace Law). Some writers have suggested that this assumption may fail at the highest levels because the atmospheric pressure there is not very large compared to the pressure of the sound wave itself. If that is so there will be a blast-wave (Riemann) type of propagation there. But even if this were so some wind information would still be available from (b) above. A further suggestion is that the increased sound speed necessary at the top of the trajectory could be produced by a large change in the composition of the atmosphere e.g. if the atmosphere were largely hydrogen or helium at 40 to 50 Km. This has also been largely disapproved and it is generally believed that the increased horizontal sound speed necessary at the high levels to explain the audibility results is due to high temperature or following wind components there. The calculations made in this work were therefore carried out on the assumption that the anomalous recordings could be explained in terms of pure refraction by wind and temperature.

MATHEMATICAL AND COMPUTATIONAL METHODS.

The Refraction of Sound in the Atmosphere.

Throughout this work it was assumed that all receptions of the anomalous sound waves could be explained in terms of refraction by wind and temperature. It was further assumed that the atmosphere could be regarded as stratified. The propagation of sound under these conditions has been fully treated by Dr. E.T. Goodwin of the Cambridge University Mathematical Laboratory in E.B.D. Reports Nos. 16,17,18 and 19. "The calculation of meteor corrections for use in the location of enemy guns by sound-ranging methods" and the results obtained in these reports are outlined below using the same notation.

The source of sound S and the microphone M are assumed to lie at the same level, so that the sound path will be symmetrical in elevation if the atmosphere is stratified. A correction for height differences between source and microphone is given below. The distance SM is denoted by X, the total sound time by T and the maximum height of sound "ray" trajectory by H. (x, h, y) are the coordinates of a typical point P on the path with respect to the axes SM, the vertical and the mutual perpendicular through S and t the time to reach P from S. The wave-normals along the path are all parallel to a fixed vertical plane ϖ the constant angle between ϖ and the xh plane is denoted by ϕ and the horizontal wind has components w , and w^1 parallel and perpendicular to this plane ϖ at any height h . The velocity of sound a = constant $\sqrt{K}$ where K is the absolute temperature ^oK at any height. The inclination of the wave normal to the horizontal is denoted by ψ
The general refraction law is

$$a \sec\psi + w = \text{constant } A \qquad \ldots\ldots\ldots (1)$$

where A is the "characteristic velocity" of the sound "ray" considered and is equal to the horizontal velocity of the sound at the top of its trajectory.

Also we have

$$\phi = \text{constant} \qquad \ldots\ldots\ldots (2)$$

and the kinematical relations

$$dx/dt = (a\cos\psi + w)\cos\phi - w^1 \sin\phi \ldots\ldots (3)$$

$$dh/dt = a \sin\psi \ldots\ldots (4)$$

$$dy/dt = (a\cos\psi + w)\sin\phi + w^1 \cos\phi \ldots\ldots (5)$$

It follows from these equations that :-

$$X\cos\phi = 2\int_o^H \frac{a\cos\psi + w}{a\sin\psi}\,dh \qquad \ldots\ldots (6)$$

$$Y = X\sin\phi = -2\int_o^H \frac{w^1}{a\sin\psi}\,dh \qquad \ldots\ldots (7)$$

$$T = 2\int_o^H \frac{dh}{a\sin\psi} \qquad \ldots\ldots (8)$$

If we write at any level :-

$$n = A - (a + w) \qquad \ldots\ldots\ldots (9)$$

it is also possible to deduce the relationship :-

$$X\cos\phi - AT = -2\int_o^H \sqrt{\frac{n}{a}\left(2 + \frac{n}{a} \right)}\,dh$$

$$\ldots\ldots\ldots (10)$$

/ and

and (6) and (7) may be rewritten as:-

$$X \cos\phi = 2\int_0^H \frac{1 + \frac{w}{a}\left(1 + \frac{n}{a}\right)}{\sqrt{\frac{n}{a}\left(2 + \frac{n}{a}\right)}}\, dh \quad \ldots\ldots\ldots\ldots\ldots(11)$$

$$Y = X \sin\phi = -2\int_0^H \frac{\frac{w^1}{a}\left(1 + \frac{n}{a}\right)}{\sqrt{\frac{n}{a}\left(2 + \frac{n}{a}\right)}}\, dh \quad \ldots\ldots\ldots(12)$$

Equation (10) can be traced to Fermat's principle that the actual path is the one of shortest time and the above reports show how the process of finding the minimum time of sound path can be utilised to find the maximum height H and the time of travel T for a given time X when the meteorological structure is known.

In the present problem the meteorological structure is unknown as is also the maximum height H , while the values of range X, time of sound path T, "angle of twist" ϕ , and characteristic velocity A are all measured.

Although it is possible using the above equations to obtain a unique determination of the sound path when the meteorological elements are known, the converse i.e. of finding the meteorological elements at all heights when the main parameters of the sound path are known does not appear to be capable of a unique determination. It is therefore necessary to postulate some form of wind and temperature variation with height (e.g. Linear, parabolic variation of w, w^1 and a with h) and then solve for the constants of the law of variation assumed. Moreover, the laws assumed will have to be comparatively simple in view of the small number of equations available in these experiments from which their constants will have to be deduced.

It thus appears that in general, experiments of this type will not lead to unique solutions for the wind and temperatures at great altitudes but on the other hand may lead to some general conclusions about the meteorological elements at great altitudes. Bearing these facts in mind the simplified methods described below were developed.

Approximate Methods.

If we assume w, w^1 and a to be even simple functions of h, equations (10) (11) and (12) lead to complicated expressions. Hence the following equations correct to first order in n/a_o were obtained. (a_o was taken to be the standard velocity of sound of 337.6 m/sec in air at 50°F and 50% Relative Humidity.)

$$X = \sqrt{(2a_o)}\int_0^H \frac{1}{\sqrt{n}}\left(\frac{A}{a_o} + \frac{5}{4}\frac{n}{a_o} - \frac{a - a_o}{2a_o}\right) dh \quad \ldots\ldots (13)$$

$$T = \sqrt{(2a_o)}\int_0^H \frac{1}{\sqrt{n}}\left(1 + \frac{3}{4}\frac{n}{a_o} - \frac{a - a_o}{2a_o}\right) dh \quad \ldots\ldots (14)$$

$$Y = \sqrt{(2a_o)}\int_0^H \frac{1}{\sqrt{n}}\left(\frac{w^1}{a_o}\right) dh \quad \ldots\ldots\ldots(15)$$

n/a_o may be as much as 0.25 in the lower stratosphere but usually it is much smaller than this.

Moreover since the largest contributions to X and T occur at levels where n is small it was considered to be sufficiently accurate to write:-

$$X \doteqdot T \doteqdot \sqrt{(2a_o)} \int_o^H \frac{dh}{\sqrt{n}} \qquad \ldots\ldots\ldots\ldots\ldots (16)$$

for the calculations in the "unknown layer" and

$$T - X = \sqrt{(2a_o)} \int_o^H \frac{1}{\sqrt{n}} \left\{ \left(1 - \frac{A}{a_o} \right) + \frac{2n}{a_o} \right\} dh \quad \ldots\ldots(17)$$

In the "known layer" i.e. from the surface to the greatest height of radio-sonde measurements where the wind and temperature values were known, the value of range X', time T', and sideways deflection Y' of the sound path in this layer were computed using formulae (13), (14) and (15). The unit for time T used throughout was the sound metre (1/337.7 sec) for symmetry in these expressions and the integrands were evaluated in 5000 ft. layers using Simpson's Rule.

This allowed values of $X - X^1$, $T - T^1$, and $Y - Y^1$ to be ascribed to the sound paths in the "unknown layer" above the greatest height of radio-sonde measurement.

In the unknown layer various laws of variation of wind and temperature with height were postulated and equations (15), (16) and (17) adapted to find the constants of these laws.

Laws of Variation in the "Unknown Layer".

Various systems were attempted and these are all outlined below:-

From (16) and (17) we have:-

$$\frac{X}{\sqrt{(2a_o)}} \doteqdot \frac{T}{\sqrt{(2a_o)}} \doteqdot \int_o^H \frac{dh}{\sqrt{n}} = g \text{ , say.} \qquad \ldots\ldots\ldots (18)$$

$$\frac{(T - X) + \left(\frac{A}{a_o} - 1 \right) X}{\sqrt{(2a_o)}} \frac{a_o}{2} = \int_o^H \sqrt{n}\, dh = f \text{ , say.} \quad \ldots (19)$$

A two straight line structure for the variation of n with h .

If at

$$h = 0 , \quad n = n_o$$

$$h = h_1 , \quad n = n_1$$

$$h = h_1 + h_2 , \quad n = 0$$

/ with

with straight line variation between (n_o , 0) and (n_1 , h_1) and between (n_1 , h_1) and (0 , $h_1 + h_2$) the integration of (18) and (19) gives:-

$$n_1 = \frac{a_o}{2X}\left\{ 3T - 3X + 3X\left(\frac{A}{a_o} - 1\right) - \sqrt{\frac{3}{2}\frac{2n_o}{a_o}}\, h_1 \right\} \quad \ldots\ldots \quad (20)$$

$$h_2 = \sqrt{\left(\frac{n_1}{a_o}\right)}\left\{ \frac{X}{2\sqrt{2}} - \frac{h_1}{\sqrt{\frac{n_1}{a_o}} + \sqrt{\frac{n_o}{a_o}}} \right\} \quad \ldots\ldots\ldots\ldots \quad (21)$$

Hence for any values of h_1 , values of n_1 , and h_2 can be found and a relationship between n and h is thus determined which will approximately satisfy the values of X, T and A along any trajectory.

Since

$$n = A - (a + w)$$

$$= A - a - D_N \cos\theta - D_E \sin\theta \quad \ldots\ldots\ldots\ldots \quad (22)$$

where θ is the azimuth of line SM with respect to true north and D_N, D_E are the northerly and easterly components of wind flow, this process has to be carried out for three values of θ i.e. there has to be three azimuths of reception to provide solutions for a (and hence temperature), D_N and D_E at various heights from three curves of n against h obtained by this method.

This method will produce a family of values as h_1 , is varied.

Parabolic variation of n with h .

The parabolic required will be of the form

$$n = \frac{n_o}{1 - \alpha}\left(1 - \frac{h}{H}\right)\left\{1 - \alpha^1\left(1 - \frac{h}{H}\right)\right\} \quad \ldots\ldots\ldots \quad (23)$$

i.e. it has :-

$n = n_o$ when $h = 0$

$n = 0$ when $h = H$

and α^1 and H have to be determined.

The type of solution required will generally be that having d^2n/dh^2 negative, i.e. α^1 positive and less than 1 .

By integrating (18) and (19) with this condition we obtain

$$g = \frac{2\rho H}{\sqrt{(n_o \tan\rho)}} \quad , \text{ where } \alpha^1 = \sin^2\rho, \quad \ldots\ldots\ldots\ldots \quad (24)$$

$$\frac{2f}{gn_o} = \operatorname{cosec}^2 2\rho - \frac{\cot 2\rho}{2\rho} \quad \ldots\ldots\ldots\ldots \quad (25)$$

Hence H and α^1 are determinable and a parabolic variation of n against h is found which will approximately satisfy the condition of sound travel.

As in (a) above we require three azimuths of reception between which we can obtain the values of D_E, D_N and temperature at the various heights.

Straight-line and Parabola Method. Temperature only.

It has often been assumed in previous work of this type that the refraction of the sound at high levels is due to the variation of temperature with height only and that the effect of wind can be ignored in certain cases.

Some of the "Single Azimuth" Experiments were worked out on this assumption and as well as the two straight-line type of variation and the parabolic type of variation of n with height described above a temperature structure was determined on the assumption that it remained isothermal up to a height h_1, and then varied parabolically to a height $(h_1 + h_2)$ - the parabola being vertical at h_1. In this case we obtain

$$h_1 = \frac{1}{\sqrt{n_o}} (2f - gn_o) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

$$h_2 = \frac{4}{\pi\sqrt{n_o}} (gn_o - f) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

and $\alpha^1 = 0.5$ always in this case for the parabolic portion.

This of course produces a solution for temperature against height on the assumption that there is no following or head wind along the line considered.

Use of the Deflection Value Y.

The above methods have utilised X, T and A and so far the other measurement Y has not been utilised.

There are difficulties involved in attempting to work in this measurement with the others and a simple method of doing this was not evolved. However the measurement was made use of in the following manner:-

If we postulate an "equivalent constant crosswind" $\overline{w}^1$ (15) such that

$$\overline{w}^1 = \frac{\int_o^H \frac{w^1}{n}\, dh}{\int_o^H \frac{dh}{\sqrt{n}}}$$

Then

$$\overline{w}^1 = \frac{Y}{T} a_o$$

$\overline{w}^1$ is thus a wind component weighted according to the time spent by the sound in the various layers at different heights.

If it is compounded with the equivalent constant crosswind across a second azimuth it is possible to obtain "(time)" mean values of D_E and D_N, from two equations of the form

$$\bar{w}^1 = D_E \cos\theta - D_N \sin\theta \qquad \ldots\ldots\ldots\ldots(29)$$

These values D_E D_N will have little quantitative significance as they are derived from two components referring to different trajectories and maximum heights. As such they will merely reveal the main wind trends at the upper level.

Further if we write

$$\frac{X}{T} = \bar{a}\cos\bar{\psi} + \bar{w} \qquad \ldots\ldots\ldots\ldots(30)$$

$$A = \bar{a}\sec\bar{\psi} + \bar{w} \qquad \ldots\ldots\ldots\ldots(31)$$

where $\bar{a}$, and $\bar{\psi}$ are mean speeds of sound and inclinations of the wave normal to the horizontal and

$$\bar{w} = \bar{D}_N \cos\theta + \bar{D}_E \sin\theta$$

we obtain:-

$$\bar{a}^2 = (A - \bar{w})\left(\frac{X}{T} - \bar{w}\right) \qquad \ldots\ldots\ldots\ldots(32)$$

which may give a certain amount of information about the temperatures to be expected at the higher levels.

This system of using (time) mean values in this manner has to be used with great care as it may lead to incorrect results when two trajectories of very different shapes or maximum heights are used to obtain the values of D_E and D_N.

Measurement of Characteristic Velocity A and twist angle ϕ by means of a square or diamond of microphones on the ground.

If the true azimuth of travel of the wave normal at the microphones is β at the ground and the sound sweeps over the microphone lattice with characteristic velocity A, considering first a square, 4 microphone readings m_1, m_2, m_3, m_4 of time of sound arrival are obtained.

The diagonal differences $m_1 - m_3 = l$ and $m_2 - m_4 = r$ are obtained and if L = length of diagonal

$$l = \frac{L\sin\beta}{A} \qquad \ldots\ldots\ldots\ldots(33)$$

$$r = \frac{L\cos\beta}{A} \qquad \ldots\ldots\ldots\ldots(34)$$

Hence A and β are determined and

$$\phi = \alpha - \beta \qquad \ldots\ldots\ldots\ldots(35)$$

Where α is the survey azimuth of the line MS

To convert similar readings m_1, m_2, m_3, m_4 for a diamond of microphones on the ground it is merely necessary to multiply the values l and r by L_1/L and L_2/L where L_1, L_2 respectively are the lengths of the two diagonals of the diamond.

It was shown that in these experiments using a square or diamond lattice the correction necessary on account of the assumption of a plane instead of a spherical wavefront was negligible.

Correction for height of microphone.

If the microphones had heights h_1, h_2, h_3, h_4 and $h_3 - h_1 = \delta h_1$ and $h_4 - h_2 = \delta h_2$ then corrections

$$\delta l_n = \delta h_1 \sin\psi_1 \quad , \quad \delta r_n = \delta h_2 \sin\psi_1 \qquad \ldots\ldots\ldots(36)$$

were applied to l and r respectively.

Where $A = a_1 \sec\psi_1 + w_1$

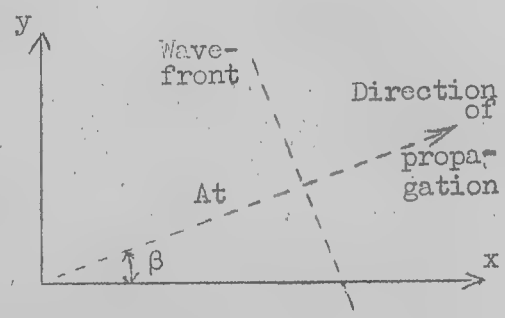
a_1 and w_1 being the sound speed at surface temperature and following component of surface wind respectively.

Measurement of Characteristic Velocity A and twist angle ϕ from readings obtained from triangular and irregular layouts of microphones on the ground.

The systems used for obtaining A and ϕ from readings of an irregular layout of microphones (e.g. the Larkhill base) were due to Mr. H.J.Sisson of A.O.R.G. and are outlined below.

Computing Method

Using the notation shown :- If a plane wavefront travels across the base with velocity A in direction β the equation of the wavefront at any time t is



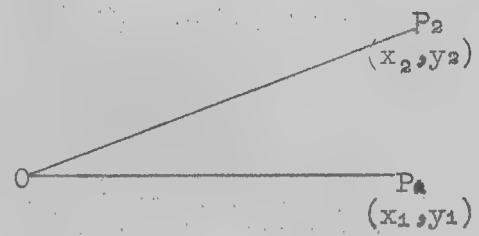
$$r x + l y = Lt \qquad \ldots\ldots (37)$$

where $l/r = \tan\beta$

and $L/r \cos\beta = A$

For a triangle

$$\left.\begin{aligned} r x_1 + l y_1 &= Lt_1 \\ r x_2 + l y_2 &= Lt_2 \end{aligned}\right\} \qquad \ldots\ldots\ldots(38)$$



and $l : r : L$ can be found by solutions.

In terms of differences of eastings and northings of P_1, P_2 from the reference microphone 0, $l : r : L$ is given by

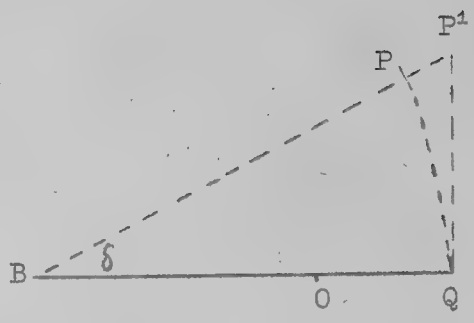
$$\begin{bmatrix} \Delta t_1 & \Delta E_1 & \Delta N_1 \\ \Delta t_2 & \Delta E_2 & \Delta N_2 \end{bmatrix}$$

omitting successively columns 2,3,1 and evaluating the determinant.

The bearing β is then given by

$$\tan\beta = -1/r$$

Curvature Corrections



If B is the explosion point, and O , P are two microphones, the correction to the position of P relative to O is $\overrightarrow{PP^1}$

Its length is

$$(\sec\delta - 1) \quad \ldots\ldots\ldots\ldots (39)$$

Hence it may be applied to ΔE and ΔN of P relative to O in the form:-

$$(E_P - E_B)(\sec\delta - 1)$$

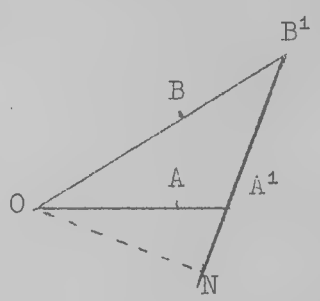
$$(N_P - N_B)(\sec\delta - 1) \quad \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

(These are second order quantities in δ . The effect of a first order twist ϕ is a third order quantity and is in practice negligible for the ranges used in this work).

A graphical method of obtaining A and ϕ with irregular layouts of microphones.

Consider three microphones O , A , B with sound arrival times, t_O , t_A , t_B respectively.
Let

$$\left.\begin{aligned} t_A - t_O &= \delta t_A \\ t_B - t_O &= \delta t_B \end{aligned}\right\} \quad \ldots\ldots\ldots (41)$$



in sound metres.

Then if $OA^1 = \dfrac{a_o\, OA}{\delta t_A}$ and $OB^1 = \dfrac{a_o\, OB}{\delta t_B}$ $\quad \ldots\ldots\ldots (42)$

The line A^1B^1 gives the position of the wavefront after unit time. Hence the perpendicular distance ON to A^1B^1 gives the velocity A and the direction ON gives the bearing of the arriving sound.

Using this principle a chart can be constructed for any lattice of microphones relative to any reference point O (which need not be a microphone).

A chart of this type is particularly useful when there are several microphones with poor inceptions of sound arrival. If all the readings are plotted on this chart the best straight line that can be fitted to the plotted points will give the most probable values of A and ϕ .

The values of A and ϕ obtained by the methods described above could also be obtained with better accuracy by means of a Least Squares Solution in which is assigned to each sound arrival time at the various microphones a relative weight according to the sharpness of its inception. Although such a solution was proposed it was however decided not to proceed with it as the extra computation involved for a comparatively small increase of accuracy would not have been justified in view of the other errors inherent in the system.

Experiments April 1944 - April 1945.

During the period April 1944 - April 1945 the opportunity arose of performing set experiments to use the phenomena of anomalous sound propagation to obtain some further information about wind and temperature at high levels and these experiments are described in detail below and their results discussed. In all 16 experiments were made throughout this period, the design of the experiments being changed as more information was obtained.

(a) Recording Bases (Fig. 2)

(1) A large lattice of at least 12 microphones was always available in the Canterbury - Hastings area in S.E.England. These microphones were arranged in squares or diamonds of 10 Km. side.

(2) A lattice of 12 microphones at the School of Survey, Larkhill, Salisbury Plain, was also available whenever required. This was an irregular lattice because the microphones were installed for other purposes but it proved very satisfactory in action.

(3) A lattice of 4 and later 6 microphones was installed at Friskney (Wash area). These were arranged in squares of 5 Km. side.

(4) A small lattice was installed near Malvern for the first experiment only but did not record anything.

(5) Lattices in the Low Countries were also used with headquarters at Malines, Belgium but these had negligible successful recordings.

(b) Explosion points (Fig. 2)

The explosion points were varied throughout the experiments to get the best conditions and in order of their utility were

(a) Friskney (Wash area)
(b) Larkhill (Salisbury Plain)
(c) St. Margarets Bay (Dover)
(d) Trawsfynydd (N. Wales)
(e) Okehampton (Devon)
(f) Thetford) Used only in Experiment No. 7
(g) Spurn Head)
(h) Fylingdales (Yorkshire Moors))

The various recording installations and explosion points used were dictated by operational necessities e.g. questions of communications, maintenance, proximity of buildings to explosion points etc. as well as the most likely zones of audibility.

A map is given (Fig. 2) showing the locations of the various explosion points and recording installations.

(c) Short Description of Experiments.

Series 1. For this experiment (being the first) eight charges were exploded at Friskney, over a period of three days and at various times of the day and night. Of these, only three were well recorded (Nos. 1, 4 and 8) and these were all at night or early morning and on the Canterbury - Hastings recorders. Nothing was recorded at either of the other recording points at Malvern and Larkhill. Subsequent experiments were all carried out at night.

Series 2. For this experiment there were two explosions at about 3 hours interval at Okehampton and Friskney. All four bangs were recorded on the Canterbury - Hastings microphone layout.

Series 3. There were two explosions at half an hours interval at Friskney and both were recorded at both Canterbury - Hastings and Larkhill.

Series 4. There were two explosions at half an hours interval at Friskney and St. Margarets Bay (Dover) with recorders at Canterbury, Larkhill and Friskney. At Canterbury, only the sound of troposphere paths (no time delay) was received from Friskney; the Larkhill recordings if any were masked by local gunfire, and although there were possible traces on the Friskney records of sound from the St. Margarets Bay explosion they were not of sufficient amplitude to be identified definitely.

Series 5. There were two explosions at half an hours interval at Friskney and St. Margarets Bay (Dover) with recorders at Canterbury, Larkhill and Friskney. At Canterbury the Friskney sound was received by anomalous paths; at Larkhill both the Dover and Friskney sounds; and at Friskney the Dover sound was received by troposphere paths only. Two responses were worked out on the Friskney-Canterbury line but one was useless because the troposphere "a+w" values along that azimuth exceeded the measured characteristic velocities.

Series 6. There were two explosions at half an hours interval at Friskney, Dover and Larkhill with recorders at Canterbury, Larkhill and Friskney. At Canterbury the Friskney sound only by anomalous paths was received on Bang 1. At Larkhill both the Friskney and Dover sounds were recorded for both bangs. Nothing was recorded at Friskney. The number of values of characteristic velocity and sideways deflection obtainable from the films was lower than on most other occasions and this tended to make the results less reliable.

Series 7. An effort was made to record anomalous path sound waves over different ranges from that from Friskney to the recording bases, but although some traces were observed, the bangs at Thetford, Fylingdale and Spurn Head were generally unsuccessful and it was considered that the ranges were either too small or too large. These Bang Points were therefore abandoned in later experiments.

Series 8. There were two explosions at half an hours interval at Friskney, Dover and Larkhill with recorders at Canterbury, Larkhill and Friskney. The Larkhill Explosion was recorded for the anomalous paths at Canterbury and Friskney, the Friskney explosion at Canterbury and Larkhill but no recordings were made of the Dover explosion. Results were generally good and allowed determinations of temperature and wind up to 45 Km.

Series 9. There were two explosions at half an hours interval at Friskney, Dover and Larkhill with recorders at Canterbury, Larkhill and Friskney. The only recordings of anomalous paths made were at Canterbury of the Friskney and Larkhill explosions.

Series 10. There were two explosions at half an hours interval at Friskney and Larkhill and one at Okehampton. Recorders were at Friskney, Larkhill and Canterbury. The only recordings of anomalous sound waves made were at Canterbury of both Larkhill explosions, Friskney both Larkhill explosions and Larkhill of the Okehampton explosion. Sound was also received by troposphere paths at Canterbury from Friskney and Larkhill and at Larkhill from Okehampton.

Series 11. There were two explosions at half an hours interval at Friskney, Larkhill and Trawsfynydd with recorders at Friskney, Larkhill and Canterbury. Recordings were obtained on all possible azimuths by anomalous paths, Bang 2 being the more successful. Sound was received also by troposphere paths along the azimuths Friskney-Canterbury, and Trawsfynydd-Larkhill.

Series 12. There were two explosions at half an hours intervals at Friskney, Dover, Larkhill and Trawsfynydd with recorders at Friskney, Canterbury and Larkhill. Recordings of anomalous path sound waves were made on all possible azimuths except Dover - Friskney and Dover - Larkhill. Sound was received also along troposphere paths from Friskney - Larkhill, Trawsfynydd to Larkhill and Friskney to Canterbury.

Series 13. There were two explosions at half an hours interval at Friskney, Larkhill, Dover and Trawsfynydd with recorders at Friskney, Larkhill and Canterbury. Recordings of anomalous sound waves were noted between azimuths of 090° and 163° only. Sound was received by ordinary troposphere paths at Canterbury from Larkhill and Friskney.

Series 14. Two explosions of ammunition dumps on Trawsfynydd Ranges were recorded at Canterbury in the daytime about 1500 G.M.T.

Series 15. There were two explosions at about 45 mins. interval at Friskney, Dover, Larkhill and Trawsfynydd with recorders at Friskney, Canterbury and Larkhill. Recordings of anomalous sound waves were obtained on all possible azimuths although the Friskney recorder was out of action in Bang 2. Sound was received by troposphere paths along the Friskney - Canterbury azimuth only.

Series 16. There were two explosions at about 30 mins. interval at Friskney, Larkhill, Trawsfynydd, and Dover with recorders at Friskney, Canterbury and Larkhill. Recordings of anomalous sound waves were obtained on all azimuths except Friskney - Canterbury where there was a recording with a troposphere path. Certain recordings were obtained on Bang 1 but not on Bang 2 and vice versa. Troposphere values of "a+w" appeared to exceed the measured characteristic velocity value in two cases.

Methods Used.

(a) Details of Equipment.

The microphones used were of the hot-wire type and were placed in surveyed positions to form squares or triangles of sides 5-10 Km. long. They were connected by line to an Einthoven six-stringed galvanometer complete with optical projecting system, tuning fork timing control and moving film camera. Synchronising marks were arranged on the films either controlled automatically from a chronometer or manually by electrically producing a sudden deflection of one of the strings. The timing motor produced vertical lines on the films at intervals of 10, 100 and 1000 sound metres so that the time of sound inception at each microphone could be read to an accuracy of a few sound metres, the accuracy depending on the "signal-noise" ratio on the records and definition of the point of sound inception. Electrical types of interference and string deflections due to local effects e.g. wind at the microphones made film reading difficult at times.

(b) Film Reading.

The accuracy of the film reading determines the accuracy of determination of the values of characteristic velocity and apparent angle of approach of the Sound Wave at the microphone lattices, and is therefore an important limiting factor in this work. Great care was therefore taken to determine as accurately as possible the times of sound inception on the various records but even so in some cases the value obtained was doubtful. These doubtful values were indicated often by the calculated Characteristic Velocity being less than values of "a+w" at some levels in the troposphere which is impossible theoretically for a path of continuous refraction. Errors of 1 metre per second or even less in the value of characteristic velocity or the accuracy of wind and temperature measured by the radio-sonde methods up to 18 km. became important in some cases, and this may account in some measure for the large scatter of results obtained in some of the characteristic velocity determinations. As many microphones were used as possible at each recording base. In all cases readings were taken of all the multiple arrivals possible.

(c) Computations Made.

In all cases when the film readings had been made the values of range, time and deflection below 18 Km. and thence the values to be ascribed to levels above 18 Km. were determined. The methods then used were varied according to the number of azimuths along which reception had been made.

(1) For Single Azimuths only a determination of the "a+w" values above 18 Km. was made together with a mean(time) cross wind across that azimuth.

(2) For Double Azimuths a mean(time) cross wind and a mean(time) temperature were determined.

(3) For three or more azimuths determinations of wind and temperature between "a+w" curves along each three azimuths were made and in addition the best mean(time) wind and temperatures were obtained from the cross wind values.

All these calculations were made along the lines indicated in the section on mathematical and computational methods.

(d) Tables of Results.

A summary of the results obtained is given in Tables 1 to 3 which are appended.

(1) Table 1 gives Results of Wind Values up to 45 Km. obtained in cases where it was possible to compare the "a+w" values above 18 Km. along three azimuths. Comparison figures of wind values of 30 Km. measured by observing the movement of a smoke burst produced at that level by a high velocity gun ("Bruce") at about the same time are included.

(2) Table 2 gives Results of Temperature Values up to 45 Km. obtained with the Wind Values in Table 1.

(3) Table 3 gives the mean(time) values above 18 Km. of Wind and Temperature obtained in the various experiments.

Summary of Results.

The results of the experiments are summarised in Tables 1 to 3 and Figs. 3 to 7 and these results give the most probable values of wind and temperature between 18 Km. and 40 to 50 Km. height required to satisfy the observations of the sound travel. It is emphasized that these results are dependent on the physical assumptions made as well as the simplifying mathematical assumptions, so that while it is believed that they give a picture of the main features of the wind and temperature structure within these levels, no claim is made with regard to the accuracy of detail. Moreover in the writer's opinion all previous work of this type must have suffered in this respect e.g. while most writers have agreed that a high temperature in summer is necessary between 40 and 50 Km. to explain the observed parameters of the sound path, small differences between results obtained have very little meaning. Whipple gave 310°K and Gutenburg 330°K at 45 Km. and this report suggests about 300°K at that level but in fact all that can reasonably be concluded is that it must be in order of 100° more than 220°K, the known mean stratosphere temperature at 20 Km. in these latitudes.

Considering the diagrams:-

Fig. 3 shows a series of diagrams, one of each experiment in which the points of good and bad audibility obtained are given with the explosion point at the centre of the circles. In general it is seen that the good audibility points lie to the west of the explosion point in summer, and to the east in winter with good audibility all round in the spring. The simplest explanation of this is that there is a monsoon effect of wind with easterly winds in summer and westerly winds in winter and that the temperature is high

at the levels of the top of the sound trajectories in spring. Successful receptions in diametrically opposite directions suggest in general high temperatures. This is most probably the case although it must be remembered that the real criteria of audibility according to the refraction theory is that the function "a+w" along any azimuth shall have its maximum value at the top of the sound trajectory; and very large variations in wind directions with large wind speeds could conceivably produce all round audibility without the necessity of postulating an increase in the value of "a" and hence temperature at the greatest heights. Moreover large values of "a+w" in the troposphere can also have a large effect on the audibility diagram because they may exceed "a+w" at the greater heights and so prevent refraction down from the greater heights even though "a+w" is comparatively great there. However it will be seen that the other results obtained tend to confirm the monsoon effect of wind and high temperatures at great heights inferred from the audibility diagram.

Figs. 4, 5 and 6 give all the individual results of the determinations of vertical distribution of temperature, easterly and northerly wind components up to heights of about 45 Km. in the seven experiments where there was sound reception along three or more azimuths. In addition these diagrams give suggested annual variations of wind and temperature at a height of 40 Km.

One of the main features of these diagrams is the very large scatter between individual determinations in the same experiment due probably to inconsistences introduced by the difficulty of reading the sound films to the required accuracy as well as to the assumptions which had to be made in the work. The assumption of a parabolic variation of "a+w" with height above 18 Km. has probably produced the typical form of temperature curve shown with minimum temperatures around 30 Km. before the rise in temperatures towards 40 Km. It is very doubtful if this is in fact real but it is considered that the increase towards the highest levels is definite. Moreover the curves of annual variation of temperature at 40 and 45 Km. both show consistently higher temperatures at these levels in summer than in winter and it seems likely that there is considerable annual variation at these heights. As regards the graphs of wind components these show a well marked westerly component in winter and a lighter easterly component in summer; while the northerly component exhibits no well marked seasonal tendency. Fortunately it is possible to compare the annual variations of wind calculated from the sound results with some direct observations of wind at 30 Km. made over South Eastern England over the same period. These wind observations were obtained by visual tracking of the movement of a smoke puff produced by a high velocity gun ("Bruce") and consequently are entirely independent of these sound experiments. As can be seen in the figures showing the variation of wind components throughout the year there is good agreement between the two sets of results and this is regarded as a confirmation of the methods used and results of the sound work. Moreover since the temperature results of Fig. 4 were obtained from the same calculations as the wind results of Figs. 5 and 6, the results of Fig. 4 also receive some confirmation from this wind comparison. In these calculations the values of wind components and temperature are interdependent - one ft/sec of wind having the same effect as 1°F approx. of temperature on the speed of sound.

The final figure Fig. 7 shows the results of the mean(time) calculations. It has been explained that this type of calculation must of necessity be "rough and ready" but even so the agreement between the sound results and the "Bruce" measurements are good. Furthermore the mean(time) temperature values obtained while not appearing very probable as regards magnitudes do give some support to the idea of a large annual temperature variation at a height of 40 - 50 Km. as shown in Fig. 4.

Discussion of Results.

The main results obtained from this work were:-

(a) The wind at levels of 30 to 50 Km. above the earth's surface appears to exhibit a monsoon effect in these latitudes. In winter the direction is generally between South West and North West with speeds of 40 - 80 metres per second. In summer the direction is between North East and South East with speeds of less than 20 metres per second.

(b) The temperature at 40 - 45 Kms. appears to increase in summer to values approaching surface values. This increase is not evident below 45 Kms. in winter and a 20 - 40°C variation of temperature between summer and winter is likely at these heights.

As has been explained above the data about wind is much better supported than that about temperature. The evidence for a monsoon variation of wind at levels of 30 - 50 Km. is now considerable.

(a) This theory was put forward by Christy in a paper on the audibility in Chelmsford of gunfire on the Western Front in the 1914-1918 Great War. It was supported by Dr. Whipple in an address to the Royal Meteorological Society (Q.J. Royal Met. Soc. 1935. The Propagation of Sound to Great Distances).

(b) Direct evidence of westerly winds in winter and easterly winds in summer over S.E. England was obtained in the work of the observation of the drift of smoke produced by shell bursts from a high velocity gun.

(c) Three independent sets of data in this report support the theory.

(1) The audibility diagrams show audibility to be good to the west in summer and to the east in winter.

(2) The calculations of wind based on times of sound travel and comparison of "a+w" curves above 19 Km. give data corresponding closely to the "Bruce" results.

(3) The calculations based on observations of sideways deflection leading to the time(mean) winds above 18 Km. also show the same annual variation.

Observations of wind by radio-sonde methods up to a maximum of about 20 Km. almost invariably have shown that the wind velocity in the stratosphere decreases with height from a maximum around the tropopause and the monsoon winds must therefore develop between 20 and 30 Km. probably between 25 and 30 Kms. There is very little other evidence on this point - observations of the movements of mother of pearl clouds and exceptionally high balloon ascents being few. The reason for the monsoon effect must be a variation of the horizontal temperature gradient between summer and winter, the atmosphere at these levels towards the poles being warmer in summer and colder in winter than over the temperate regions. This might be due to direct absorption of solar radiation by the ozone in these levels; and the annual variation of ozone content between 20 and 40 Km. together with the absence of solar radiation at the poles in the winter will be the controlling factors.

As regards the temperature results the high temperatures towards 40 Km. in summer and the large annual variation will also be due to direct absorption of solar radiation in the summer and decrease of solar radiation in the winter. The idea of high temperatures there in summer has been put forward by many writers in different fields e.g. meteors entering the atmosphere, atmospheric oscillations, theoretical calculations on the heating of the atmosphere by direct absorption of solar radiation by ozone etc. but as far as the writer is aware no previous suggestions have been made of the large annual variation given in this report.

ANOMALOUS SOUND RECEPTION. Bibliography.

1. Q.J.Roy.Met. Soc.44, 1918.p.281 - Christy, (Paper on hearing guns from the Western Front in Chelmsford).
2. Met.Glossary 1930, p.25 (Silvertown Explosion).
3. Proc.Roy.Soc. Edinburgh, 1917-18, (Silvertown Explosion).
4. Nature, London 118, 1926, p.308 (La Courtine Explosion).
5. Arbeiten Preuss. Aeron.Obs.Lindenburg 16.1927.H.B. (Juterborg Explosion).
6. Duckert.Zeit fur Geophys.10 1934.p.119 (Oldebroch Explosion).
7. Maurain Ann.de l' Inst.de Phys.du Globe Paris 1926 (La Courtine Explosion).
8. Ergebnisse d. Kosmischen Physik.Leipzig 1 p.272 (1931) (Juterborg Explosion).
9. Regulá.Zeit fur Geophys.10. 1934.p.185 (Oldebroch Explosion).
10. Q.J.Roy.Met.Soc.1935 - Propagation of sound to great distances - Whipple.
11. Explanation of the upper air by means of sound waves - Ramanathan.
12. Bulletin American Met.Soc. May 1939 - Velocity of sound waves and temperature of the stratosphere in S. California - Gutenberg.
13. Journal of the Aeronautical Sciences, Vol.9, Dec.1941 - Propagation of sound through the atmosphere - Haurwitz.
14. Zeit.fur Geophys.Vol.10, 1934. Schalluntersuchengen in Polargebiet.K.Wolchen.
15. Deutsches Met.Jahrbuch, 1921 (Bayern) H.Zierl (Oppau Explosion).
16. Comptes Rondus. 7th July 1926 p.507.
17. Temperature of the atmosphere at levels accessible to air waves - Whipple.
18. Temperature in Summer of the upper atmosphere as indicated by the study of air waves - Whipple.
19. Audibility of explosions and constituents of upper atmosphere - Whipple.
20. Exploring the atmosphere by air waves - Whipple.
21. Met. Zeit. Band 58. Heft 8. Aug. 1941 (Wegener).
22. Investigation of audibility of Burton on Trant Explosion (MRP 284).
23. Office National Meteorologique. La Temperature en Altitude M.L. Auberger 25. 3. 41.
24. Zur Kenntnis Der Windverhaltnisse in der Oberen Stratosphare E.Ekhart Berlin 1940.
25. The physical state of the Upper Atmosphere. Haurwitz.
26. Sveriges Meteorologiska Och Hydrologiska Institut. Series B. No.3. Lindholm (Explosion at Oslo 1943).

Acknowledgements.

These experiments were performed by a War Office organisation commanded by Col. Brooke O.B.E. with headquarters at Canterbury. Included in that organisation were the 11th Survey Regiment commanded by Col. S. Attenborough R.A., H.Q.R.A. (Sound Ranging) commanded by Major P.W.E.Taylor R.A. and an A.O.R.G. Section including Dr. G.J.B. Clews and Mr. Bunten all of whom cooperated in the work.

The recordings at Larkhill were made by the Survey Wing, School of Artillery and R.E. Units conducted the explosions.

Great assistance was given by Mr. H.J.H. Sisson of A.O.R.G. to whom the methods of film reading described above are due, and by Sgt. H.E. Davies of H.Q.R.A. (Sound Ranging) who did most of the film reading work.

The results were worked out by the Meteorological Sections attached to H.Q.R.A. (Sound Ranging) and at the Meteorological Office, Larkhill.

TABLE 1. Suggested Wind Values Above Medium Radio-Sonde Heights (No Results on Experiments 1,2,3,4,7,9,10,13,14.)

Series	Date	25 Km.	30 Km.	35 Km.	40 Km.	45 Km.	30 Km. "Bruce"	Degrees true. Speed metres per sec. REMARKS
5. Bang 1	26. 7.44.	055°16	065°24	070°23	090°18	133°14		Bang 1 more reliable than Bang 2. Veer of wind with height contradicts audibility results.
" 2	" " "	060°25	065°37	070°36	090°24	124° 8	080° 9	
6. " 1	8. 8.44.	075°13	090°12	125°15	160°19	195°25	085°10	Southerly winds at high levels disagree with audibility results. Considered wind mainly NE or ENE at high levels.
" 2	" " "							
8. " 1	13. 9.44.	080° 9	065° 9	010° 7	300°18	300°31	Light NE	The light NE'ly winds at 30 Km agree with Bruce values. The NW'ly winds at 40 Km agree with audibility results.
" 2	" " "	090°12	085°15	070° 9	300° 8	285°25		
11. " 1	6.12.44.	252°19	215°48	229°61	250°71	279°95	263°49	These are mean values. Individual determinations spread considerably from these. Bang 2 results more trustworthy than Bang 1. Agreement with Bruce and audibility results reasonably good.
" 2	" " "	203°23	235°48	252°67	266°83	268°95		
12. " 1	6. 1.45.	332°40	320°48	312°53	313°59	308°54	320°58	Large scatter of individual results about the mean values quoted.
" 2	" " "	331°57	321°68	315°62	316°72	317°87		
15. " 1	6. 3.45.	301°36	295°50	297°68	295°72	288°78	297°49	Large scatter of individual results about mean values quoted.
" 2	" " "	295°35	288°48	292°63	-	-		
16. " 1	6. 4.45.	360° 2	360° 3	022° 8	017°16	360°23	112°17	Generally light variable or N'ly.
" 2	" " "	360°12	360°14	360°14	014°13	039°10		

TABLE 2. SUGGESTED TEMPERATURE VALUES °C AT GREAT HEIGHTS (No results on Experiments 2,3,4,9,10,13, and 14).

Series	Date	Heights in Kilometres						Remarks
		25	30	35	40	45	Greater	
1. Bang 1	15. 4.44.	- 65	- 68	- 59	- 46	- 23	+ 48 at 52 Km.	Assumed sound results explained by parabolic variation of temperature with height above 18 Km. and following or head wind component neglected.
" 4	" " "	- 68	- 62	- 48	- 32	- 1	+ 38 at 48 Km.	
" 8	17. 4.44.	- 73	- 68	- 51	- 32	+ 4	+ 38 at 47 Km.	
5. " 1	26. 7.44.	- 78	- 71	- 54	- 24	+ 21		Bang 1 values more probable than Bang 2.
" 2	" " "	- 90	- 93	- 73	- 34	+ 21		
6. " 1	8. 8.44.	- 70	- 68	- 46	- 7	+ 32		High level inversion probable but values affected by errors in wind values (Table 5)
7. " 1	23. 8.44.	- 62	- 59	- 48	- 29	- 1	+ 35 at 50 Km.	Same assumptions as in Experiment 1
" 2	" " "	- 71	- 62	- 48	- 23	+ 16		
8. " 1	13. 9.44.	- 73	- 73	- 62	- 39	- 7		Generally consistent results showing inversion at higher levels.
" 2	" " "	- 73	- 73	- 59	- 32	- 1		
11. " 1	6.12.44.	- 90	- 46	- 40	- 51	- 59	- 54 at 50 Km.	No temperature inversion below 45 Km. and only small evidence of inversion above 45 Km. These are mean results and scatter of individual results is large.
" 2	" " "	- 84	- 90	- 84	- 73	- 57		
12. " 1	6. 1.45.	- 79	- 78	- 68	- 51	- 23		Some possibility of inversion towards 45 Km. but big scatter of individual results makes conclusion doubtful.
" 2	" " "	-106	-101	- 84	- 73	- 57		
15. " 1	6. 3.45.	- 87	- 87	- 73	- 59	- 32		Same remarks as for Experiment 12.
" 2	" " "	- 84	- 84	- 79				
16. " 1	6. 4.45.	- 84	- 79	- 62	- 40	- 12		Reasonably good agreement showing inversion around 40 Km. agreeing with audibility results.
" 2	" " "	- 78	- 73	- 59	- 40	- 4		

TABLE 3. BEST MEAN(TIME) VALUES.

EXPERIMENT No.	1	2	3	5	6	7	8	9	10	11	12	13	15	16
DATE	16/4	6/5	13/6	26/7	8/8	23/8	13/9	11/10	8/11	6/12	6/1	6/2	6/3	6/4
D_E metres/sec	+ 25	+ 11	- 12	- 14	- 8	+ 2	+ 4	+ 9	+ 40 ?	+ 67	+ 53	+ 118 ?	+ 37	0
D_N metres/sec		- 4	- 2	- 1	- 8		- 2	- 6	+ 20 ?	- 19	- 57	- 24 ?	- 20	0
Temperature °C		- 47	- 32	- 29	- 35		- 37	- 46		- 96	- 95	- 140 ?	- 47	-30

Experiments 1-11 in 1944, 12-16 in 1945.

D_E positive denotes a westerly component negative, easterly

D_N " " " southerly " " , northerly.

These values are all very approximate and individual determinations showed considerable scatter from these.

Zones of Audibility.

Fig. 1.A.

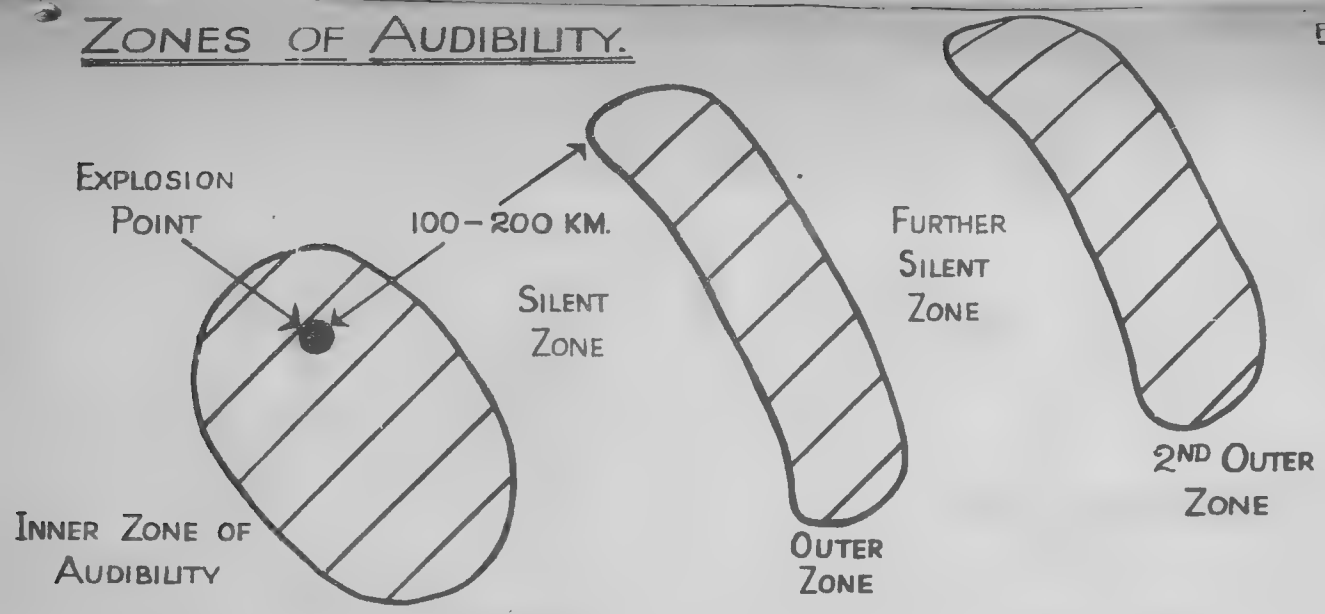
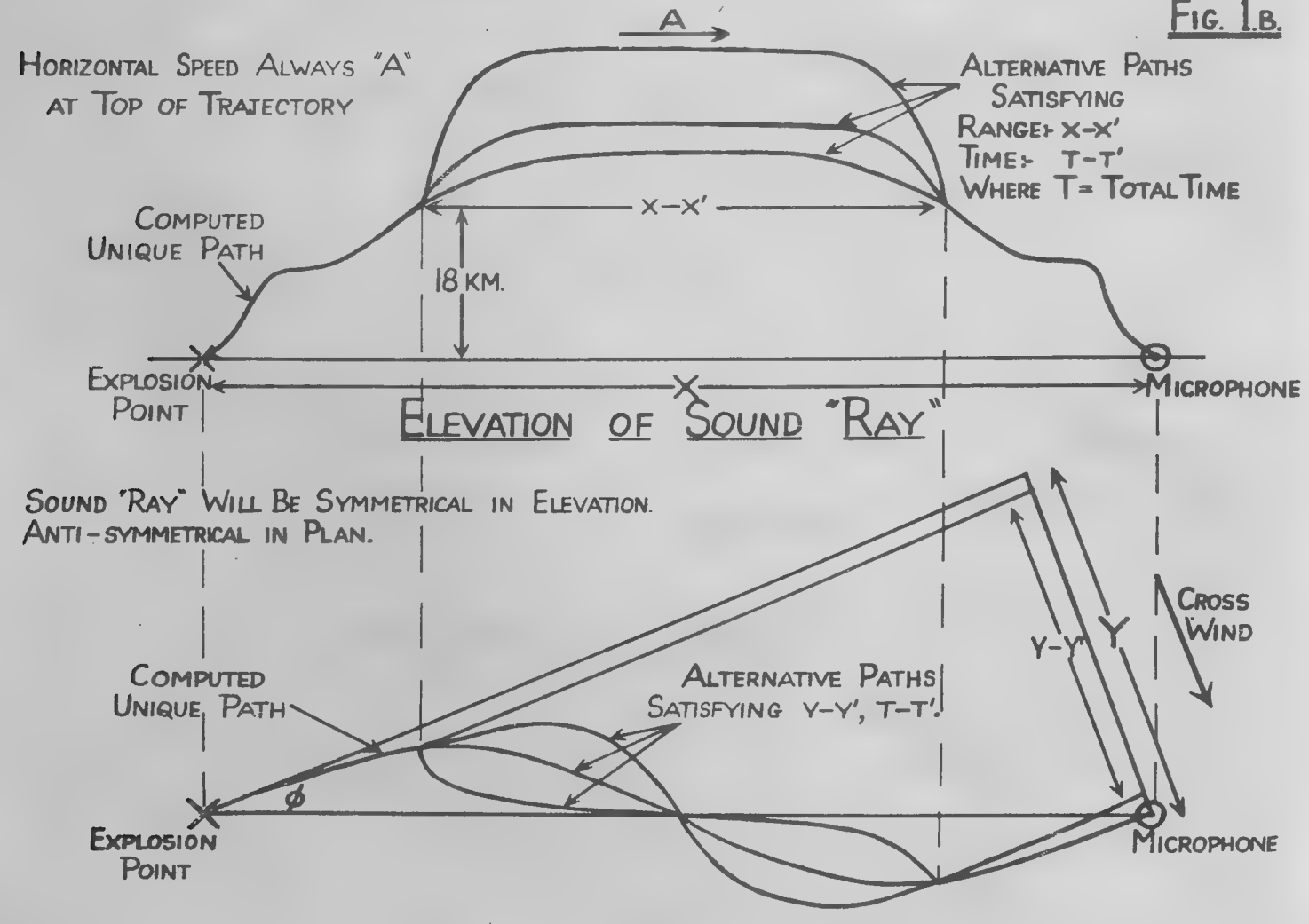


Fig. 1.B.



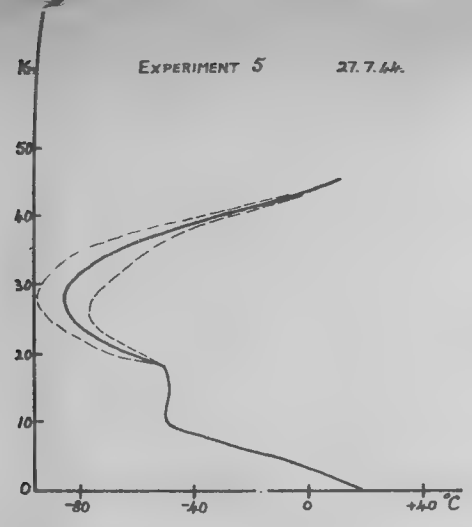
Elevation of Sound "Ray"

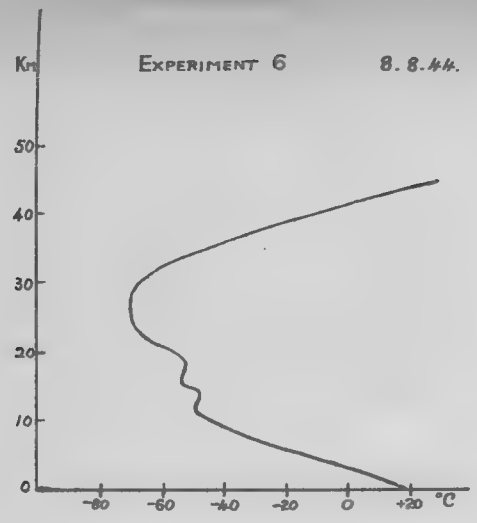
Plan of Sound "Ray"

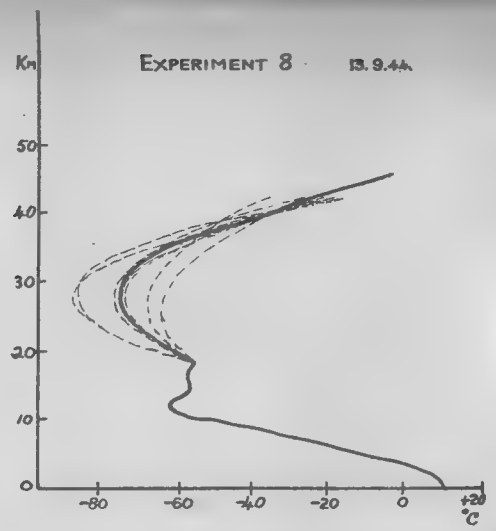
FIG. 2.

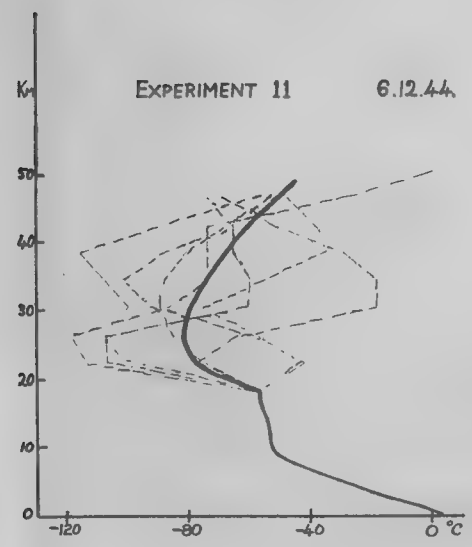
DIAGRAM OF LOCATIONS.

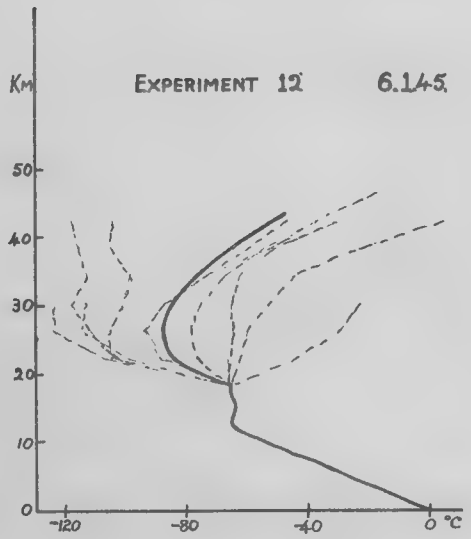
AUDIBILITY DIAGRAM OF ANOMALOUS SOUND PROPAGATION EXPERIMENTS.

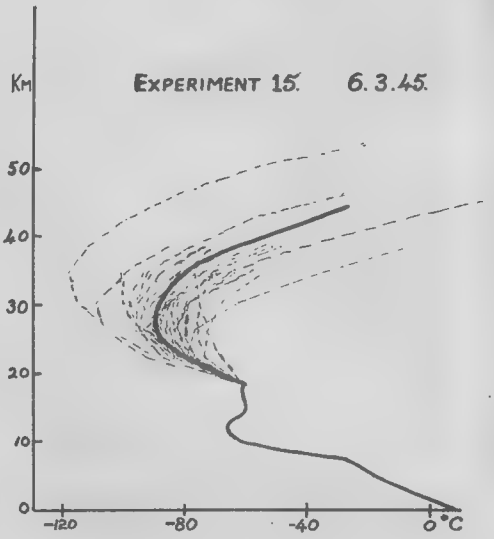


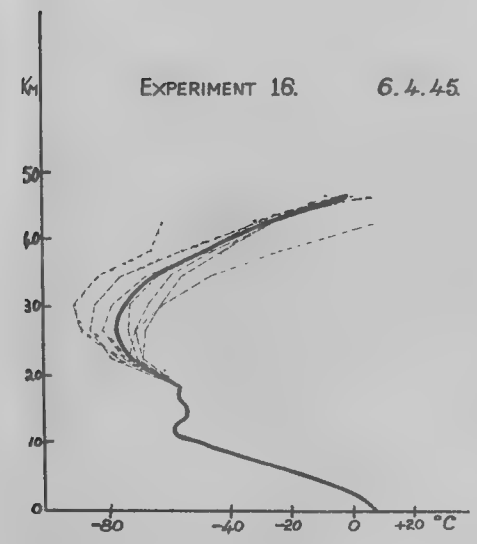


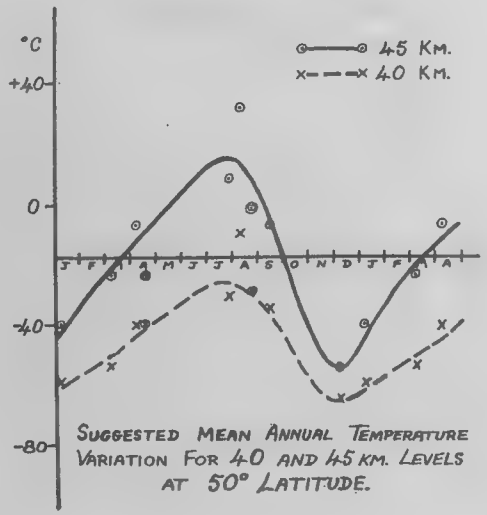










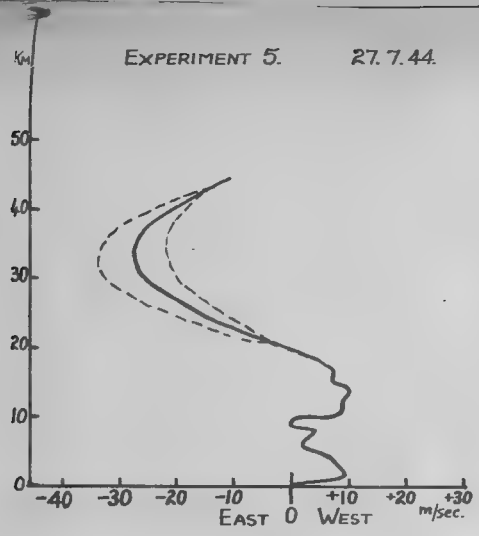


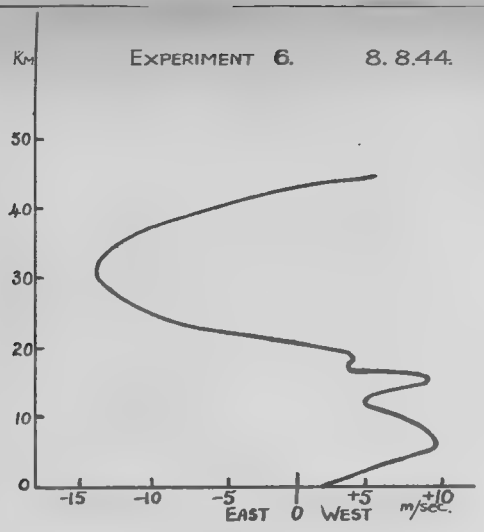
Suggested Mean Annual Temperature Variation For 40 and 45 km. Levels at 50° Latitude.

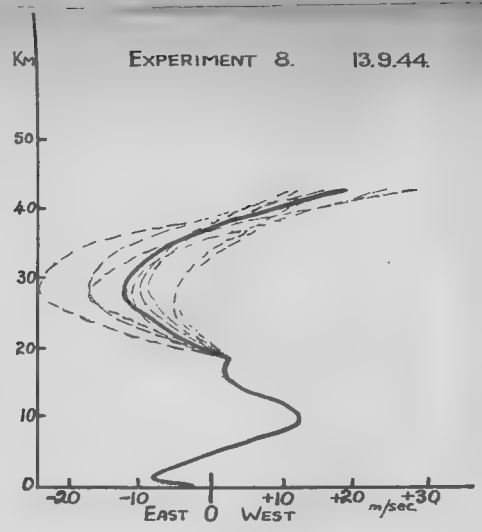
Vertical Temperature Distributions Obtained in the Different Experiments.

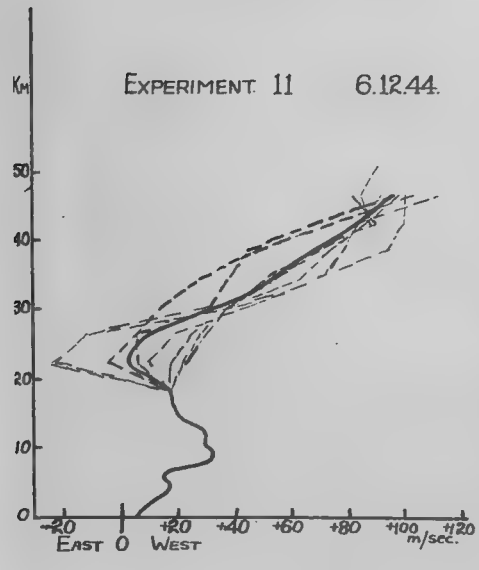
Continuous lines indicate Mean Values.
Broken lines indicate Individual Determinations, where several were possible. Large scatter in Experiments 11, 12 and 15 coincides with high wind speed values.
Values below 18 km. are Radio-Sonde Values.

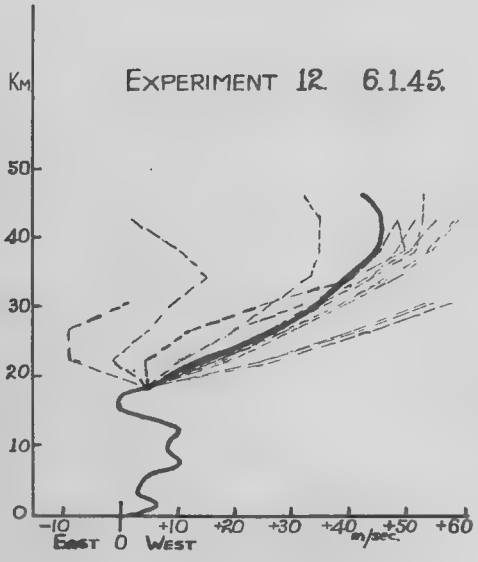
Fig. 4.

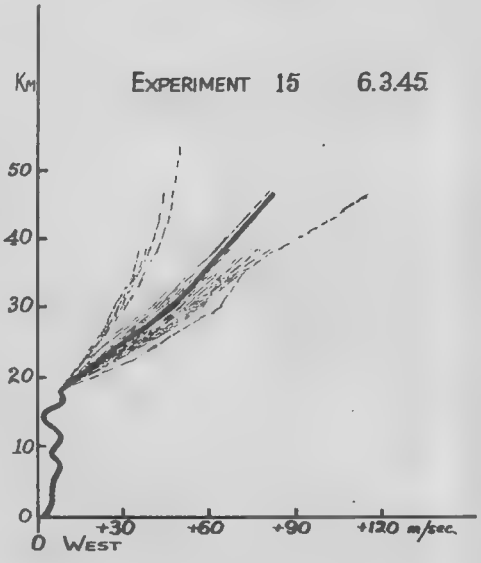


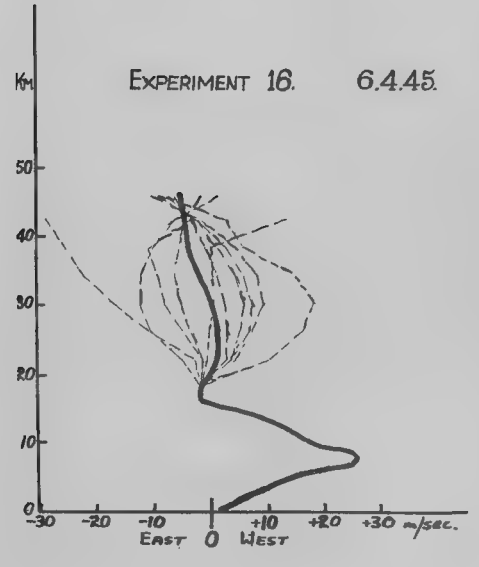


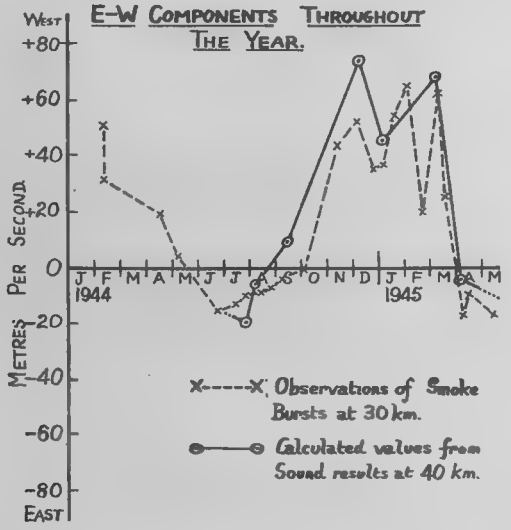












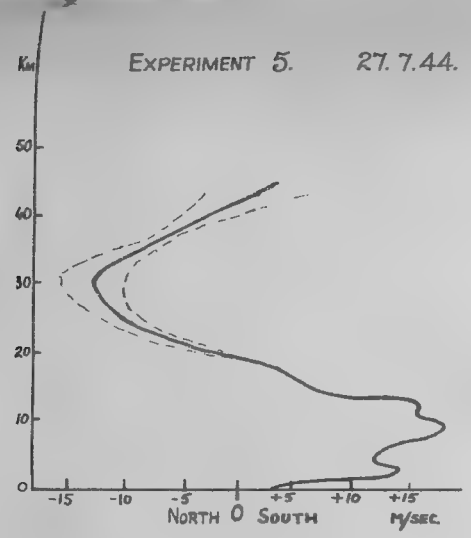
East — West Wind Component Distributions Obtained in the Different Experiments.

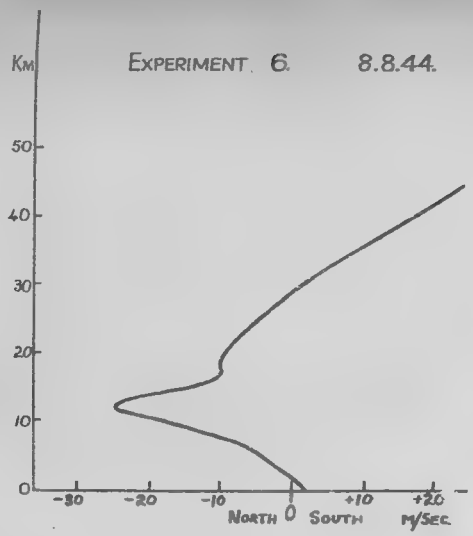
Continuous lines indicate Mean Values.

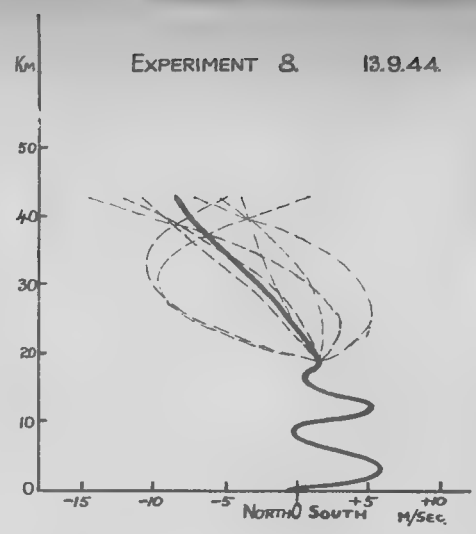
Broken lines indicate Individual Determinations where several were possible.

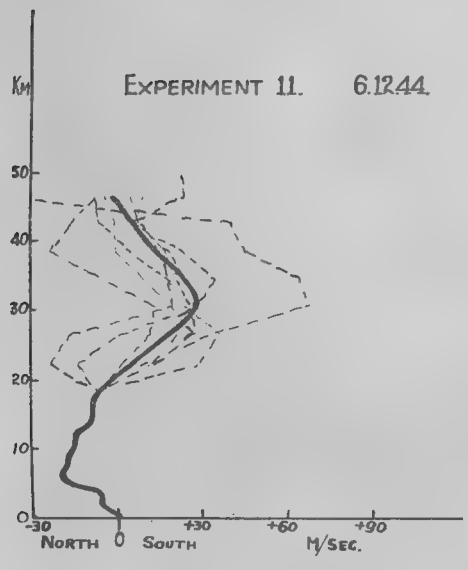
Values below 18 km. are Radio-Sonde values.

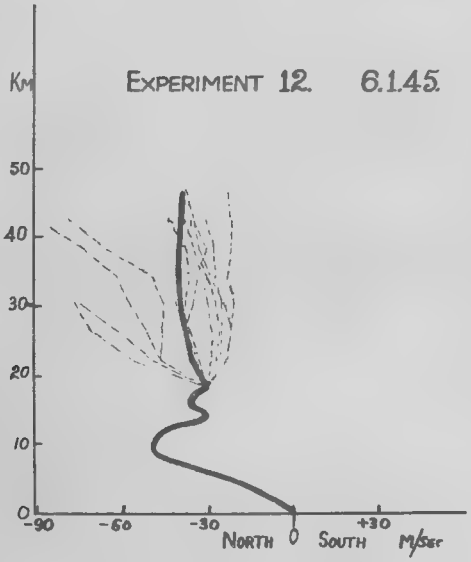
Fig. 5.

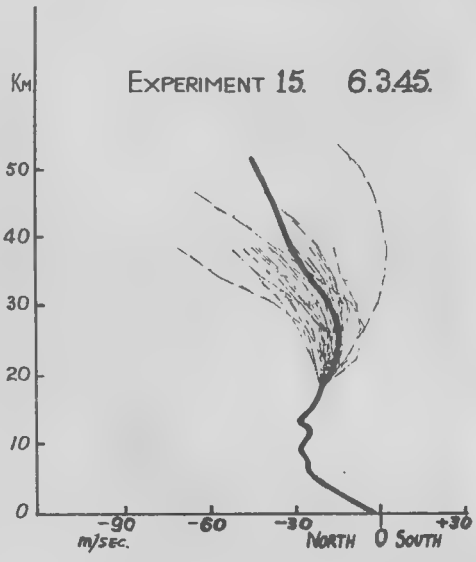


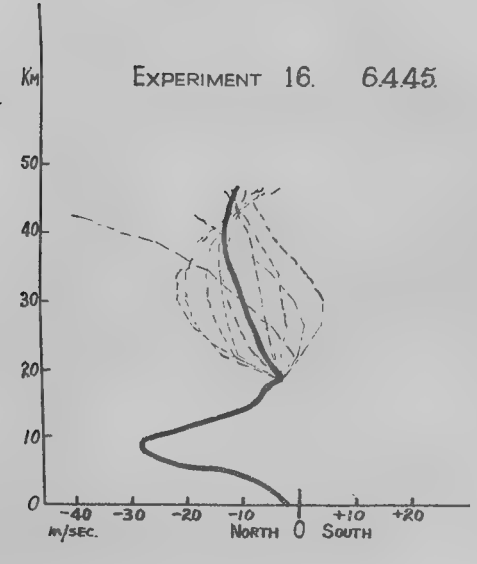


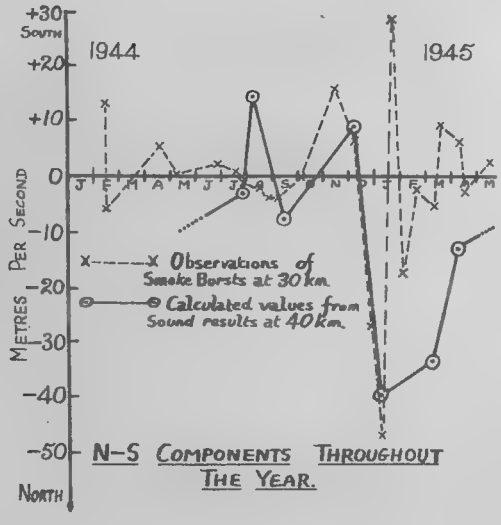












N-S Components Throughout The Year.

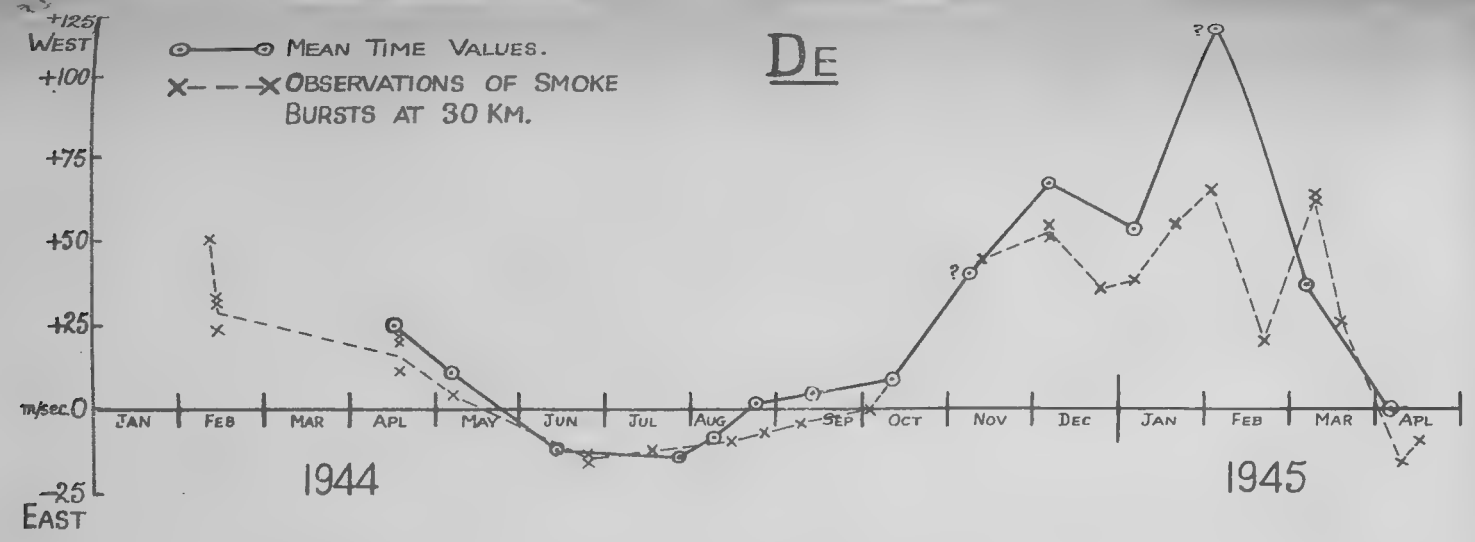
North — South Wind Component Distributions Obtained in the Different Experiments.

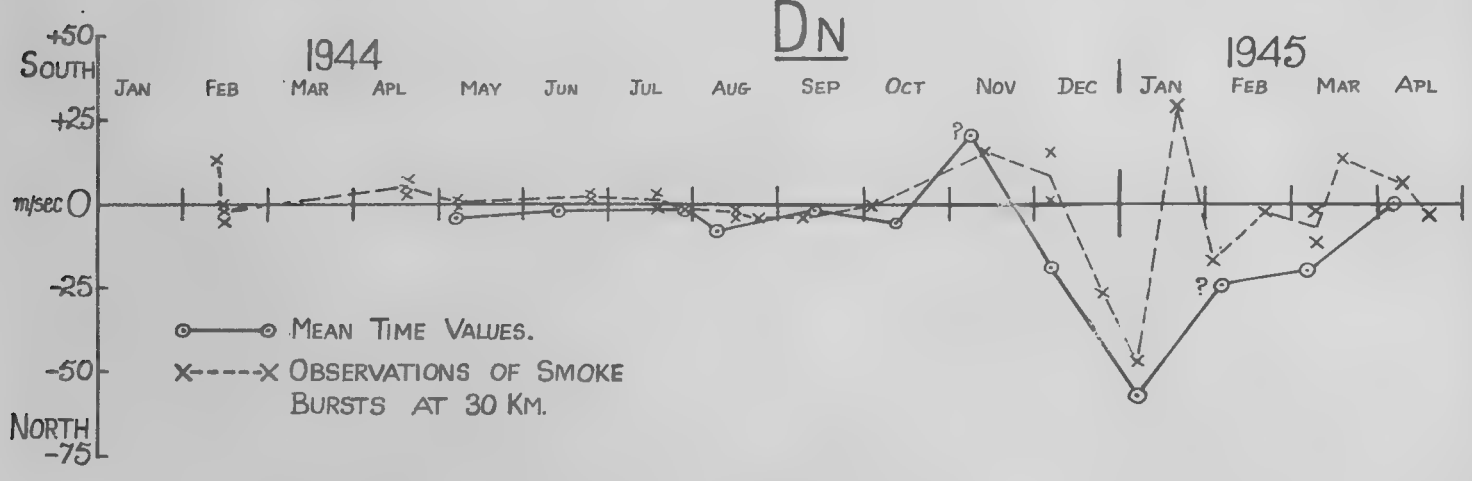
Continuous Lines Indicate Mean Values.

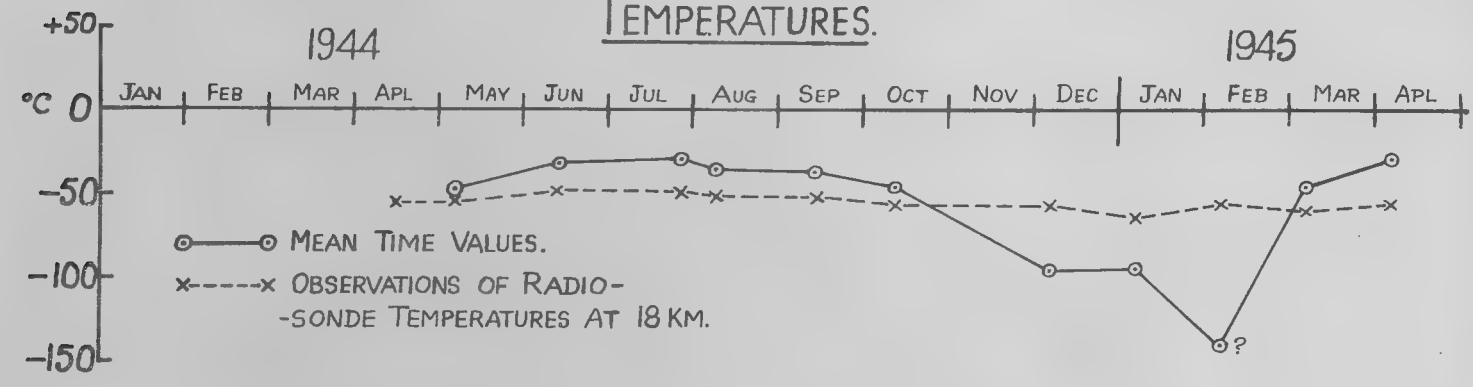
Broken Lines Indicate Individual Determinations Where Several Were Possible.

Values Below 18 km. Are Radio-Sonde Values.

Fig. 6.







BEST MEAN TIME VALUES.

THESE VALUES ARE ALL VERY APPROXIMATE AND INDIVIDUAL DETERMINATIONS SHOWED CONSIDERABLE SCATTER FROM THESE.

FIG. 7.

1/2 964

1947 SEP 11 AM 9 28

7th July 1947

(Appendix No.3. of N.P.L. 760)

tion of the Stability of Radio — C.F.M. Amerasun
L.F.B. = 71°

CONFIDENTIAL Its contents must not be quoted.

AIR MINISTRY

M.R.P. 350
24th July 1947

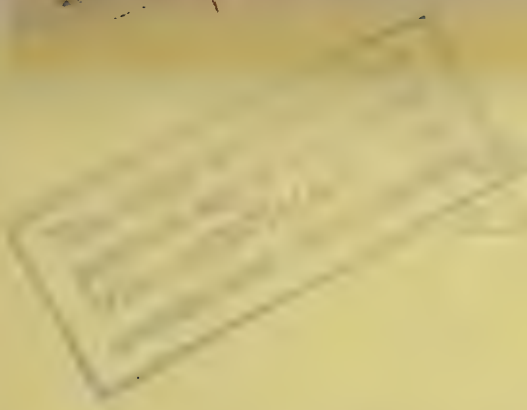
METEOROLOGICAL RESEARCH COMMITTEE

INDEX OF M.R.P.'s APPROVED FOR GENERAL DISTRIBUTION

(Appendix No.3. of M.R.P. 200)

M.R.P.	TITLE	AUTHOR
284	The Audibility of the Burton-on-Trent Bomb Dump Explosion	R.J. Murgatroyd
293	An Investigation of the Stability of Radio - Altimeter S.C.R. - 718	C.K.M. Aanensen
297	A Preliminary Note on Summer Atmospherics	C.K.M. Douglas
298	The Theory of Tropical Cyclones	J.S. Sawyer
305	Security Category of M.R.P.'s.	
309	The Size Distribution of Cloud Particles	A.C. Best
318	The Present Position of Weather Forecasting in Tropical Regions	A.G. Forsdyke
319		
320	The Diurnal Variation of Fog at Sea	H. Jameson
321	Meteorological Research Programme	
328	A Review of Progress in the Development of some Meteorological Instruments.	
329	Report of the Tropical Met. Panel.	
330	Shape of Raindrops and mode of Disintegration of large drops.	A.C. Best
331	An Approximate Method of Estimating the Pressure Pattern Flight Path for Minimum Flight time in a Stationary System of Geostrophic Winds.	J.S. Sawyer
334		
338	The Fine Structure of Wind in the Free Air.	C.S. Durst
341	High Altitude Sounding Balloons.	O.M. Ashford
344	A Note on the Adiabatic Lapse Rate for Saturated Air at Temperatures below 0° C.	J.K. Bannon

Encl #15



[illegible]

[illegible] observations were made by Mr. L. J. Lewis at the Ynyslas [illegible] by Mr. A.R. Lewis, both of the Meteorological Office. A [illegible] were also made at Aberporth (Cardiganshire) by Mr. Lucas and these

[illegible] purpose of this note is to compare the results obtained at Ynyslas and

[illegible] was charged across [illegible] and then either by [illegible] the power [illegible]

[illegible] is a set of [illegible]

This report has not been published and is to be treated CONFIDENTIALLY. Its contents must not be quoted.

16th June, 1947

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

THE SIZE DISTRIBUTION OF RAIN DROPS

BY

A. C. BEST.

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES
ACC.47.S12.

1. Introduction. During the latter half of 1944 and the first half of 1945 a number of observations of the size distributions of rain drops were made at Shoeburyness (Essex) and Ynyslas (on the coast of Cardiganshire). The Shoeburyness observations were made by Mr. L.G. Hemens and the Ynyslas observations by Mr. A.R. Laird, both of the Meteorological Office. A few observations were also made at Aberporth (Cardiganshire) by Mr. Laird and these have been combined with the Ynyslas observations.

The purpose of this note is to compare the results obtained at Ynyslas and Shoeburyness both with one another and with results obtained in similar investigations by other workers.

2. Experimental Details. The same methods were used at both stations. A Whatman No. 1 filter paper, having a diameter of 27 cms, was clamped across a circular aluminium framework having a diameter of about 20 cms. The paper had been previously impregnated with Rhodamine Red dye either by dusting it onto the paper with a tuft of cotton wool or by dipping the paper in carbon tetrachloride in which the dye was suspended. Drops of water falling on paper prepared in this way leave a permanent record consisting of red stains more or less circular in shape. The relation between drop diameter and stain diameter was obtained from a calibration curve determined experimentally by the Physics Section, C.D. Experimental Station, Porton.

In each experiment one or more such papers were exposed horizontally to the rain for a measured time. Subsequently the stain diameters were assessed individually by comparison with either a set of circles or a set of straight lines representing drops of specified diameters. Normally all stains on the paper were assessed but in a few cases the number was so great that only a limited area was examined. The drop size distribution was, of course, determined from the number of drops having diameters within specified limits. From a knowledge of the area of the paper and the time of exposure (which varied from a few seconds to one or two minutes according to the rate of rainfall) the rate of rainfall could also be computed.

At Shoeburyness the normal practice was to expose only one paper at a time but at Ynyslas three papers were usually used simultaneously in any one exposure. At both stations exposures were usually made about every two or three minutes during the period of rain.

It is of interest to note that the "magnification" introduced by measuring a stain diameter instead of a drop diameter is not linear over the range of drops found. This can be seen from the following readings taken from the smoothed calibration curve.

Drop Diameter (mm)	0.5.	1.0	2.0	3.0	4.0	5.0	6.0
Stain Diameter (mm)	1.9	4.4	10.8	19.0	28.2	37.8	48.0

A practical difficulty of this method lies in the fact that large drops tend to splash on hitting the paper, thus adding a number of small stains. Frequently however, such stains could be recognised by their shape and proximity to the large stain.

Although not an experimental difficulty the tremendous labour involved in measuring some thousands of stains may also be mentioned here.

3. Data Available. The data available from each station for each exposure consisted of:-

(a) Date and time of exposure
(b) Number of papers exposed
(c) Duration of exposure
(d) Number of stains measured resulting from drops having diameters falling between specified pairs of values.
(e) Rate of rainfall based upon (c) and (d).

In the case of the Ynyslas data the earlier measurements gave numbers of drops with diameters less than 0.25 mm, 0.25 to 0.75, 0.75 to 1.25 mm. etc. and the later observations used the series 0 to 0.5, 0.5 to 1.0, 1.0 to 1.5 mm. etc. The Shoeburyness data were all based upon the latter diameter series.

The amount of data available may be seen from the following table.

	Shoeburyness	Ynyslas and Aberporth
Number of rain periods	6	13
Number of exposures	90	90
Number of papers used	90	299
Total number of drops measured.	55,934	145,123

4. Results by Other Workers. There has been comparatively little detailed information published concerning the size distribution of raindrops. The following paragraphs indicate the workers whose results have been used in later sections for comparison with the observations at Shoeburyness and Ynyslas.

P. Lenard. Account published in Met. Zeit XXI 1904. The method adopted was similar to that at Ynyslas and Shoeburyness.

E. Niederdorfer. Account published in Met. Zeit XLIX 1932. The method adopted was similar to that at Ynyslas and Shoeburyness.

A. Defant. Full account published in Wiener Sitzungsber; Math. - naturwiss. Klasse, CXIV, 1905 and a summary in Met. Zeit XXII 1905. Method similar to that of Ynyslas and Shoeburyness.

J.O. Laws and D.A. Parsons. Account published in T. Amer. Geoph. Union, Washington 1943. In this work the raindrops were caught in pans of sifted flour. The resulting pellets were baked and the size distribution of the raindrops assessed from the size distribution of the pellets.

Some results obtained by W.A. Bentley, who used a method similar to Laws and Parsons, published in the U.S. Monthly Weather Review for 1904 were also examined but the measurements seem to have been made with no very great attempt at accuracy and no use has been made of these results.

Of the four investigations mentioned above there is no doubt that the results of Laws and Parsons are the most comprehensive. Lenard does not state the number of drops measured and some of his figures seem rather irregular. Defant and Niederdorfer were both greatly concerned with the question of whether the volumes of the larger drops were exact multiples of the volumes of the smaller drops. Their accounts contain a good deal of information relevant to this point, but other information which might have been given has been omitted. Because of this it has not been possible to make as complete a comparison as is desirable between the results of Laws and Parsons and of the present investigation on the one hand and the results of Defant, Niederdorfer and of Lenard on the other hand.

5. Presentation of Results. The considerations governing the presentation of the results of this analysis were similar to those operating in the presentation of the results of an analysis of cloud particle sizes (M.R.P.309) and accordingly the same method has been adopted here. Attention has been concentrated on the volume of water comprised by drops having diameters falling between certain limits rather than upon the numbers of such drops and the basic analysis has aimed at providing percentage cumulative volume curves, i.e. curves showing the relation between drop diameter and the percentage volume of the water reaching the ground in the form of drops having smaller diameters.

It seemed probable that the type of rainfall would affect the drop size distribution and the weather conditions associated with the various measurements at both stations were examined with this in mind. It was not possible, however, to devise any systematic grouping of the occasions according to type of rainfall. Probably this results from the comparatively small number of occasions on which measurements were made. More adequate data would probably show that dropsize distribution does vary with type of rainfall.

Even if it had been possible to group the occasions according to type of rainfall it would still have been necessary to examine the drop size distribution in relation to rate of rainfall. The failure to group the occasions by type simplifies the problem to the extent that the rate of rainfall is the only other relevant factor against which drop size distribution could be assessed.

When dealing with rate of rainfall as a factor in drop size distribution it is necessary to consider over what period the rate of rainfall shall be averaged. In the present case there are only two practicable possibilities. The rate of rainfall could be averaged over the duration of rain during which exposures were made. In this case the drop size distribution would have to be averaged over the same period. Alternatively the drop size distribution obtained from any one exposure could be related to the rate of rainfall during the exposure period. The former alternative was examined but gave rather scattered results. In the following paragraphs the rate of rainfall was accordingly computed for each exposure and an attempt has been made to relate drop size distribution to the rate of rainfall so calculated.

In addition to knowing the distribution of drop sizes it is also useful to know the average drop size. There are several ways of defining the average drop size and, as in the case of cloud particles (M.R.P.309) two parameters have been defined as follows:-

D_V = Mean volume diameter = diameter of a drop having volume equal to total volume of water reaching ground divided by the number of drops.

D_{50} = 50% diameter = diameter of drop whose size is such that 50% of the water reaching the ground is comprised by smaller drops.

Later it has also been found convenient to define a "predominant drop size" which differs from both the mean diameters mentioned above.

6. Variation in Mean Drop Size for the same rate of Rainfall. In order to determine the constancy of the drop size distribution for a given rate of rainfall all exposures at both stations when the rate of rainfall was between 2.4 and 3.0 mms./hour were selected. For each exposure the mean volume drop diameter D_V (i.e. $(6V/\pi N)^{1/3}$ where V is volume of water corresponding to N drops, N being sufficiently large to give a representative sample) and the 50% diameter D_{50} were computed. The results are given in Table I below.

TABLE I

Mean volume diameter D_v and diameter D_{50} corresponding to 50% of volume of water reaching ground, for precipitation intensity between 2.4 and 3.0 mm/h.

Station	No. of Exposures	Mean	Max.	Min.	Standard Deviation
		Mean Volume Diameter D_v (mm).			
Shoeburyness	8	1.38	1.48	1.24	0.09
Ynyslas	11	0.90	1.25	0.63	0.20
		50% Diameter D_{50} (mm).			
Shoeburyness	8	2.07	2.50	1.59	0.30
Ynyslas	11	1.32	1.94	0.72	0.38

The figures in this table are to be regarded as giving an indication of the reliability of the factor "rate of precipitation" as a guide to drop size distribution. For each of the two stations considered separately the results are reasonably satisfactory but the comparison between the two stations is disappointing. It will be noticed that not only are the mean values at Shoeburyness substantially greater than those at Ynyslas but the variation at the latter station is greater than that at the former. The fact that the mean value at each station approximates so closely to the average of the corresponding maximum and minimum values indicates a symmetrical variation.

7. Variation of Mean Volume Diameter and 50% Volume Diameter with rate of precipitation. The total range of rate of precipitation was divided into steps and the exposures grouped according to the rate of precipitation. For each group of exposures (at each station) a percentage cumulative volume curve was then constructed as follows.

The numbers of drops for a given range of diameters were added for all exposures in one group. The total volume of the drops in the range of diameters was computed by assuming all drops in the range to have a diameter corresponding to the mid point of the range; for each group these volumes were added in succession, thus giving corresponding values of drop diameter and volume of all drops having a smaller diameter. Finally the volumes were expressed as percentages of the total value for the group. For these computations the Ynyslas observations were necessarily divided into two sets according to the ranges of diameters employed in the original measurements. Cumulative volume curves were plotted for each group of each set and values of the cumulative volumes read from these curves for the same diameters. Corresponding cumulative volumes from the two sets were then added and finally percentage values found for the composite curves.

From the percentage cumulative volume curves described above the drop diameters D_{50} corresponding to 50% of the volume of water were read. The mean volume diameter D_v was also computed for each group. The results are given in Table 2.

/Table 2.

TABLE 2.

Variation of Mean Volume Diameter and 50% Volume Diameter with Rate of Precipitation.

Ynyslas					
Range of Rate of Precip. (mm/hr)	0.0 to 0.6	0.6 to 1.8	1.8 to 3.6	3.6 to 6.6	6.6 to 12.0
Mean Rate of Precip. (mm/hr)	0.36	1.32	2.76	4.68	8.88
Mean Vol. Diam. (mm) D_V	0.65	0.75	0.86	0.93	1.23
50% Vol. Diam. (mm) D_{50}	0.91	1.11	1.22	1.41	1.78
Number of Exposures	8	23	29	21	9
Number of Drops	7411	31323	46344	41214	18831
Shoeburyness					
Range of Rate of Precip. (mm/hr)	0.0 to 0.6	0.6 to 1.8	1.8 to 3.6	3.6 to 4.38	-
Mean Rate of Precip. (mm/hr)	0.18	1.14	2.58	4.2	-
Mean Vol. Diam. (mm) D_V	0.68	0.88	1.33	1.46	-
50% Vol. Diam. (mm) D_{50}	1.19	1.65	1.96	2.61	-
Number of exposures	37	26	24	3	-
Number of Drops	21906	20112	11893	2073	-

It is clear that both D_{50} and D_V increase with increasing rate of precipitation.

8. Mean Diameters from other workers.

(a) Lenard. As already mentioned the figures given by Lenard show various irregularities, due undoubtedly to a small number of measurements. Lenard gives the size distribution of drops in 13 separate measurements (two measurements were made in one rainfall and three in another but the other eight measurements were in different rainfalls). For each measurement he also quotes the rate of rainfall. Fortunately the rates of rainfall fall into two well defined groups. In series 1 to 7b (series numbers as in Lenard's paper) the intensity of rainfall varied from 0.08 to 7.8 mm/hour and in the remaining series from 15.6 to 43.2 mm/hour. Accordingly the drop distributions for series 1 to 7b have been added together and D_V, D_{50} and rate of rainfall computed from the result and the remaining series (omitting No. 9 which had a very irregular distribution of drop sizes) treated similarly. The results are as follows:-

Rate of Rainfall	D_V	D_{50}
3.24 mm/hr	1.26 mm	1.94 mm
22.8	2.13	3.3

/(b)

(b) Niederdorfer. In this paper the rates of rainfall are not quoted. Instead the author divides his results into "general rains" (5 series) and "thunder rains" (3 series). Moreover the author does not give the sizes of the largest drops. In the "general rains" a total of 8005 drops were measured of which three had unspecified sizes greater than 2.59 mm. In the following paragraphs these three drops have been ignored. In the "thunder rains" a total of 1647 drops were measured but the sizes of 95 drops exceeding 2.59 mm are not given. From the text, however, it appears that the biggest drop had a diameter of 5.42 mm. From this information a "cumulative number" curve was plotted and the size distribution of the 95 large drops assessed. Inevitably this procedure introduces some errors but in effect it is equivalent to "smoothing" the upper part of the cumulative number curve and it is probable that the errors are not serious.

The values of D_V and D_{50} for both "general rains" and "thunder rains" were computed from Niederdorfer's data with the following results.

	D_V	D_{50}
General rains	0.89mm	1.15mm
Thunder rains	1.71	2.38

(c) Defant. The difficulties in using Niederdorfer's data are present in using Defant's data in exactly the same form. Defant divided his results into those obtained from "continuous rains" (7 series) and those obtained from "shower or thunder rains" (13 series). The "continuous rain" samples contained a total of 2438 drops of which no sizes are given for the 13 drops with diameter exceeding 2.15mm. These 13 drops have been ignored in the following paragraphs. The "thunder rains" samples contained a total of 4049 drops with 260 drops having unspecified diameters exceeding 2.15mm. A search through the text of the paper found the statement that the largest drop had a weight of 140 mgm. (d = 6.45mm) and the sizes of 87 other drops which exceeded 2.15mm in diameter were mentioned in the text in discussion of the suggestion that raindrop sizes occur in multiples of a basic volume. It was not possible, however, to ascertain the sizes of all the 260 large drops. A cumulative number curve was accordingly constructed as for Niederdorfer's data and a size distribution estimated from that. Finally D_V and D_{50} were computed with the results indicated below. It was not possible to assess the rates of rainfall.

	D_V	D_{50}
Continuous Rain	0.94 mm	1.18 mm
Thunder rain	1.52	2.97

(d) Laws and Parsons. These workers presented their results in the form of a table showing the percentage amount of water reaching the ground comprised by drops of different sizes for each of eight different rates of rainfall. There was thus no difficulty in computing D_V and D_{50} for each of the intensities of rainfall tabulated in their paper. The results were as follows.

Rate of rainfall mm/hr.	0.25	1.27	2.54	12.7	25.4	50.8	101.6	152.4
Mean vol. diam. D_V mm	0.50	0.64	0.69	1.02	1.50	1.66	1.81	1.86
50% diam. D_{50} mm.	0.95	1.30	1.45	1.95	2.20	2.50	2.85	3.10

9. Comparison of mean diameters.

The values of D_v and D_{50} have been plotted logarithmically against rate of precipitation I for all available data in figs 1 and 2 respectively. The values obtained from the data of Niederdorfer and of Defant are also indicated though for these two workers the rate of rainfall is not known.

In both figures there is considerable scatter of the points but it is possible to draw a straight line in each case to give a reasonable fit. These two lines indicate the relationships.

$$D_v = 0.71\ I^{0.25} \qquad D_{50} = 1.21\ I^{0.19}$$

where the two diameters are measured in millimetres and I in mm./hour.

It will be observed that, in both cases, the Shoeburyness and Lenard data give consistently larger diameters than indicated by these two equations. In fact these data alone would give different straight lines, almost parallel to those drawn in figs. 1 and 2, leading to similar equations to those above with different coefficients but with exponents not significantly different.

There is a puzzling feature about these curves. Each set of data gives a smooth curve but there are considerable differences between the various curves. In the case of Laws and Parsons data this may be explained by the fact that these authors smoothed their experimental data and published results obtained from smoothed curves. This, however, does not apply to the Shoeburyness and the Ynyslas data. The differences between the Laws and Parsons curves and the Ynyslas curves are small; the Lenard data are also in reasonable agreement with the Shoeburyness data but there are appreciable differences between these two pairs of sets of data. As already indicated the equipment and methods employed at Shoeburyness and Ynyslas were, as far as could be ensured, identical. The differences in the results must then be due either to the small number of rain periods not giving representative results or to geographical differences in drop size distribution. The former explanation might be expected to lead to a considerable scatter of points on each curve. Thus it seems probable that there may be a consistent difference between drop size distribution for the same rate of rainfall at different stations. Obviously this may be associated with differences in the climatology leading to different frequencies of various types of rainfall.

10. Drop Size Distributions.

From the percentage cumulative volume curves for each set of data the percentage volumes corresponding to diameters 0.25, 0.50, 0.75mm etc. were read off. These were differenced in succession giving the drop size distribution figures in table 3. The figures for the Laws and Parsons data are of course taken direct from the original paper. In that paper the authors made it clear that they had smoothed the original experimental data before compiling the table. They also gave corresponding figures for three higher rates of rainfall which have been omitted from table 3, since there are no other figures with which to compare them.

A certain amount of approximation has been necessary in arriving at the cumulative volume curves from the published data of Lenard, Niederdorfer and of Defant and accordingly it was considered preferable to draw a smoothed curve through the results based on the work of these authors. In the case of the Ynyslas and Shoeburyness data, however, the percentage cumulative volume curves were drawn to pass through each one of the experimental points.

It is clear that all the curves represented by the data in table 3 are of the same general shape i.e. roughly of the "normal error" type with the diameter corresponding to the predominant drop size increasing with increasing rate of rainfall. Figure 3 shows four sets of data, all for approximately a rainfall intensity of about 2.5 mm/hr, plotted as illustrations.

/It

It is interesting that the highest rainfall intensity data for Ynyslas, Shoeburyness and Defant all show a double peak but the data is probably inadequate to be sure that this feature is significant and not due to experimental error.

Table 3 has been given in some detail because it represents the results of the basic analysis of all the available data. For many purposes however the size distribution of the drops while in the air is more useful than the size distribution of the drops on the ground. Accordingly no further use will be made of table 3 until we consider the size distribution of drops in the air in section 13 below.

/TABLE 3.

TABLE 3

Percentage volume of water reaching ground comprised by drops having diameters within various ranges.

Author	Ynyslas					Shoeburyness				Laws and Parsons					Lenard		Niederdorfer		Defant	
Rate of Rainfall (mm/hr)	0.36	1.32	2.76	4.68	8.88	0.18	1.14	2.58	4.2	0.25	1.27	2.54	12.7	25.4	3.24	22.8	CR	TR	CR	TR
Drop Diameter (mm)																				
0 - 0.25	0.1	0.0	0.3	0.0	0.0	1.0	1.0	0.0	0.0	1.0	0.5	0.3	0.1	0.0	0.2	0.0	0.0	0.0	0.0	0.3
0.25 - 0.50	4.1	1.9	0.9	0.0	0.3	2.7	6.8	0.2	0.2	6.6	2.5	1.7	0.7	0.4	1.0	0.1	2.3	0.4	1.1	1.8
0.50 - 0.75	20.3	11.0	4.2	5.5	2.1	8.3	9.3	1.9	1.1	20.4	7.9	5.3	1.8	1.3	2.2	0.2	11.1	0.8	10.1	3.7
0.75 - 1.00	36.9	25.0	23.5	16.7	7.5	16.1	11.8	3.5	2.6	27.0	16.0	10.7	3.9	2.5	4.6	1.0	22.3	2.2	21.3	4.9
1.00 - 1.25	25.0	24.8	23.7	15.1	11.5	22.9	11.4	6.3	4.7	23.1	21.1	17.1	7.6	5.1	7.6	1.6	20.4	4.2	20.5	6.8
1.25 - 1.50	11.4	15.5	14.9	17.4	10.7	13.3	14.6	10.3	7.0	12.7	18.9	18.3	11.0	7.5	9.5	2.7	15.9	6.0	18.5	7.0
1.50 - 1.75	1.6	10.1	11.6	12.5	19.6	11.7	12.8	15.2	10.2	5.5	12.4	14.5	13.5	10.9	13.1	3.4	12.8	7.9	15.5	5.5
1.75 - 2.00	0.5	6.1	7.9	9.6	10.9	6.8	10.3	14.9	10.1	2.0	8.1	11.6	14.1	11.8	15.1	5.0	8.2	9.4	11.0	5.5
2.00 - 2.25		3.2	5.2	7.4	13.5	6.2	8.9	9.5	4.9	1.0	5.4	7.4	11.3	12.1	12.2	5.3	5.0	10.1	2.0	4.6
2.25 - 2.50		1.0	3.0	5.5	6.1	4.8	6.9	9.2	5.5	0.5	3.2	4.7	9.6	11.2	8.9	5.5	1.7	10.5		5.4
2.50 - 2.75		0.7	2.4	3.3	4.2	3.2	2.5	9.4	9.7	0.2	1.7	3.2	7.7	8.7	6.9	6.2	0.3	9.5		5.0
2.75 - 3.00		0.3	1.1	2.1	3.9	1.9	0.9	7.8	10.5		0.9	2.0	5.9	6.9	5.2	8.5		8.0		5.5
3.00 - 3.25		0.2	0.9	1.7	3.3	0.6	1.0	5.4	10.9		0.6	1.3	4.2	5.9	4.5	8.5		6.0		5.6
3.25 - 3.50		0.2	0.3	0.8	2.8	0.5	1.0	4.0	7.6		0.4	0.7	2.6	5.0	3.2	8.6		5.0		5.5
3.50 - 3.75			0.3	0.7	1.0		0.4	1.8	4.3		0.2	0.4	1.7	3.2	2.3	8.6		4.2		5.5
3.75 - 4.00			0.1	0.6	0.9			0.6	2.5		0.2	0.4	1.3	2.1	2.3	7.8		3.8		5.6
4.00 - 4.25				0.4	1.0				6.8			0.2	1.0	1.4	1.2	7.6		2.7		5.3
4.25 - 4.50				0.1	0.5				2.4			0.2	0.8	1.2		6.9		2.2		4.5
4.50 - 4.75					0.2								0.4	0.9		6.1		2.1		3.9
4.75 - 5.00													0.4	0.7		4.2		1.8		2.9
5.00 - 5.25													0.2	0.4		2.2		1.8		2.0
5.25 - 5.50													0.2	0.3				1.4		1.2
5.50 - 5.75														0.2						0.7
5.75 - 6.00														0.2						0.7
6.00 - 6.25														0.1						0.6

11. Number frequency distribution of drops.

The volume distribution figures given in table 3 can be easily converted to give actual frequencies of drops of various sizes reaching the ground but the converted figures are liable to be in error for the very large and the very small diameters. Accordingly table 4 has been constructed to show the percentage numbers of drops of various sizes measured at Ynyslas and Shoeburyness.

TABLE 4.

Percentage number of drops of various sizes reaching the ground.

Rate of Rainfall mm/hour	0.36	1.32	2.76	4.68	6.88
Drop Size mm	Ynyslas				
0 - 0.25	33.0	21.0	21.0	18.8	31.2
0.25 - 0.75	49.0	57.6	37.8	43.0	25.2
0.75 - 1.25	17.1	16.8	32.7	25.7	26.4
1.25 - 1.75	1.07	3.96	6.75	10.0	12.3
1.75 - 2.25		0.44	1.34	1.66	3.88
2.25 - 2.75		0.03	0.24	0.49	0.60
2.75 - 3.25			0.055	0.108	0.229
3.25 - 3.75			0.017	0.014	0.092
3.75 - 4.25				0.020	0.023
4.25 - 4.75					0.015
Total No. of Drops	5338	15170	23525	14749	13096

Rate of Rainfall mm/hour	0.36	1.32	2.76	4.68	6.88	0.18	1.14	2.58	4.2
Dropsize mm	Ynyslas					Shoeburyness			
0 - 0.5	48.2	51.3	48.1	39.1	24.6	74.7	61.7	31.1	33.6
0.5 - 1.0	47.5	37.3	41.2	46.9	36.2	18.0	25.4	29.5	27.4
1.0 - 1.5	4.20	9.62	9.32	9.53	23.4	5.81	8.02	19.9	18.9
1.5 - 2.0	0.145	0.978	1.47	2.96	10.4	1.03	3.41	13.2	11.8
2.0 - 2.5		0.217	0.333	0.824	3.43	0.301	1.13	3.74	2.85
2.5 - 3.0		0.025	0.096	0.212	1.17	0.078	0.313	1.96	2.89
3.0 - 3.5		0.006	0.009	0.057	0.506	0.009	0.030	0.64	1.69
3.5 - 4.0			0.004	0.019	0.087		0.020	0.012	0.386
4.0 - 4.5				0.004	0.052				0.386
Total No. of drops	2073	16153	22819	20465	5755	21906	20112	11893	2073

In table 4 the Ynyslas data have been divided into two groups because the grouping of the drop sizes was changed during the course of the work. In the upper part of the Ynyslas table the percentage numbers in the smallest class should of course be doubled for purposes of comparison with the numbers of the larger drops.

The rates of rainfall given in table 4 have been taken from table 2. In the case of the Ynyslas data this is not exact as the values in table 2 are means of the rainfall rates over the two periods of the Ynyslas work; the discrepancy, however, was trifling.

An outstanding feature of table 4 is the large number of small drops. It is clear from the table that if the figures were plotted it would be impossible in most cases to assess the size of maximum frequency owing to the fact that this size is below the lower limit of measurement. This emphasizes the convenience of dealing with the volume distribution rather than the number distribution.

12. Concentration of water.

If the rate of rainfall is I mm/hour then the volume of water reaching the ground is $I.10^4/36$ mm^3/sec./square metre. This volume of water is comprised of groups of drops as indicated in table 3. Let the terminal velocity of the drops in one group be V metres/second and let this group account for P% (the entries in table 3) of the water reaching the ground. Then the amount of water reaching the ground due to drops in one group is

$$\frac{P}{100} \cdot \frac{I.\ 10^4}{36} \quad \text{mm}^3/\text{sec./square metre.}$$

This water falls with a terminal velocity V metres/sec. so that the amount of water per unit volume in the air due to this group of drops is

$$\frac{PI}{V}\ \frac{10^2}{36} \quad \text{mm}^3/\text{cubic metre.}$$

For each column in table 3, I is a constant so that the total volume of water suspended in the air is

$$\frac{I.\ 10^2}{36} \cdot \Sigma\ \frac{P}{V} \quad \text{mm}^3/\text{ cubic metre.}$$

the summation being taken over the various groups. Values of $\frac{P}{V}$ were formed for each entry in table 3 (after some of the larger drop groups had been added together) using the following values for the terminal velocities (based upon M.R.P.40).

Drop Diameter (mm)	.125	.375	.625	.875	1.125	1.375	1.625	1.875
Term. Vel. (M/sec.)	.33	1.43	2.5	3.51	4.4	5.1	5.66	6.16

Diam.	2.25	2.75	3.25	3.75	4.25	4.75	5.25	5.75	6.25	6.75
Vel.	6.85	7.60	8.13	8.55	8.85	9.01	9.10	9.13	9.15	9.17

By summing over the groups the value of the liquid water content of the air

$$W = \frac{I.\ 10^2}{36} \cdot \Sigma\ \frac{P}{V} \quad \text{mm}^3/\text{ cubic metre.}$$

was computed for each value of I for each author. These values are given in table 5.

In figure 4 log I has been plotted against log W. It can be seen that the points from the four different sets of data employed all approximate reasonable closely to a straight line whose slope and position indicate that W and I are connected by the relation

$$W = 66.1\ I^{0.87}$$

13. Size distribution of drops in the air.

By adding $\frac{P}{V}$ successively for each group (for a given value of I) and expressing as percentage values of the total, percentage cumulative volume figures are obtained for the distribution of the drops in air. Instead of tabulating the results so obtained it has been found possible to devise a formula which, within the limits of the experimental error, appears to fit the observational data reasonably well.

Let F be the fraction (not percentage) of water in the air comprised by all drops having diameters less than d write

$$1 - F = \exp. \left\{ - \left(\frac{d}{a} \right)^n \right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where a and n are constants. Then it is easily seen that

$$\log. \log. \frac{1}{(1 - F)} = -0.36 + n(\log. d - \log. a)$$

where the logarithms are to base 10. Thus if log log $(\frac{1}{1 - F})$ is plotted against log d the points should lie on a straight line if equation (1) can be used to describe the drop size distribution in the air, and the values of n and a can be obtained from the slope and position of the line. In fig. 5 log log$(\frac{1}{1 - F})$ has been plotted against log d for each of the precipitation rates of each author (including the three higher precipitation rates from Laws and Parsons not hitherto specified). It can be seen that each set of plotted points approximates reasonably well to a straight line. Naturally the divergencies from straight lines are greatest for very small values of F. This probably arises from the fact that the volume of water reaching the ground has been divided by the terminal velocity. For the smallest drops the terminal velocity is small and if there is a discrepancy between the assumed mean diameter of the smallest group (0.125mm) and the actual mean diameter the corresponding error in the terminal velocity may be an appreciable fraction of the actual terminal velocity, thus leading to a large error in P/V .

From fig, 5 values of n and of a have been computed and are given in table 5.

TABLE 5.

Parameters in the formula describing drop size distribution in the air

$$1 - F = \exp. \left\{ - \left(\frac{d}{a} \right)^n \right\}$$

F = fraction of water in the air comprised by drops having diameter less than d.

Author	I	W	n	a	D'_{50}	D'_p
	mm/hr	mm^3/m^3			mm	mm
Ynyslas	0.36	30.2	3.29	1.00	0.89	0.89
	1.32	97	2.33	1.21	1.03	0.95
	2.76	180	2.43	1.36	1.16	1.09
	4.68	270	2.0	1.60	1.33	1.13
	8.88	451	2.38	1.95	1.67	1.55
Shoeburyness	0.18	13.1	1.79	1.24	1.01	0.79
	1.14	86	1.56	1.14	0.90	0.59
	2.58	121	3.02	2.11	1.87	1.85
	4.2	169	2.61	2.37	2.07	2.03
Lenard	3.24	163	2.16	1.95	1.64	1.47
	22.8	840	3.0	3.31	2.92	2.89
Niederdorfer	C	-	2.45	1.20	1.03	0.97
	T	-	2.08	3.34	2.79	2.44
Defant	C	-	3.00	1.23	1.09	1.07
	T	-	2.05	2.82	2.35	2.03

Author	I	W	n	a	D'_{50}	D'_p
	mm/hr.	mm^3/m^3			mm	mm
Laws and Parsons	0.25	22.6	2.26	0.90	0.77	0.69
	1.27	91	1.96	1.37	1.14	0.95
	2.54	155	2.19	1.51	1.27	1.14
	12.7	620	2.24	2.04	1.73	1.57
	25.4	1126	2.44	2.50	2.15	2.01
	50.8	2110	2.46	2.77	2.38	2.24
	101.6	4010	2.41	3.12	2.67	2.50
	152.4	6110	2.34	3.23	2.75	2.54

Considering the values of n in table 5 it will be seen that Shoeburyness and Lenard show n increasing as I increases, Niederdorfer and Defant indicate a reverse relationship. Laws and Parsons and Ynyslas (if the first value of n for Ynyslas be neglected) indicate no substantial dependence of n upon I. It is difficult to resist the conslusion that, substantially, n is independent of I at any rate between the values of 1 and 150 mm/hour for I. Accepting this view the mean value of n has been computed as 2.38 with a standard deviation of 0.41. It will be noticed that, if n is treated as a constant, the drop size distribution is identical for all rates of rainfall provided the drop diameters are expressed as fractions of "a". It will be seen later that there is a simple proportionality between "a" and the predominant drop diameter so that if all drop diameters are expressed in terms of the predominant drop diameter the drop size distribution is independent of the intensity of precipitation.

In figure 6 log a has been plotted against log I. Some of the Shoeburyness points and the Lenard points depart rather markedly from the general run of the other points. A straight line has been drawn in this figure to represent the best average relation between a and I, for the other points. From this line we deduce that a and I are connected by the following relation.

$$a = 1.21\ I^{0.21}$$

It will be noticed that a line through the neglected points would be parallel to the line drawn and so would affect only the coefficient of I and not the exponent.

14. Some Average Diameters for Airborne Drops.

(i) 50% Diameter.

Assuming that the relationship between F and d in section 13 is true we can obviously obtain the 50% diameter by putting F = 0.5, leading to

$$\log. D'_{50} = \log. a + \frac{1}{n}(0.36 + \log. \log. 2)$$

or

$$D'_{50} = (0.69)^{\frac{1}{n}} a$$

where D'_{50} represents the 50% diameter of the drops in the air. This has been done and values of D'_{50} are given in table 5.

(ii) Predominant Diameter.

It is clear that the predominant drop diameter, that is the diameter of the drops which account for the greatest volume of water in the air, is the diameter for which $\partial F/\partial x$ is greatest, i.e. the point at which $\partial^2 F/\partial x^2 = 0$ Differentiation of the equation for F shows that this occurs when

$$x = a\left(\frac{n-1}{n}\right)^{1/n} = D'_p$$

This predominant drop diameter has been computed for each set of data and is included in table 5.

15. Number frequency distribution of drops in the air.

The fractional volume of water corresponding to drops having diameters between x_1 and x_2 is clearly

$$e^{-(x_1/a)^n} - e^{-(x_2/a)^n}$$

and if x_1 and x_2 are sufficiently close together the number of drops of this size per unit volume of air can be obtained by dividing this quantity by $\pi x^3/6W$ where x is the mean of x_1 and x_2 and W is the total volume of water per unit volume of air.

Alternatively we may proceed as follows. The fraction of water comprised by drops having diameter between x and x + dx is $\partial F/\partial x.dx$ Hence, if W is the volume of water per unit volume of air the number of drops with diameter between x and x + dx in unit volume of air is

$$N. \frac{6}{\pi x^3} \frac{\partial F}{\partial x} dx$$

and so the number of drops having diameter between x_1 and x_2 is

$$\frac{6W}{\pi} \int_{x_1}^{x_2} \frac{1}{x^3} \frac{\partial F}{\partial x} dx$$

Differentiating the equation for F and making the appropriate change of variable this easily reduces to

$$\frac{6W}{\pi} \int_{(x_1/a)^n}^{(x_2/a)^n} e^{-t} t^{\frac{n-3}{n} - 1} dt$$

Since, however, n is less than 3 this expression is not readily integrable except by arithmetical methods.

16. Summary.

A number of measurements of raindrop sizes by the filter paper method have been made at Ynyslas and at Shoeburyness. These have been analysed the results compared with those of Lenard, Defant, Niederdorfer and of Laws and Parsons. The basic analysis is based upon the conception of the percentage volume of water comprised by the drops having diameters between certain limits and the analysis has aimed at showing how the size distribution of the drops varies with the rate of precipitation.

For the size distribution of drops on the ground two mean diameters have been defined viz.

D_v = the mean volume diameter
D_{50} = the diameter such that 50% of the water is composed of smaller drops.

It is shown that, for a rate of rainfall of 2.7mm/hr., the standard deviation for both D_v and D_{50} is about one quarter of the mean value at Ynyslas. At Shoeburyness it is about 1/15th for D_v and 1/7th for D_{50}.

The variation of D_v and D_{50} with rate of precipitation I is examined on the basis of all the available data and it is shown that approximately

$$D_v = 0.71\ I^{0.25} \qquad D_{50} = 1.21\ I^{0.19}$$

if the diameters are measured in mm. and I in mm/hour.

A table is given showing the percentage volume distribution of drop sizes for various rates of precipitation for various investigations.

A amount of liquid water per unit volume of air while the drops are still falling is examined and it is shown that, if W is the amount of water in the air in cubic millimetres per cubic metre

$$W = 66.1\ I^{0.87}$$

It is shown that the distribution of drop sizes in the air can be expressed by

$$1 - F = \exp.\left\{ -\left(\frac{d}{a}\right)^n \right\}$$

where F is the fraction of the volume of water comprised by drops having diameter less than d, and 'a' and 'd' are constants. It appears than n is a constant for all rates of precipitation and has a value 2.38 (standard deviation 0.41) but that 'a' is connected with the rate of precipitation by the relationship

$$a = 1.21\ I^{0.21}$$

The 50% Diameter for drops in the air is given by $D'_{50} = (0.69)^{\frac{1}{n}}\ a$ and the predominant diameter by

$$D'_p = \left(\frac{n-1}{n}\right)^{\frac{1}{n}} a.$$

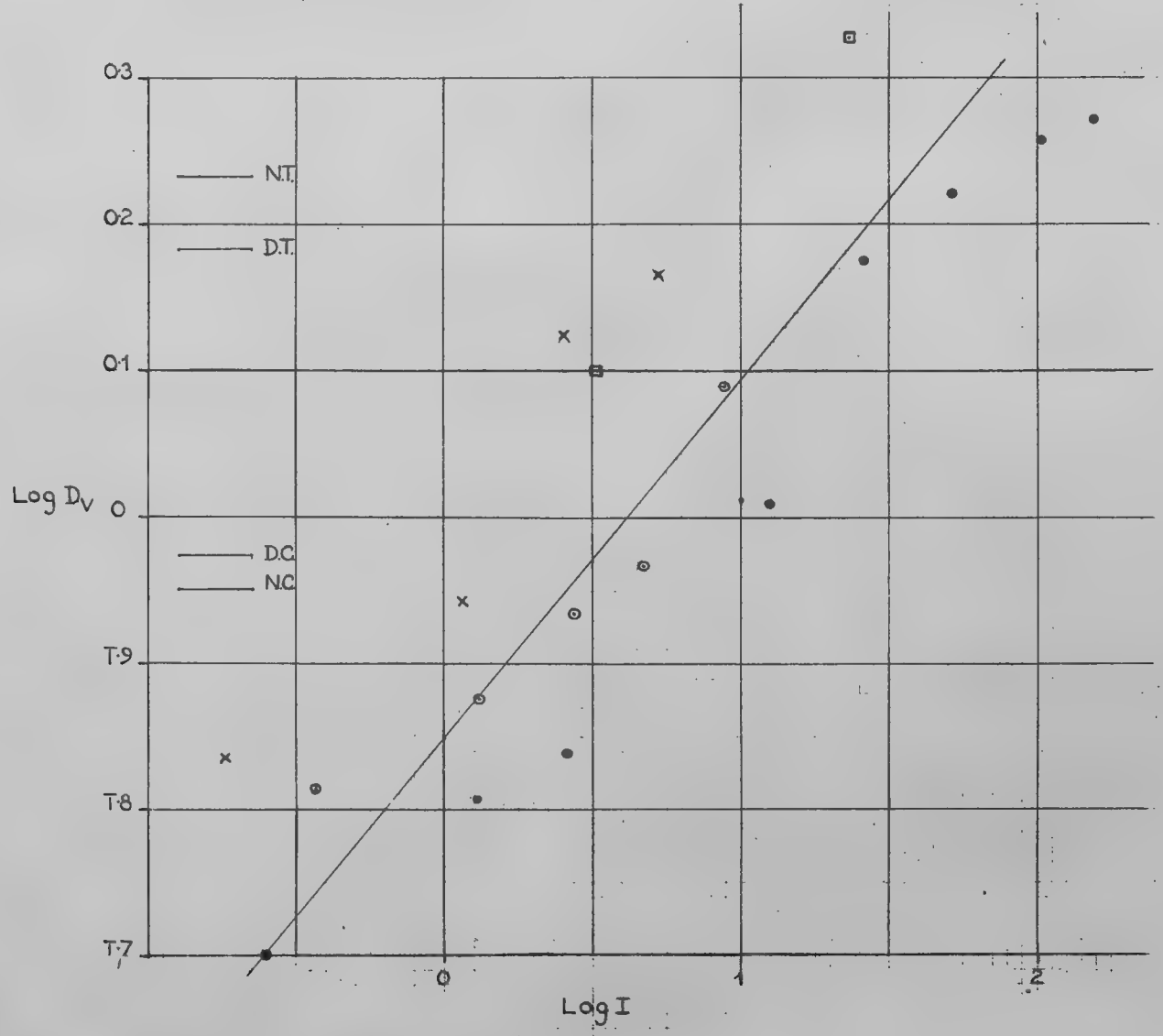


FIG 1.

Mean Volume Diameter D_V as a function of intensity of rainfall I (mm/hr)

⊙ Ynyslas
X Shoeburyness
⊡ Lenard
● Laws and Parsons

N = Niederdorfer C = Continuous rain
D = Defant T = Thunder rain.

1/3104

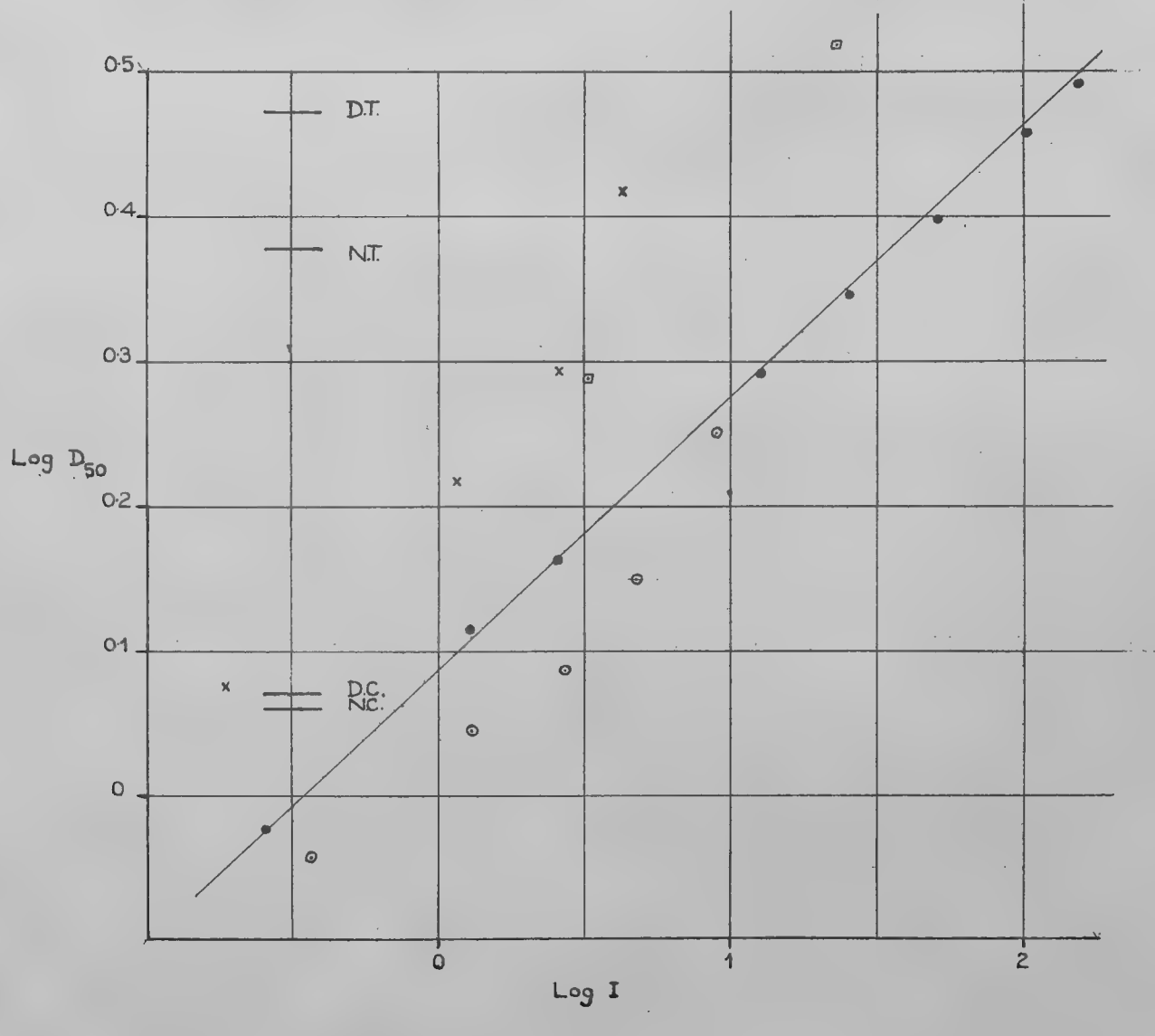


FIG.2.

50% Diameter D_{50} as a function of intensity of rainfall I (mm/hr)

- ⊡ Ynyslas
- X Shoeburyness
- ⊞ Lenard
- ● Laws and Parsons

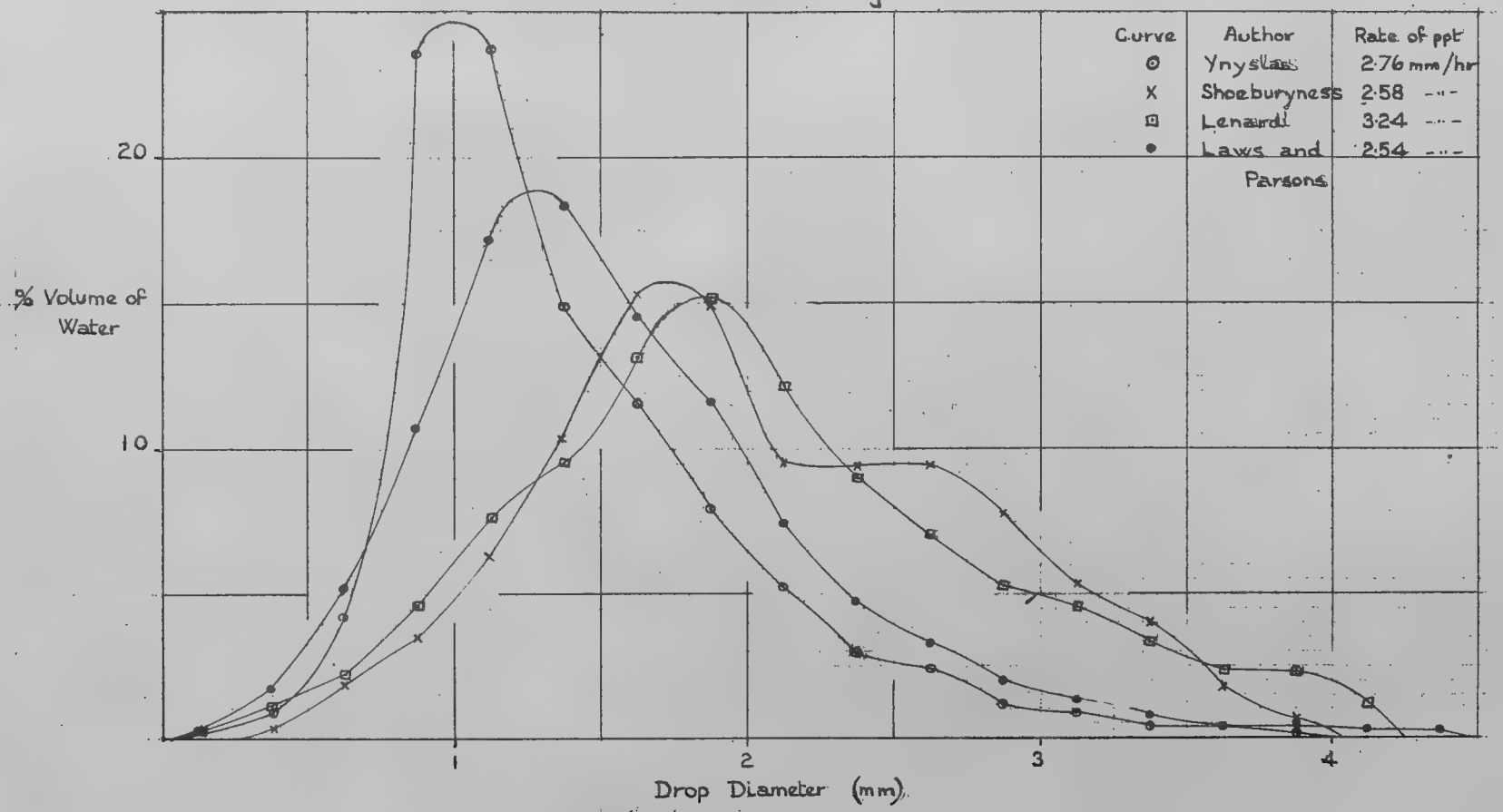
N = Niederdorfer C = Continuous Rain
D = Defant. T = Thunder Rain.

1/3104

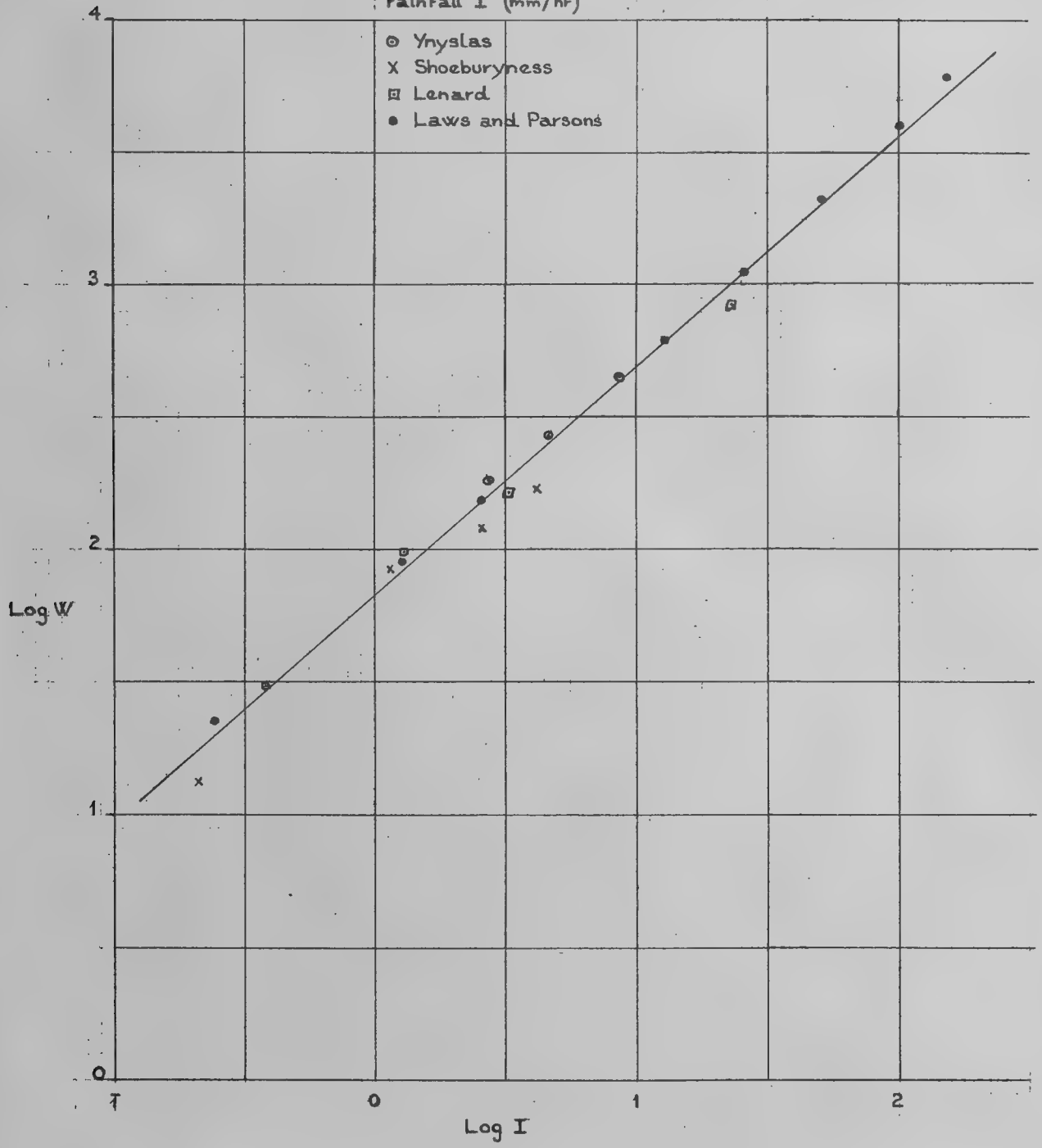
FIG.3.

MRP 352

Percentage Volume of water reaching the ground comprised by drops having diameters within a range of 0·25 mm.



Liquid Water Content of Air W (cubic mm/cubic metre) as function of rate of rainfall I (mm/hr)

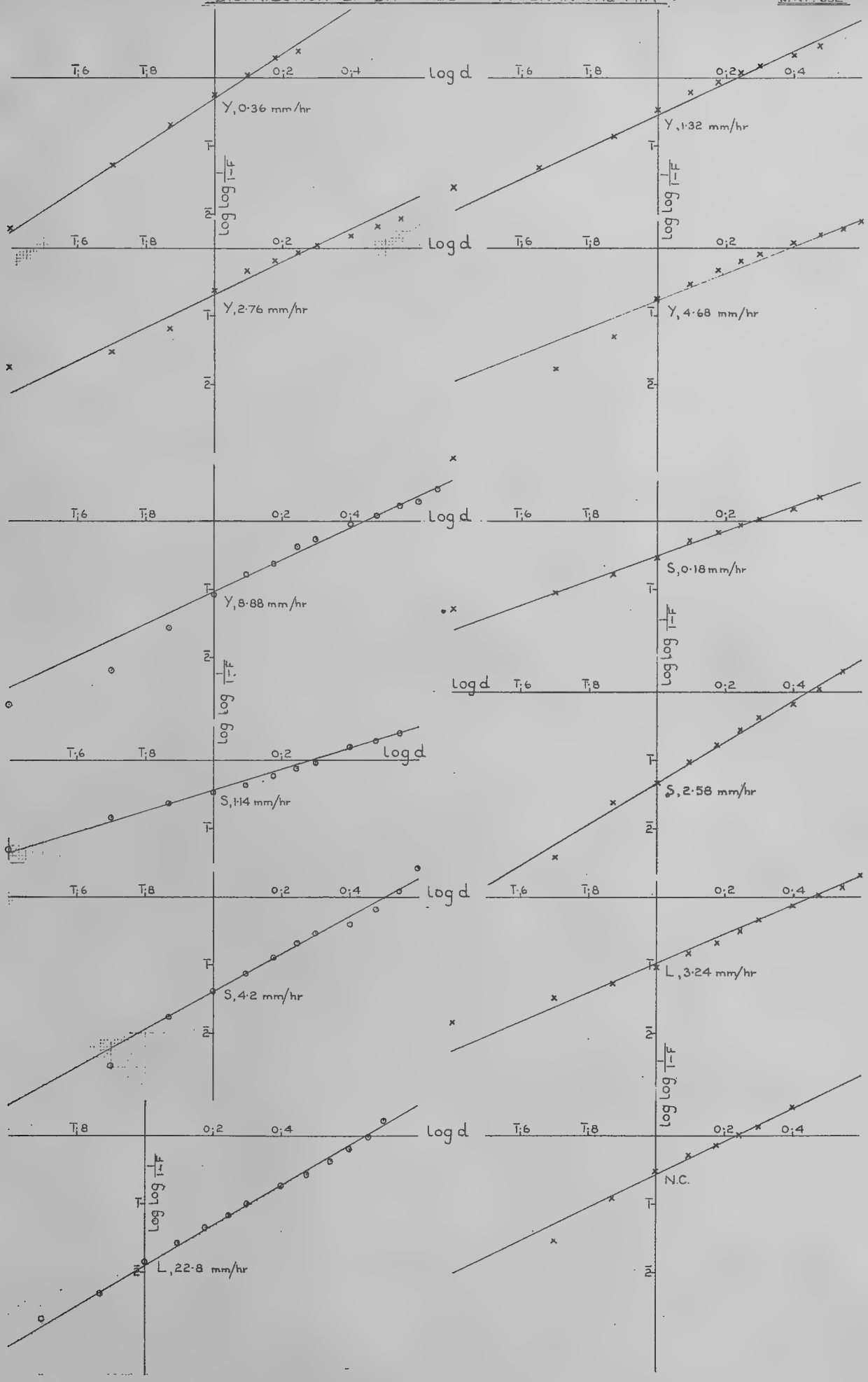


1/3104

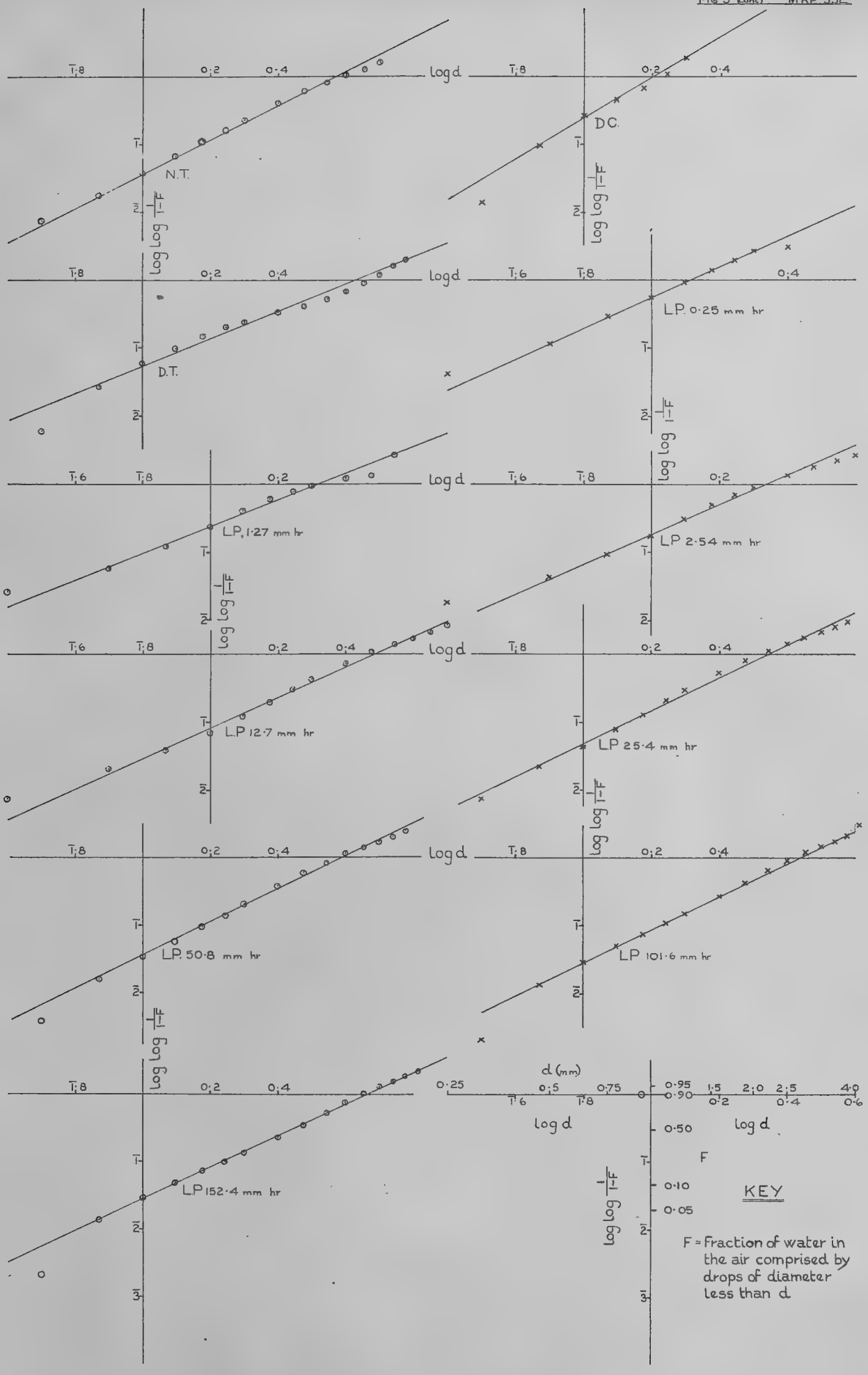
FIG. 5

DISTRIBUTION BY DROP SIZE OF WATER IN THE AIR

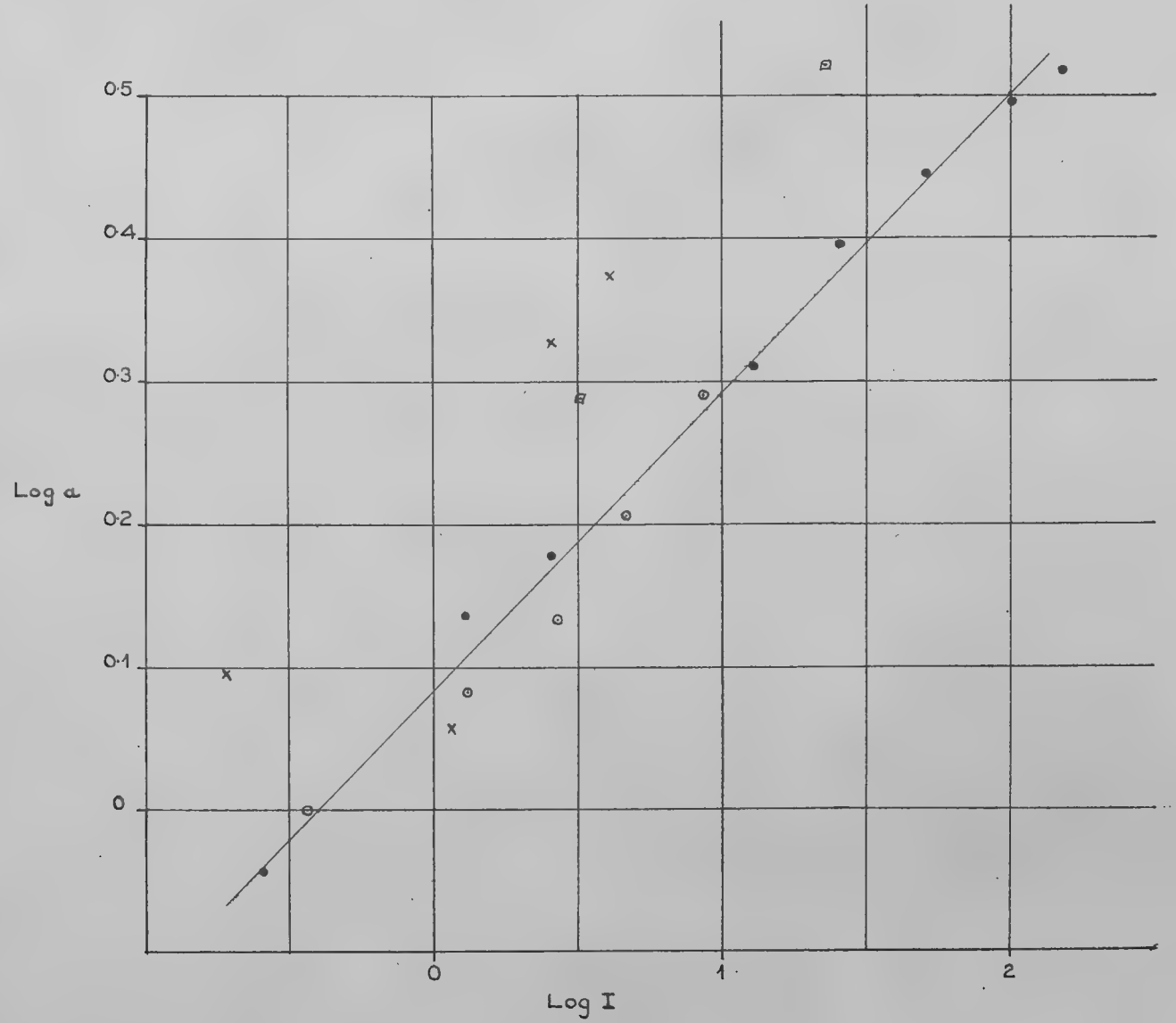
M.R.P. 352



1/3104


N.T.
D.C.
D.T.
LP 0·25 mm hr
LP 1·27 mm hr
LP 2·54 mm hr
LP 12·7 mm hr
LP 25·4 mm hr
LP 50·8 mm hr
LP 101·6 mm hr
LP 152·4 mm hr
log d
Log Log 1/1-F
d (mm)
0·25
0·5
0·75
1·5
2·0
2·5
4·0
0·95
0·90
0·50
0·10
0·05
F
KEY
F = Fraction of water in the air comprised by drops of diameter less than d

1/3104



MRP 352

FIG 6

Parameter "a" as a function of intensity of rainfall I (mm/hr)

- ⊙ Ynyslas
- X Shoeburyness
- ⊡ Lenard
- ● Laws and Parsons

(22)

1/3104

rge Scale Vertical Rate of Motion.

by

Crown Copyright reserved its contents may not be quoted.
AIR MINISTRY

2nd July, 1947.

METEOROLOGICAL RESEARCH COMMITTEE.

The Estimation of Large Scale Vertical Currents from the Rate of Rainfall.

by

J.K. BANNON.

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILE
ACC. 47.511.

Introduction and Summary

The depressions of middle and high latitudes cause the raising of large masses of air from low to high levels in the troposphere.

As the air is lifted it soon becomes cooled below its saturation temperature and its surplus water vapour condenses into cloud particles and later into rain or other form of precipitation.

Thus the rate of rainfall is a rough measure of the rate of lifting, or the upward velocity, of the air above.

It is shown here how this measure may be made more precise when it is certain that the lifting process is on a large scale and comparatively uniform and not influenced by local instability effects.

Examples of the application of the method are given, and it appears that upward velocities of from 10 to 20 cms per second (i.e. 9 to 17 kms per day), are usual in the active depressions often experienced over the North Atlantic.

1. Ascending Saturated Air - Disposal of its Water Content.

As saturated air is raised, its temperature will fall and it will lose its capacity to hold water vapour. The original water content, (as vapour), of the air can, after raising, be accounted for under three headings:-

(a) Part remaining as water vapour.

(b) Part in the form of cloud particles.

(c) Part released as precipitation.

Thus the rate of precipitation from a rising mass of air can be expressed by the equation:-

Rate of precipitation = Rate of decrease of water vapour content + rate of decrease of water content of cloud particles. ..... (1)

2. Water Vapour Content of Ascending Air

The water vapour content of ascending air initially saturated, will differ little from the saturated water vapour content. Super-saturation above more than 1 or 2 per cent cannot exist, otherwise the cloud particles would grow indefinitely. Hygroscopic nuclei can induce fog or cloud particles to condense at between 90 and 100 per cent relative humidity; condensation at relative humidities below 100%, however, would be of very short duration in cloud formed in ascending air which will have a steadily decreasing temperature.

Thus the water vapour content of the air will, to within 1 or 2 per cent, be the saturated water vapour content at that temperature, and the rate of decrease of water vapour content will be the rate of decrease of saturation vapour content due to the falling temperature.

/3.

3. Liquid Water Content of Cloud Particles.

The liquid water (or ice) content of clouds varies considerably, the type, height or temperature of the cloud may be influencing factors, but observational evidence is inadequate to determine this. The cumulo-nimbus cloud, for example, probably has a high liquid water content as it can sustain large drops in its strong up-currents.

The nimbo-stratus cloud, however, formed by the much less vigorous but large scale up-currents, probably can support mechanically only drops of small size. The largest drops in non precipitating cloud observed by Diem, for example, (see MRP 309), were about 0.1 mm in diameter and were infrequent; a drop of this size would have a terminal velocity of 25 cm/sec. (Calculated from figures given by Castleman, N.A.C.A. Technical Note No. 231, Washington 1926). As will be seen, (section 10), up-currents in disturbed conditions in this country are estimated to be of the order of 10 to 20 cms/sec, so that it is reasonable to assume that the particles of a nimbo-stratus cloud are little, if any, larger than the largest drops of non precipitating cloud.

No data are available on the liquid or solid water content of a precipitating cloud; the results of tests of non-precipitating cloud by Mazur and Diem are given in MRP 309. On one occasion only, out of 44 observations, did the liquid water content exceed .5 gms/metre^3, the majority of cases giving values .1 to .3 gms/metre^3. (The exception was in a strato-cumulus cloud where Mazur found .82 gms/m^3).

Mazur's and Diem's observations were made at temperatures varying between 14.6 and -11°C and though the data are meagre, there does appear to be a tendency for low water content to occur at low temperature. The fact that visibility in alto-stratus cloud is usually much better than in strato-cumulus or other low cloud, (see for instance, M.O.400, Met. Aspects of Cloud or Blind Flying), also indicates that clouds, (other than cumuloform), at low temperatures have a smaller water content than clouds at temperatures not far from 0°C.

Thus, though the available evidence is not at all conclusive, it does appear likely that the particles of a precipitating cloud, (not cumuloform), not counting any precipitation which may be falling through it, contain only something of the order of .5 gms/m^3 of liquid or solid water at low levels and considerably less than this amount at the low temperatures of the upper troposphere.

It certainly appears unlikely that the rate of change of liquid or solid water content of an ascending cloud mass is anything but very small compared with the rate of change of the saturated water vapour content of the air, except in the initial stages of its formation, since the overall range of the liquid or solid water content is only of the order of 1 gm/m^3.

In general, therefore, the second term on the right hand side of equation (1) is very small compared with the first.

4. Rate of change of saturation water vapour content due to ascent.

Consider the ascent of saturated air, at temperature T° A, pressure p millibars and density ρ. If z is the height (measured upwards), the state of the air will be governed by:-

$$\left.\begin{aligned} pv &= RT \\ \frac{\partial T}{\partial z} &= \Gamma_m \\ \frac{\partial p}{\partial z} &= -g\rho \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where $v = 1/\rho$ and Γ_m is the saturated adiabatic lapse rate [4] and R is the gas constant appropriate to the mixture of air and water vapour.

Let r = the water vapour content of 1 cc of air. Then since the air is saturated

$$r = r(T)$$

On raising a cc of air through a small height δz, its volume will become :-

$$1 + \frac{1}{v}\frac{\partial v}{\partial z}\delta z = 1 + \frac{\delta z}{T}\left(\frac{g}{R} - \Gamma_m\right)$$

since

$$\frac{1}{v}\frac{\partial v}{\partial z} = \frac{1}{T}\frac{\partial T}{\partial z} - \frac{1}{p}\frac{\partial p}{\partial z}$$

$$= -\frac{1}{T}\Gamma_m + \frac{g}{RT}$$

An original cc of air, after raising δz will thus be able to hold the following weight of water vapour at saturation :-

$$\left\{1 + \frac{\delta z}{T}\left(\frac{g}{R} - \Gamma_m\right)\right\}\left\{r + \frac{\partial r}{\partial T}\frac{\partial T}{\partial z}\delta z\right\}$$

$$= \left\{1 + \frac{\delta z}{T}\left(\frac{g}{R} - \Gamma_m\right)\right\}\left\{r - \Gamma_m\frac{\partial r}{\partial T}\delta z\right\}$$

$$= r + \delta z\left\{\frac{gr}{RT} - \Gamma_m\left(\frac{r}{T} + \frac{\partial r}{\partial T}\right)\right\}$$

to first order of δz.

If the saturated air is rising at rate w, therefore, it will be losing the capacity to hold water vapour at saturation at the rate, per unit volume :-

$$w\left\{\Gamma_m\left(\frac{r}{T} + \frac{\partial r}{\partial T}\right) - \frac{gr}{RT}\right\} \qquad \ldots\ldots\ldots\ldots\ldots (3)$$

Extending Equation (3), the rate at which a column of air of unit cross section, is losing the capacity to hold water vapour at saturation is :-

$$P = \int w\left\{\Gamma_m\left(\frac{r}{T} + \frac{\partial r}{\partial T}\right) - \frac{gr}{RT}\right\} dz \qquad \ldots\ldots\ldots\ldots (4)$$

where the integral is taken over all the rising column.

---

[4] If the unit of air, here considered, is part of one large ascending mass of air, the changes may be considered to take place without outside influence and thus adiabatically.

/ 5.

5. Rate of Precipitation from a rising column and its measurement at the ground.

From Equation (1) and the considerations of sections 2 and 3 it appears that the rate of release of precipitation from a column of unit cross section of rising air, is, to a good degree of approximation, equal to P given by equation (4) above.

The rate of precipitation released from a unit column of air, however, is not in general, the same as the rate of rainfall measured at the ground, for the following reasons:-

(a) some of the precipitation may evaporate before reaching the ground, into a dry layer of air,

(b) the horizontal wind may cause the precipitation to be scattered over more than 1 sq. cm of the ground.

Regarding (a) there are many occasions when the whole column is saturated or nearly so, and losses due to evaporation will be negligible.

If the vertical motion, producing the rain, and the thermal and moisture structure of the air is comparatively uniform over a wide area, (b) will be unimportant. The size of the area over which the conditions should be similar can be gauged by the fact that a medium sized rain drop, originating at 15,000 ft, would be carried 10 miles, during its fall, by a mean wind of 40 mph.

The best way of assessing uniformity is from the rainfall observations themselves; if the rates of rainfall at stations 20 to 40 miles apart are similar, it should be safe to assume that the factor (b) will not seriously affect the deduction that the rainfall rate at the ground is the rate of loss of a unit column above it, neglecting evaporation effects.

6. Estimation of w when the Rate of Rainfall is known.

With the proviso that the conditions are comparatively uniform horizontally and that no precipitation is lost by evaporation during its fall, we have seen that:-

Rate of rainfall as measured at the ground

$$= P$$

$$= \int w \left\{ \Gamma_m \left(\frac{r}{T} + \frac{\partial r}{\partial T} \right) - \frac{gr}{RT} \right\} dz$$

$$= \int w I \, dz \qquad \text{say.}$$

$$= \bar{w} \int I \, dz \qquad \ldots\ldots\ldots\ldots (5)$$

where $\bar{w}$ is an average, (not an arithmetical average), upward velocity of the rain-producing air above the observing station.

Thus when the rate of rainfall is known, and $\int I \, dz$ is known, $\bar{w}$ can be calculated.

7. The Function I.

Γ_m can be calculated for any distribution of temperature and pressure, though the presence of cloud particles makes its accurate estimation impossible unless the free water or ice content of the cloud is known. It can, however, be calculated to within 1 per cent in conditions likely to be encountered in temperate latitudes. (MRP 344 refers).

/The

The other variables in I are known when the temperature structure of the air is known.

Table I below gives values of I for various temperatures and pressures, and these are also illustrated by the curves in Figure I which gives the variation of I with temperature, at constant pressures.

In the computation of Table I, the values for Γ_m were calculated in the manner described by Brunt, (Q.J.Roy. Met. Soc., 59, 1933, pages 351 et seq), from the Tables I (for temperatures above freezing) and II (temperatures below freezing) of MRP 344. This assumes that the condensation in the rising air is to super-cooled water, above freezing level.

The values for r were taken from the Smithsonian tables, for temperatures above freezing, and, for temperatures below freezing, were calculated from the pressures of saturated water vapour over super-cooled water, given by Harrison (U.S. Weather Review, July 1934).

If I is calculated on the assumption that condensation is to ice at temperatures below freezing, the values, as shown in Table I, to the degree of accuracy to which they are given, are unaltered.

Table I

I in c.g.s. units multiplied by 10^{11}

Temperature in °F	Pressure in millibars 950	850	750	650	550	450	350
90	2.80						
80	2.70						
70	2.57	2.37					
60	2.40	2.23	2.07				
50	2.17	2.03	1.93	1.75			
40	1.90	1.77	1.70	1.57			
30	1.57	1.50	1.45	1.35	1.23	1.10	
20	1.25	1.40	1.15	1.10	1.03	0.93	.80
10	.90	.88	.87	.85	.80	.73	.65
0		.65	.65	.63	.60	.55	.50
-10		.47	.47	.45	.43	.40	.37
-20				.30	.30	.30	.27
-30					.17	.17	.17
-40					.10	.10	.10

8. Application of Equation (5) in Practice

If the rate of rainfall is known and also the upper air structure, Equation (5) may be used to evaluate $\bar{w}$ when:-

(a) the rain forming processes are known to be uniform horizontally i.e. to change little within 20 to 40 miles,

(b) the rain is not of an instability type but is caused by the mechanical lifting of a large mass of air in a more gradual manner,

(c) the evaporation of the falling rain is known to be small.

A study of the synoptic situation and upper air temperatures and humidities should be sufficient, in most cases, to determine if this method can be applied. Rate of rainfall records themselves will be the most useful determining factor in doubtful cases. For example, instability type of rain can often be detected in a very irregular hyetograph trace which has no counterpart at a neighbouring reporting station.

In many cases the whole of the lower troposphere may have an upward motion and the integral in Eq.(5) will have to be taken over all this depth, and thus $\bar{w}$ will be an average over a very deep layer. But there are occasions when narrower limits can be fixed for the rain-bearing layer, e.g. the base and top of the main nimbo-stratus cloud may have been determined by aircraft reconnaisance, or the height of a frontal surface above the station may be known.

It should be noted that, since I falls off quite rapidly with temperature, below freezing point, any uncertainty as to the upper limit of vertical motion will affect the evaluation of $\bar{w}$ but little and in fact $\bar{w}$ is mainly related to the up-currents in the lowest part of the rising air-mass.

In any given case the curves in Fig.I can be used to make an approximate evaluation of $\int I\,dz$, taking layers of 100 mbs thickness.

9. Evaluation of $\int I\,dz$ in Standard Cases

$\int I\,dz$ has been computed for saturated atmospheres of potential wet-bulb temperatures 60° , 50° , 40° and 30° F and the results are set out in Table II, giving values for each case assuming that the air up to 300 mbs is all contributing, above the various pressure levels stated.

Table II

$\int I\,dz$ in c.g.s. units multiplied by 10^6

All troposphere above	Potential wet-bulb temperature (F) 60°	50°	40°	30°
1,000 mbs	9.69	7.25	5.06	3.57
900 .	7.62	5.45	3.57	2.37
800 .	5.68	3.86	2.29	1.41
700 .	3.82	2.38	1.26	.71
600 .	2.26	1.25		
500 .	1.04			

Thus if the rate of rainfall is P and the air is of potential wet-bulb temperature 60° F and is being lifted at all heights above 1,000 mbs, the average upward velocity $\bar{w}$ is given by :-

$$P = \bar{w} \times 9.69 \times 10^{-6} \text{ gms of water /sec/cm}^2$$

where $\bar{w}$ is in cms/sec.

Or if P is measured im mms of rain per hour,

$$\bar{w} = 2.9\,P \text{ cms/sec.}$$

Table III below has been computed from Table II and gives the factor necessary to multiply P , the rate of rainfall in mms / hour, to obtain $\bar{w}$ in the various cases.

Table III

Multiplying Factor for Rate of Rainfall (mm/hour) to give $\bar{w}$

All troposphere above	Potential Wet-Bulb Temperature (F) 60°	50°	40°	30°
1,000 mbs	2.9	3.8	5.5	7.8
900 .	3.7	5.1	7.8	11.7
800 .	4.9	7.2	12.1	19.7
700 .	7.3	11.7	22.0	25.5
600 .	12.3	22.2		
500 .	26.7			

/In

In many cases the upper air temperatures approximate to a constant potential wet-bulb temperature curve, and the required factor can then be interpolated from the above Table III.

If the temperature distribution is markedly different from a constant potential wet-bulb curve, the value of $\int I\,dz$ can be easily calculated from Table I or Fig. I, and the known temperature distribution.

10. Examples of the Method.

The hyetograph records for Stornoway, Tiree, and, when available, Benbecula, Ardnamurchan Point and Glenbrittle, have been analysed, in conjunction with synoptic charts and upper air information (as available in the D.W.R.), for a number of occasions during the year 1946, and estimates made of the vertical up-currents, care being taken to ensure that instability rains were excluded.

The Hebridean stations were chosen so as to eliminate, as much as possible, any orographic effects. It is considered, however, that this method of estimating vertical currents may be used in most places in the British Isles if the possible errors are realised and allowances made for them.

Occasions of comparatively high rates of rainfall only have been considered and the up-currents estimated are thus those of disturbed conditions. The synoptic situations were not, however, exceptional or rare, and the fronts and depressions examined were normal for disturbed conditions over the North Atlantic.

The tilting-syphon raingauges in general use are probably more suited to measuring large rates of rainfall than small and thus cases of rainfall of 0.5 mm/hour or less have not been examined.

Houghton and Austin, (Journal of Meteorology, Sept. 1946), have used rainfall records to estimate vertical velocities but their method has not been published; values of from 1 to 15 cms/sec are quoted by them for various occasions.

Table IV lists the various cases investigated. The average vertical velocities deduced are given in column (5). Taking this mean up-current as applicable to 10,000 ft, the slopes of the air trajectories at that height have been calculated from observed or estimated wind speeds and are given in column (6).

The average slopes of the frontal surfaces, deduced from the radio-sonde observations available and surface maps, are given in column (7), where applicable.

Vertical currents near the earth's surface have been deduced for three cases of warm-frontal development by the method described by Petterssen in Geof. Pub. Vol. XI, Oslo, 1936, and also in his book "Weather Analysis and Forecasting", page 408, and are given in column (8). The difference between the gradient wind in the warm sector, resolved at right angles to the warm front, and the speed of the front, has been taken as the speed at which the warm air begins to climb up the warm-frontal surface, the mean slope of which is known.

In some cases it was possible to fix approximate lower limits to the rain-bearing air mass, (column (4)). In all cases the upper limit of the up-currents was taken as between 25,000 and 30,000 ft; as pointed out in Section 8, this upper limit is relatively unimportant.

As an example of the application of the methods, the case of 4th July, 1946, is discussed in detail. On that occasion the very slow movement of a warm front made conditions ideal for the application of the rainfall method of estimating up-currents.

/4th

4th July, 1946.

Synoptic Situation:-

A depression, over Ireland early in the day, moved NNE across Scotland and was near Wick at 1800 hours.

The warm front at 0600 hours lay from Rattray Head to Lough Neagh and moved 60 miles nearer Stornoway in the following 12 hours by which time it was beginning to be occluded.

The 0600 and 1800 hour upper air observations at Stornoway showed the warm frontal surface, well marked, at 13,500 and 9,500 ft. respectively, and the average slope of the frontal surface will, thus, have been 1/55 at 0600 hrs. and 1/42 at 1800 hours. It is possible, therefore, to estimate, roughly, the height of the frontal surface above any of the reporting stations in the Hebrides during the day, and hence, from their rates of rainfall to calculate the up-currents above the frontal surface. This has been done for four stations but it is not possible, of course, to estimate losses in the rain during its fall from the rain-bearing cloud to the ground, and thus the values obtained may be under-estimates.

Stornoway rate of rainfall in the afternoon was between 2.2 and 3.0 mms/hour and this gives an up-current of between 18 and 25 cms/sec. above 10,000 ft.

Tiree was nearer the centre of the depression, (50 miles from the warm front), and the frontal surface was thus lower over that station than over Stornoway. From the rate of rainfall recorded there, the average up-current, above 6,300 ft, must have been 33 cms/sec for about an hour, and 19 cms/sec for longer than that.

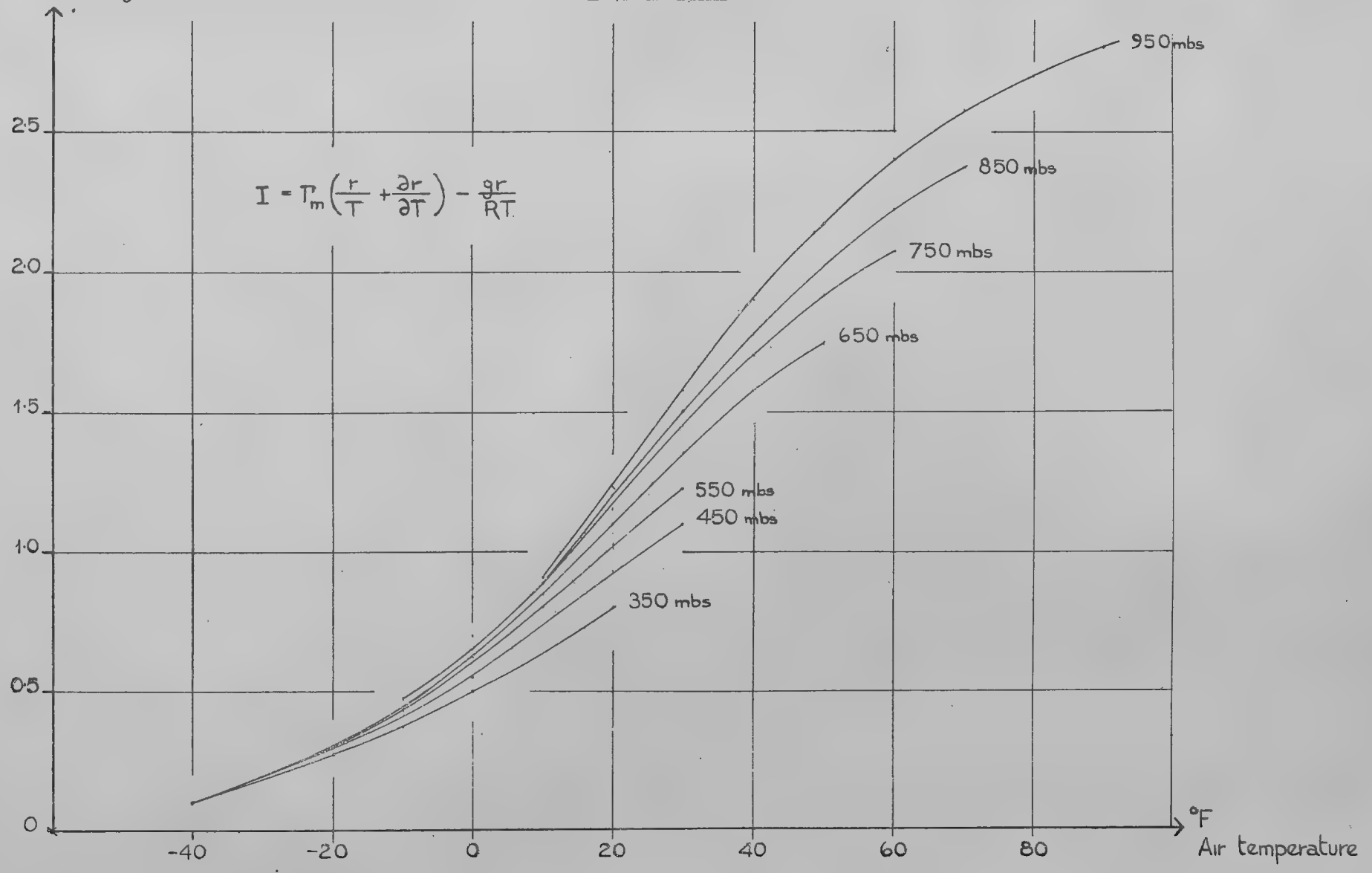
Glenbrittle and Ardnamurchan were within about 35 miles of the warm sector i.e. the frontal surface must have been only 4,400 ft. above and their rainfall records indicate up-currents of 17-19 cms/sec above that height over 2 to 3 hours.

Table IV — Examples of Vertical Velocities and Slopes of Trajectories.

Date and Time (Year 1946) (1)	Type of Disturbance (2)	Rate of rainfall (mm/hour) (3)	Lower limit of vertical current (4)	Calculated up-currents (cms/sec) (5)	Approx. slope of trajectory at 10,000 ft. (6)	Mean slope of warm front surface. (7)	Up-current near the ground (Petterssen's method). (8)
22nd January, 1700-2100 hours	Stationary occlusion	2 to 3	Surface	10 to 15	1/210	N.A.	-
22nd February, Afternoon and evening.	Depression	2 to 3	Surface	11 to 16	1/140	N.A.	-
17th March Morning	Warm front	1	8,000 ft.	7 (but may be greater - possible evaporation errors)	1/190	1/90	7 cms/sec.
		3	Surface	17	1/76	do.	do.
5th June Early morning	Warm sector depression	3	5,000 ft	15	1/170	1/85	7
		4	Surface	12	1/280 (both these doubtful - wind inaccurate)	do.	do.
24th June Early morning	Shallow secondary depression	3 to 4	Surface	9 to 13	1/180	N.A.	-
4th July Midday and afternoon	Warm sector depression	2 to 3	10,000 ft.	18 to 25	1/50	1/55 (0600 hrs.)	9
		3.4	6,300 ft.	19	1/53		do.
		3.8 to 4.2	4,400 ft.	17 to 19	1/56	1/42 (1800 hrs.)	do.
17th November Early morning	Occluding depression	1.1	7,000 ft.	9	1/220	N.A.	-
22nd December Early morning	Wave secondary (not very active)	1.3 to 1.8	Surface	5 to 7	1/400	N.A.	-

FIGURE 1

M.R.P. 353



(9)

1/3120

AIR WEATHER SERVICE

MRP's 36[illegible]3, 364, 372

AIR WEATHER SERVICE

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

This [illegible] published [illegible] CONFI[illegible] contents [illegible] Its [illegible] ...ued.

AIR MINISTRY M.R.P.360

METEOROLOGICAL RESEARCH COMMITTEE 10th September, 1947

The Upper Atmosphere
Estimated Distribution of Temperature, Pressure and Wind up to the 45 Km level.

by A.H.R. Goldie

1. Introduction

This paper is an attempt to set out the temperature distribution of the upper air in relation to latitude so far as it has been observed or can be estimated with reasonable confidence, to compute therefrom the density and pressure distributions, to deduce the air circulations to which these distributions should give rise in and perpendicular to meridian planes and to examine the effect which those circulations may have on the temperature of the high atmosphere. Reasons are given for believing that estimates of temperature where used cannot be substantially different from the reality.

The picture given here is for the solstices. The picture for the equinoxes would be somewhat different and would need to be independently computed.

2. The probable temperature distribution.

The first objective is to get a cross-section over the globe from winter pole to summer pole, but all the data used relate to the northern hemisphere in winter (Nov.-Jan.) and summer (May-July) respectively and predominantly to observations made in the evening or at night. The data used are (a) the actual means from balloon sondes or radio sonde results for Abisko, India and Batavia and the means from radio sonde and balloon sondes for England so far as they go (b) the upper air temperatures for levels up to 45 km for England as estimated from wartime experiments on propagation of sound but somewhat smoothed(1). Except for England the figures beyond 20 km are mainly extrapolations or estimates made with regard to the physical considerations mentioned in Section 3. The values used appear in Table I. The pressures and potential temperatures related to a pressure of 200 mb. have been computed.

Table II gives p/T -that is figures proportional to the air density - and also the percentage departure of the density at each station from the density at the same level at the equator.

Table III gives values of Δp i.e. the difference of pressure from the value at the equator at the same level and also $\Delta p/p/T$ i.e. the pressure difference divided by a figure proportional to the air density at the same level.

In figure 1 the temperature distribution is plotted from the data of Table I. In this diagram use is also made in the summer hemisphere of an upper air observation made at latitude 80°N. by Dymond. This observation happens to fit in with a very probable run of the isotherms. Figure 2 shows the distribution of potential temperature.

3. Method of estimating or extrapolating.

The extrapolated temperatures of Table I were first deduced provisionally from the run of the isotherms of Figure I, as first plotted from directly observed data or data based on travel of sound waves. Potential temperatures were then computed and a first drawing was made of Figure 2. Examination of levels for anomalies showed the temperatures for Abisko above 26 km. in winter to be potentially warmer than those at these levels for England. This in the latitude of Abisko seemed practically impossible since (a) in winter there is no contribution of heat from solar radiation and (b) adiabatic increase of temperature beyond that arising in England is eliminated since the density distribution showed ascent rather than subsidence in the latitude of Abisko. To keep the temperature for Abisko potentially just colder above 26 km. than the air over England would have required actual temperatures of not more than 180 to 188 at Abisko from 26 km. upward distributed as shown. These values were therefore adopted as being the highest that seemed physically possible for this latitude in winter, having regard to the temperature obtained for England from sound wave experiments.

/The

The only other anomaly emerging in the isotherms of potential temperature was a minor one, namely that the temperature of 220 at 45 km. for England in winter (as estimated from propagation of sound) was apparently high and it was decided to assume for 45 km the same value as for 40 km., namely 213.

With these changes a physically consistent looking distribution of temperature was obtained. A qualitative check was, however, made by the method described in the next section.

4. Temperature Estimates from the Krakatoa data.

A way is open by which a rough estimate may be made of the way in which temperature varies over the globe at the high level associated with the long distance travel of sound waves, namely from the manner in which the explosion wave from the Krakatoa eruption of 1883 was propagated. (Apart from other reasons why the estimate so made can only be a rough one, there is the difficulty that the eruption took place on Aug. 26-27 which is considerably removed from the solstices.) Part II, Section I of the Report of the Krakatoa Committee of the Royal Society[2] deals with the travel of the air waves round the globe. The observations showed approximately "an atmospheric wave or oscillation, propagated over the surface of the globe from Krakatoa as a centre, and thence expanding in a circular form, till it became a great circle at a distance of 180° from its origin; after which it advanced, gradually contracting again, to a node at the antipodes of Krakatoa; whence it was reflected or reproduced, travelling backwards again to Krakatoa, from which it once more returned in its original direction, and in this manner its repetition was observed not fewer than seven times at many of the stations, four passages having been those of the wave travelling from Krakatoa and three those of the wave travelling from its antipodes, subsequently to which its traces were lost".

Tables VI and VII of the Krakatoa report give the time intervals and velocities in the two directions, the latter being expressed in degrees of the Equator and decimals per hour. Presumably the velocities depend on the temperatures and winds in the high atmosphere in the parts of the globe over which the waves travelled. Call the velocity within the Northern Arctic Circle V_1, that in North Temperate latitudes V_2, that within the Tropics V_3, that in South Temperate Latitudes V_4 and that within the Southern Arctic Circle V_5. Then assuming that the wave travelled approximately in great circles it is possible to form sets of equations for V_1, V_2, V_3, V_4, V_5. For example where the great circle path lies entirely or almost within the Tropics, as from the Mauritius data, the value of V_3 can be determined directly, the effect of wind being eliminated by taking the means of velocities in opposite directions round the globe. Other paths are less simple involving combinations of V_1, V_2, etc. It is possible to derive a number of equations, which though having some inconsistencies point broadly to the same general results, namely, rather higher velocities in the northern (summer) hemisphere than in the southern (winter) hemisphere and an extremely low velocity for the first wave which passed across the southern polar regions.

An attempt to get some numerical solutions led to the following results:-

	Speed in degrees per hour	Speed in m/s of wave assumed to travel on great circle at 45 km height.	Corresponding air temperature for sound travelling at this speed.
	°/hr	m/s	°A
Summer Polar Zone (V_1)	10.19	319	255
Summer Temperate Zone (V_2)	10.08	316.8	250
Tropics (V_3)	10.16	318	252
Winter Temperate Zone (V_4)	10.08	316.8	250
Winter Polar Zone (V_5)	8.29	258	165

/Thus

Thus the conclusion is again reached that the atmosphere at about the 4[illegible] level is warmest in summer polar regions and coldest in winter polar regions. Temperatures estimated in this way should give lower limits since they depend the assumption that the wave travelled by the shortest (great circle) path.

5. Density Distribution and probable average resultant circulation of air in vertical meridian planes.

Figure 3 shows the percentage departures of density at each latitude from that at the equator at the same level. The data are expressed in this way to indicate the relative accelerations upward or downward which should be impressed on the air by gravity in consequence of the non-uniformity of density. Fig. 3A shows how the mean variability changes with height. These are so definite that a connection with vertical circulations is at once suggested.

At a height of about 8 km. there is approximate uniformity of density all over the globe. This has previously been recognised. Below this height the convectional circulation should be downward in the Arctic regions and upward in the tropics. Above 8 km. and up to the tropopause there is little significant density difference in the horizontal direction and it seems likely that the upper limit of this convectional circulation is approximately the level of the equatorial tropopause and that air flows from troposphere to stratosphere in a return poleward circulation somewhere in subtropical latitudes. The vertical temperature distribution near the winter pole suggests that this is the probable locus of the return of air from stratosphere and troposphere.

In the stratosphere above the tropopause and up to about 24 km. the density differences would promote a circulation in which air subsides en masse in the tropics towards the tropopause, i.e. the tropical stratosphere temperatures should increase slowly with height. The air which subsides here should then drift polewards in the lower stratosphere finally giving mass ascent of stratosphere air in the Arctic regions, particularly at about the 16 km. level and more strongly in the winter hemisphere than in the summer hemisphere, and giving in the stratosphere near the winter pole a fall of temperature with height extending probably to at least 30 km.

In these high levels the density differences as between one latitude and another, it will be noted, are enormously greater in relation to the actual density than they are in the troposphere. Presumably therefore the vertical circulations are correspondingly more rapid.

At about 16 km. there is a maximum variation of air density over the globe and at about 24 km. the variation falls to another minimum. Above this height again the relative importance of the density variation in latitude increases enormously until at about 40 km. the density is some 30% greater at the summer arctic circle and some 35% less at the winter arctic circle than it is at the equator. Indeed above about 30 km. and very notably above 40 km. there is a continuous and relatively steep density gradient from summer pole to winter pole, with therefore a strong tendency to subsidence at summer pole and ascent at winter pole.

6. Pressure distribution and associated (geostrophic) horizontal circulation perpendicular to meridian planes.

The motion discussed above in the vertical meridian plane and in the N-S or S-N direction is complicated by the E-W and W-E horizontal movement associated with the pressure field in combination with the rotation of the earth.

Figures 4, 5 show the pressure difference at each level in relation to pressure at the equator and divided by a figure proportional to the mean density $(\bar{\rho}/\bar{T})$ at that level for the hemisphere. The idea is to produce a vertical meridian section in which the horizontal spacing of the isopleths shall be proportional to W-E or E-W component of the geostrophic wind, omitting only the latitude factor ($1/\sin\phi$). The regions of West wind and East wind and the boundaries between them are indicated on the two diagrams i.e. for the Northern Summer and Northern Winter respectively.

It will be seen that in each hemisphere there is, up to the 16 km. level and excluding shallow East wind in the subtropical regions and doldrums near the equator, a west wind circulation around the pole with maximum speed somewhere about the tropopause. This polar vortex, as it will be called for convenience, will tend

below 8 km. to counteract the gravitational circulation associated with the density field; but above 8 km. it is assisted by the density distribution.

This polar vortex is a good deal stronger in the winter hemisphere than in the summer hemisphere.

In the winter hemisphere in a narrower region from about 40° latitude up to the pole it extends (with even greater horizontal speeds) up to at least 45 km. and very possibly to a much higher level. The density field here in the stratosphere in winter tends to promote this circulation.

In the summer hemisphere the west wind vortex is similarly hindered below 8 km. by the density field, but is assisted by the density field above this level and up to perhaps 22 or 24 km. From about 24 km. it changes over to an easterly (anticyclonic) circulation which is promoted or assisted by the density field in these higher levels up to at least 45 km.

At some level above 45 km. there must be a continuous horizontal movement of air from winter arctic regions, across the equator, to summer arctic regions.

Thus altogether for the higher atmosphere above 8 km. we have a pretty consistent picture of cyclonic and anticyclonic circulations with, within them, the respective appropriate ascents and subsidences of air and the appropriately related convergences and divergences of air across parallels of latitude.

7. Computation of wind speeds up to 45 km. level.

The concentration of the isopleths (on figs.4 and 5) per 10° of latitude, multiplied by a factor $(k/\sin\phi)$, where ϕ is latitude may be used to compute the W-E or E-W component of the wind speed. The value of $k/\sin\phi$ to give the wind speed in m.p.h. in lat.50° is 5.24 and in lat.60° it is 4.63.

On this basis the mean summer or winter W-E or E-W components of wind speed at any height in any part of the globe may be readily computed from the diagrams Figs. 4 and 5.

Table IV is a specimen of results obtained in this way. For comparison with actual measurements it may be said that the mean winter and mean summer measurements at about 28 km. from the Bruce gun-experiments(3) in southern England were roughly West 105 m.p.h. and East 30 m.p.h. which is in close agreement with the values shown in the table. This agreement confirms that there cannot be much wrong with the estimated temperature distribution at least up to 30 km.

TABLE IV

Computed Mean Wind Speeds Perpendicular to Meridian(m.p.h.)

Height km.	Lat. 50°		Lat. 60°	
	Summer	Winter	Summer	Winter
45	E. 145	W. 150	E. 180	W. 190
40	E. 110	W. 135	E. 130	W. 160
35	E. 85	W. 115	E. 100	W. 140
30	E. 60	W. 100	E. 45	W. 120
28	E. 30	W. 100	E. 20	W. 105
24	E. 28	W. 45	- 0	W. 90
20	- 0	W. 22	- 0	W. 80
12	W. 26	W. 30	W. 23	W. 40
10	W. 26	W. 37	W. 23	W. 45

Table IV indicates that very high W-E or E-W winds exist in the high atmosphere at heights above 35 km., more especially in winter.

8. Wind Circulations as deduced from Krakatoa data and meteor trails

The spread of the dust and of optical phenomena considered to be due to its presence in the high atmosphere after the eruption of Krakatoa showed the existence of an east wind in tropical regions at heights of about 20 to 40 km. The speed of this wind at the time (August) was estimated as averaging 73 m.p.h.

This is of the same order as the speed indicated by Fig.4. As regards a somewhat lower level the following comment is made. "The rapidity of propagation to S. Africa and Japan was so great that we can hardly doubt the prevalence at the time of a direct upper current in each direction at a level below that of the great current which carried the dust from east to west at the velocity above mentioned." *

From the time of arrival at Japanese stations the rate of divergence from the equator in this level seems to have been about 40° of latitude in four days or nearly 30 m.p.h. It looks as if this might correspond to the maximum speed of the current indicated on Fig.3. as existing at heights between about 12 and 18 km., i.e. at around the level of the equatorial tropopause as suggested in Section 5.

At this rate air would travel from Equator to lat.50° in five days, but as the direction is likely to develop an increasing westerly component with increase of latitude the time required might be a day or two longer.

Coming to much greater heights, a recent paper by Hoffmeister(4) on determination of wind speed at heights of 90 to 180 km. from meteor trails in the northern hemisphere shows in winter a mean speed of the order of 200 m.p.h. mainly from NNW or WSW; and in summer a mean of 110 m.p.h. mainly from S.S.W. Each group of observations is spread over 6 months and a great range of heights, but it seems that the major component of the motion is usually in the sense from winter polar regions across the equator to summer polar regions.

Thus the Krakatoa observations confirm the circulations indicated in Figs. 3 and 4 and the meteor trail observations point to a continuation with increasing height of the latitude circulation indicated in the upper part of Figure 3.

9. Adiabatic temperature changes as preventing true radiative equilibrium

The changes of pressure δp associated with vertical motion of the air are, in the high atmosphere, large in relation to the actual pressure (p) and thus the adiabatic effect on the temperature, being proportional to $\delta p/p$ is liable to be enormous in relation to anything to which we are accustomed in the lower air. From this it follows that the air temperature must often depart to some extent from the condition of simple radiative equilibrium. The actual temperature will be determined by the rate of the circulation in vertical meridian sections in relation to the net rate of gain or loss of heat by radiation. In the Bakerian lecture(5) of 1946, Dobson estimated that in the 10-15 km. level the actual rises of temperature produced by the absorption of radiation are probably less than 0.6°C per day and the net changes of temperature probably considerably less. Thus only if the vertical circulation is sufficiently slow could an approximation to radiative equilibrium be maintained. But the rate of global circulation as deduced in Section 8 is not slow even in levels of the order of 12 to 18 km, where about a week might on the average bring air from tropics to lat.50°, the air subsiding the while by something not differing greatly from the subsidence of the potential isotherms of Fig.2 i.e. by some $2\frac{1}{2}$ km. Since the actual rise of temperature at say 16 km. as between Equator and latitude 50° is about 29° in summer and 20° in winter, it is difficult to see that radiation or changes in the constituents of the atmosphere, or indeed any effect other than the dynamical one of pressure change could explain this great difference of temperature unless the air were kept nearly at rest in latitude for many weeks at a time and the evidence is against this being the case. The dynamical effect of a subsidence of $2\frac{1}{2}$ km. is however of the right order.

10. Possible existence of a high cold layer.

The rate of the circulation at 100 km. (according to the meteor trails) is such as to bring air from winter arctic circle to latitude 50° in the winter hemisphere in about 6 hours, to the equator in about 26 hours and to latitude 50° in the summer hemisphere in about 60 hours. If therefore the winter arctic temperatures in these levels are of the order of 188°A or less and if the gain of heat by radiation is only of the order estimated by Dobson-which would permit a change of at most 1.5°C in the time available - there is the probability that a cold layer with temperatures of this order covers a good part of the globe at some level above 45 km. and under 100 km. The air in question would be air that originally ascended in the winter polar region mainly from about the 30 or 40 km.

* P.325 of the Krakatoa Report.

/level

level and it would finally subside and become warmed intensely by pressure change and slightly by radiation in the high latitudes of the summer hemisphere. This condition, it is to be noted, applies only in summer and winter. The picture for the equinoxes (spring in one hemisphere and autumn in the other) would probably be different, the extreme cold would not be present at either pole and there could be no circulation of extremely cold air at great heights from one pole to the other.

11. Diurnal Variation in the high atmosphere.

The pictures presented so far are of night time conditions being based on temperature measurements made predominantly at night. It is not proposed to discuss diurnal variation in detail here but only to indicate the part which dynamical causes may play in determining it also.

The primary effect of the daytime solar heating is to expand the lowest layers of air so that the amount of air above a given fixed level-say a mountain observatory - is increased to a maximum in the early afternoon. The broad effect on a uniform globe, at the equinoxes should, in the high atmosphere, be to impose upon the existing pressure distribution a centre of high pressure at the equator where the local time is 16 h and a centre of low pressure where the time is 6 to 8 h.

A number of years ago the writer[6] showed that the probable diurnal changes in the movement of air in the high atmosphere which should result from such a pressure distribution would be qualitatively rather like those required to account for on the dynamo theory the "quiet-day" diurnal variation of terrestrial magnetic force.

The effect is a high-level run-off of air from the equator in about the region of the sub-solar point. We cannot say within what range of levels this applies but at least it seems fairly certain that it must apply up to the level of the equatorial tropopause i.e. it should give a daily exaggeration of the divergence of air from the tropics in the upper troposphere and lower stratosphere as described earlier in this paper. The dynamical effect of this, especially if the off-flow is maintained to very great heights, could possibly produce a much greater daily rise of temperature in the stratosphere in middle and high latitudes than could solar radiation, the effect being proportional to $\Sigma \delta p/p$ where δp is the superposed pressure.

Data of diurnal range of temperature at levels up to 20 km. are now available for the British Isles.[7] It must be emphasized that the possibility of errors of measurement due to radiation effects on the instruments is not yet eliminated, but in the lower stratosphere the temperatures so far as they go indicate a range of temperature increasing with height in the same way as it would do if caused in this dynamical way with a steepness which it seems impossible to account for on any other basis. At 20 km. at Larkhill the diurnal range amounts to about 10°F. in the summer. The type of circulation envisaged need cause no pressure change at ground level and its temperature effect on the lower air would be too small for separation from the ordinary daytime rise of temperature at surface levels.

REFERENCES

(1) R.J. Murgatroyd. Anomalous Sound Reception Experiments. M.R.P. 346.

(2) The Eruption of Krakatoa and Subsequent Phenomena, Royal Society 1888

(3) R.J. Murgatroyd. Summary of winds deduced from Shell Bursts at 30 km. (approx.) in Dover Area. M.R.P.194.

(4) C. Hoffmeister. Met. Zeit. 1946 pp.33-41

(5) G.M.B. Dobson, Meteorology of the Lower Stratosphere. Proc. Roy.Soc. A.185, 1946.

(6) A.H.R. Goldie. Trans. Roy. Soc. Edinburgh LVII Pt.I No.4 (1931).

(7) J. Piegza. Diurnal Variation of Upper Air temperature as measured by Kew Mk.I radio sonde. M.R.P.349.

TABLE I

TEMPERATURE AND PRESSURE AND COMPUTED POTENTIAL TEMPERATURE (for 200 mb pressure)

WINTER

Height Km.	Abisko			England			India			Equator			India			England			Abisko			Height Km.
	Pressure mb	Temp. °A	Pot. Temp. °A	Pressure mb	Temp. °A	Pot. Temp. °A	Pressure mb	Temp. °A	Pot. Temp. °A	Pressure mb	Temp. °A	Pot. Temp. °A	Pressure mb	Temp. °A	Pot. Temp. °A	Pressure mb	Temp. °A	Pot. Temp. °A	Pressure mb	Temp. °A	Pot. Temp. °A	
45	.46	185	1030	.80	213	1050	1.02	232	1070	1.09	240	1089	1.16	253	1127	1.43	280	1173	1.80	295	1156	45
40	1.17	185	820	1.81	210	820	2.15	220	820	2.26	229	840	2.32	237	868	2.76	240	831	3.33	258	845	40
38	1.68	185	740	2.51	207	740	2.94	217	740	3.05	227	764	3.10	233	779	3.67	235	754	4.36	250	758	38
36	2.43	184	656	3.50	205	664	4.04	216	670	4.13	225	698	4.16	230	708	4.96	227	664	5.74	246	689	36
34	3.52	182	586	4.89	202	592	5.55	215	608	5.60	223	632	5.60	226	639	6.69	220	589	7.51	241	621	34
32	5.14	180	521	6.89	198	526	7.64	214	550	7.61	222	572	7.61	224	577	9.14	220	537	10.1	240	572	32
30	7.49	182	464	9.75	194	466	10.5	214	500	10.3	221	515	10.3	222	517	12.5	220	493	13.4	240	525	30
28	10.9	185	421	13.9	194	422	14.5	214	456	14.1	221	475	14.0	220	475	17.0	222	455	17.5	240	486	28
26	15.7	188	385	19.6	201	395	19.9	214	418	19.6	217	432	19.8	216	423	23.0	224	417	23.8	239	442	26
24	22.5	191	361	27.4	207	370	27.4	214	381	27.2	214	390	27.4	213	372	31.1	226	387	31.6	238	406	24
23	26.9	193	345	32.3	210	353	32.1	214	365	32.1	212	358	32.2	210	355	36.1	226	372	36.4	238	389	23
22	32.1	195	331	38.0	213	342	37.7	214	349	38.0	210	340	38.0	207	338	42.0	226	355	42.0	238	374	22
21	38.2	197	317	44.6	212	329	44.2	211	327	44.8	205	313	44.9	210	320	48.9	224	337	48.5	237	355	21
20	45.5	200	308	52.5	213	313	52	207	306	53.2	196	290	53	205	302	57	222	319	56	236	334	20
19	53.9	203	298	62	213	293	61	202	285	63.1	194	270	62	199	281	66	222	307	66	236	326	19
18	63.7	205	288	72	213	285	73	199	267	75.2	192	255	73	194	260	78	222	292	76	235	312	18
17	75	207	275	85	213	272	86	194	246	90	189	239	86	193	244	91	222	279	89	234	296	17
16	88	210	266	99	213	260	104	194	235	107	193	232	105	193	232	105	222	271	103	232	283	16
15	105	212	256	118	216	252	124	199	229	128	199	227	125	198	227	123	222	256	120	232	269	15
14	124	212	244	138	216	241	146	203	223	152	205	222	149	205	221	144	222	245	139	231	258	14
13	146	213	233	162	216	230	173	211	220	178	213	222	176	215	224	168	223	232	162	231	246	13
12	171	214	225	190	217	220	204	220	219	209	222	222	206	223	221	196	222	223	186	231	236	12
11	202	212	213	222	218	212	239	228	217	244	231	222	240	231	219	229	221	213	219	228	223	11
10	238	213	202	259	221	205	277	236	215	283	239	222	279	239	217	266	225	207	255	226	211	10
9	279	216	196	301	226	202	320	244	213	327	247	220	322	247	215	310	232	205	297	228	204	9
8	328	221	192	349	232	198	368	251	211	376	254	219	370	255	215	358	238	202	346	234	200	8
7	383	228	188	403	239	196	422	259	207	430	260	217	423	261	211	413	245	200	400	242	199	7
6	445	235	186	466	246	194	482	265	204	491	266	217	483	267	206	475	252	200	462	249	197	6
5	515	242	184	534	252	193	549	271	202	556	272	217	549	273	204	543	259	200	530	255	196	5
4	593	249	183	610	259	192	623	277	199	632	277	217	623	279	201	619	265	200	607	261	194	4
3	681	254	179	696	265	191	707	281	196	713	283	218	705	285	198	703	271	200	692	267	194	3
2	780	259	177	790	269	189	799	287	193	803	288	219	796	291	195	797	276	199	787	272	194	2
1	891	265	174	898	273	184	900	295	189	903	294	225	895	299	192	901	281	198	894	278	193	1
0		269			276			299		1012	299	225	943	303			287					0

SUMMER

TABLE II

Values of p/T and of $\Delta\rho$ the percentage departure of air density from density at equator at the same level.

	WINTER								SUMMER										
	Abisko		England		India		Equator		India		England		Abisko		Mean p/T		Mean Departure of Density %	
	p/T	Δ %	p/T	Δ %	p/T	Δρ %	p/T	Δ %	p/T	Δρ %	p/T	Δ %	p/T	Δ %	Winter	Summer		
45	.0025	−51	.0037	−27	.0044	−13	.0050	0	.0046	−9	.0051	+1	.0061	+21	.0040	.0052	20	45
40	.0064	−35	.0086	−13	.0098	−1	.0099	0	.0098	−1	.0115	+16	.0129	+30	.0088	.0110	16	40
38	.0091	−30	.012	−8	.013	0	.013	0	.013	0	.016	+23	.017	+31	.0120	.0142	15	38
36	.013	−28	.017	−6	.019	+5	.018	0	.018	0	.022	+22	.023	+28	.017	.020	15	36
34	.020	−20	.024	−4	.026	+4	.025	0	.025	0	.030	+20	.031	+24	.024	.028	13	34
32	.029	−15	.034	0	.036	+6	.034	0	.034	0	.041	+20	.042	+23	.033	.038	11	32
30	.041	−11	.049	+7	.048	+4	.046	0	.047	+2	.057	+24	.056	+22	.046	.051	12	30
28	.060	−10	.072	+7	.068	+2	.067	0	.064	−5	.077	+15	.073	+9	.067	.070	8	28
26	.087	−3	.093	+3	.093	+3	.090	0	.092	+2	.102	+13	.100	+11	.091	.096	6.5	26
24	.117	−8	.132	+4	.128	+1	.127	0	.129	+2	.138	+9	.133	+5	.126	.132	5	24
23	.139	−8	.154	+2	.150	−1	.151	0	.153	+1	.159	+5	.153	+1	.149	.154	3	23
22	.165	−14	.183	+1	.175	−3	.181	0	.184	+2	.186	+3	.157	−13	.176	.177	8	22
21	.194	−12	.210	−5	.210	−5	.221	0	.213	−4	.219	−1	.206	−7	.209	.215	6	21
20	.228	−16	.246	−9	.250	−8	.271	0	.258	−5	.257	−5	.237	−13	.249	.256	9	20
19	.264	−18	.287	−12	.302	−7	.325	0	.313	−4	.297	−9	.280	−14	.295	.304	11	19
18	.311	−20	.333	−15	.368	−6	.391	0	.389	−1	.351	−10	.323	−17	.351	.363	11.5	18
17	.362	−24	.394	−17	.443	−7	.476	0	.457	−4	.410	−14	.380	−20	.419	.431	14	17
16	.416	−24	.456	−17	.536	−3	.553	0	.545	−1	.473	−15	.443	−20	.491	.503	13	16
15	.496	−23	.546	−15	.626	−3	.644	0	.646	+1	.554	−14	.516	−20	.578	.591	13	15
14	.585	−21	.639	−14	.718	−3	.741	0	.726	−2	.658	−11	.602	−19	.671	.682	12	14
13																		13
12	.80	−15	.88	−6	.93	−1	.94	0	.92	−2	.88	−6	.81	−14	.89	.89	7	12
11																		11
10	1.12	−5	1.17	−1	1.17	−1	1.18	0	1.17	−1	1.18	0	1.13	−4	1.16	1.17	2	10
9																		9
8	1.48	0	1.51	+2	1.47	−1	1.48	0	1.47	−1	1.51	+2	1.48	0	1.49	1.49	1	8
7																		7
6																		6
5																		5
4	2.36	+4	2.36	+3	2.25	−1	2.28	0	2.24	−2	2.34	+3	2.33	+2	2.32	2.32	2.5	4
3																		3
2																		2
1	3.36	+11	3.20	+9	3.05	+1	3.03	0	3.00	−1	3.24	+7	3.21	+6	3.18	3.12	6	1
0																		0

TABLE III

Values of Δp (difference of pressure from pressure at equator at same level) and of $\Delta p/(p/T)$ where p/T is the mean for the level for the hemisphere

	WINTER								SUMMER								
	Abisko		England		India		Equator		India		England		Abisko				
	Δp	$\frac{\Delta p}{p/T}$	Δp	$\frac{\Delta p}{p/T}$	Δp	$\frac{\Delta p}{p/T}$	Δp	$\frac{\Delta p}{p/T}$	Δp	$\frac{\Delta p}{p/T}$	Δp	$\frac{\Delta p}{p/T}$	Δp	$\frac{\Delta p}{p/T}$			
45	-0.63	-151	-0.29	-72	-0.1	-25	0	0	+0.1	+19	+0.3	+58	+0.7	+135	45		
40	-1.09	-124	-0.5	-57	-0.1	-11	0	0	+0.1	+9	+0.5	+45	+1.1	+100	40		
38	-1.37	-108	-0.5	-42	-0.1	-8	0	0	+0.1	+7	+0.6	+42	+1.3	+91	38		
36	-1.70	-100	-0.6	-35	-0.1	-6	0	0	0.0	0	+0.8	+40	+1.6	+80	36		
34	-2.08	-87	-0.7	-29	-0.1	-4	0	0	0.0	0	+1.1	+39	+2.0	+71	34		
32	-2.47	-75	-0.7	-21	0.0	0	0	0	0.0	0	+1.5	+40	+2.5	+66	32		
30	-2.8	-61	-0.5	-11	+0.2	+4	0	0	0.0	0	+2.2	+43	+3.1	+61	30		
28	-3.2	-48	-0.2	-3	+0.4	+6	0	0	-0.1	+1	+2.9	+41	+3.4	+49	28		
26	-3.9	-43	0.0	0	+0.3	+3	0	0	+0.2	+2	+3.4	+35	+4.2	+44	26		
24	-4.7	-37	+0.2	+2	+0.2	+2	0	0	+0.2	+1	+3.9	+29	+4.2	+32	24		
23	-5.2	-35	+0.2	+1	0.0	0	0	0	+0.1	+1	+4.0	+26	+4.3	+28	23		
22	-5.9	-33	0.0	0	-0.3	-2	0	0	0.0	0	+4.0	+23	+4.0	+23	22		
21	-6.6	-32	-0.2	-1	-0.6	-3	0	0	+0.1	+0.5	+4.1	+19	+3.7	+17	21		
20	-7.7	-31	-0.7	-3	-1.2	-5	0	0	-0.2	-1	+3.8	+15	+2.8	+11	20		
19	-9.2	-31	-1.1	-4	-2.1	-7	0	0	-1.1	-4	+2.9	+9	+2.9	+9	19		
18	-11.5	-33	-3.2	-9	-2.2	-6	0	0	-2.2	-6	+2.8	+8	+0.8	+2	18		
17	-15	-36	-5	-12	-4	-9	0	0	-2	-5	+1	+2	-1	-2	17		
16	-19	-39	-8	-16	-3	-6	0	0	-2	-4	-2	-4	-4	-8	16		
15	-23	-40	-10	-17	-4	-7	0	0	-3	-5	-5	-8	-8	-13	15		
14	-28	-42	-14	-21	-6	-9	0	0	-3	-4	-8	-12	-13	-19	14		
13	-32		-16		-5		0	0	-2		-10		-16		13		
12	-38	-43	-19	-21	-5	-5	0	0	-3	-3	-13	-15	-21	-24	12		
11	-42		-22		-5		0	0	-4		-15		-25		11		
10	-45	-39	-24	-21	-6	-5	0	0	-4	-3	-17	-14	-28	-24	10		
9	-48		-26		-7		0	0	-5		-17		-30		9		
8	-48	-32	-27	-18	-8	-5	0	0	-6	-4	-18	-12	-30	-20	8		
7	-47		-27		-8		0	0	-7		-17		-30		7		
6	-46		-25		-9		0	0	-8		-16		-29		6		
5	-43		-24		-9		0	0	-9		-15		-28		5		
4	-39	-17	-22	-9	-9	-4	0	0	-9	-4	-13	-6	-25	-11	4		
3	-32		-17		-6		0	0	-8		-10		-21		3		
2	-23		-13		-4		0	0	-7		-6		-16		2		
1	-12	-4	-5	-2	-3	-1	0	0	-8	-3	-2	-1	-9	-3	1		
0							0	0							0		

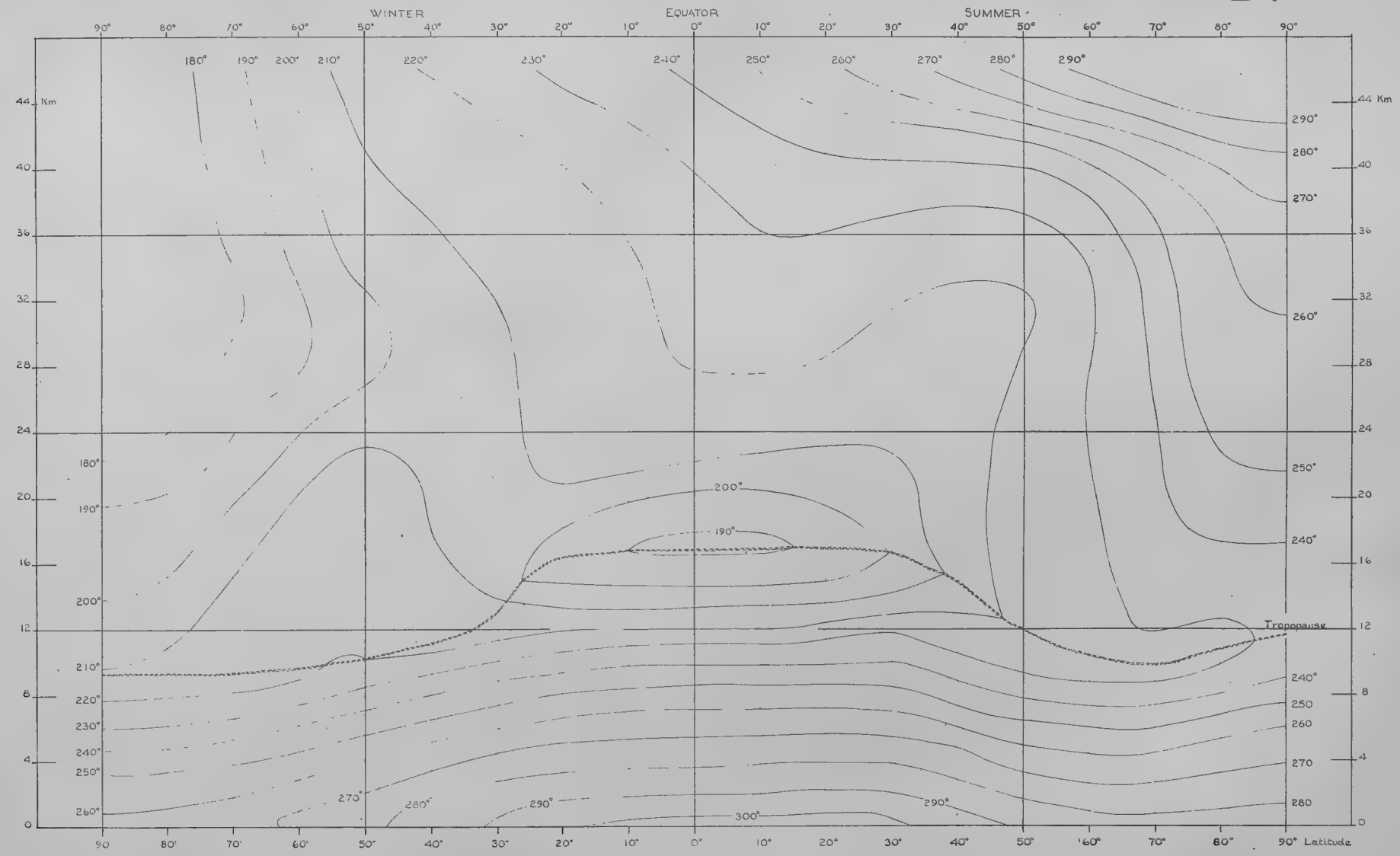


FIG 1 DISTRIBUTION OF TEMPERATURE WITH HEIGHT AND LATITUDE

1/3537

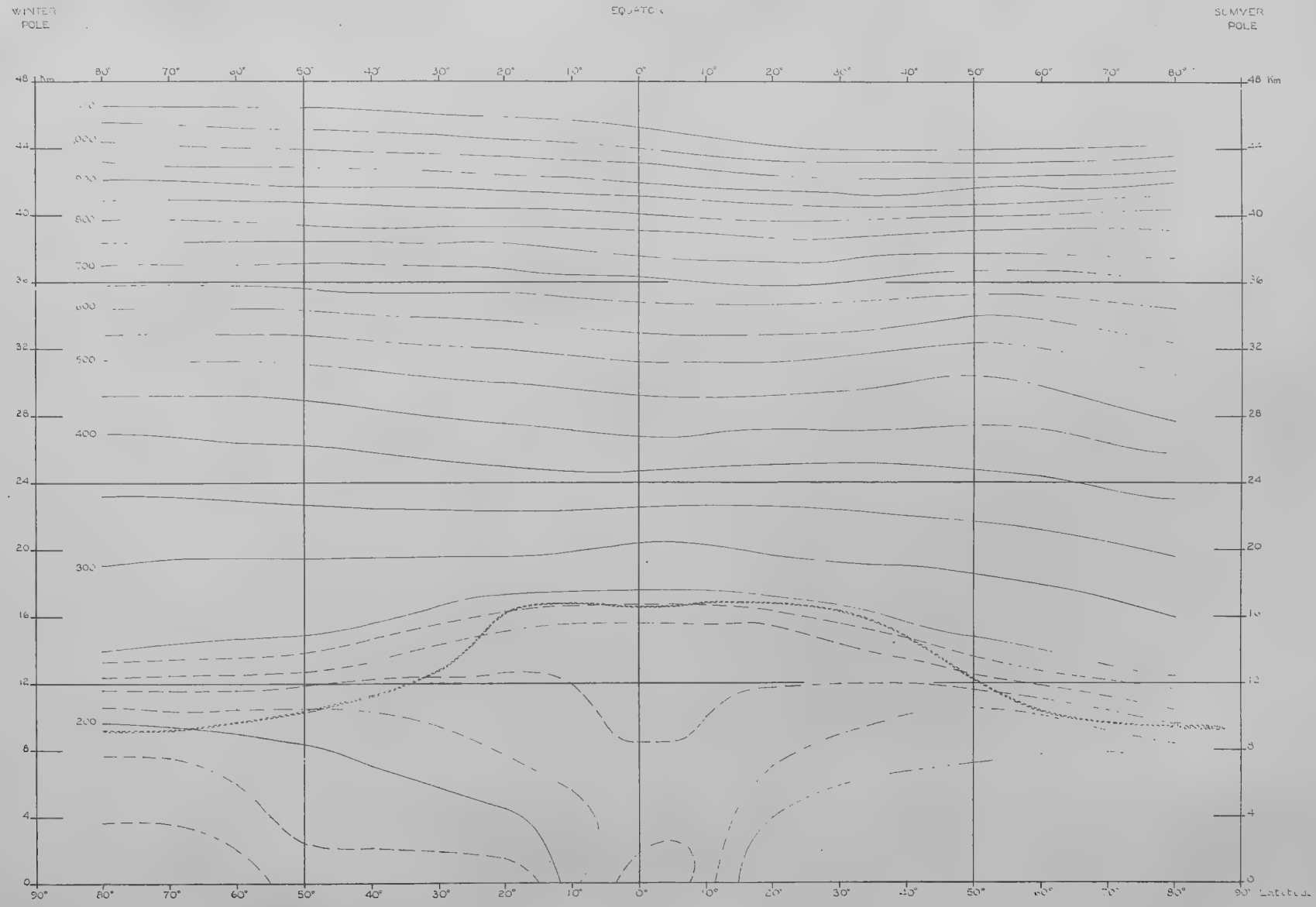


FIG 2. DISTRIBUTION OF POTENTIAL TEMPERATURE

(referred to a pressure of 200 mb)

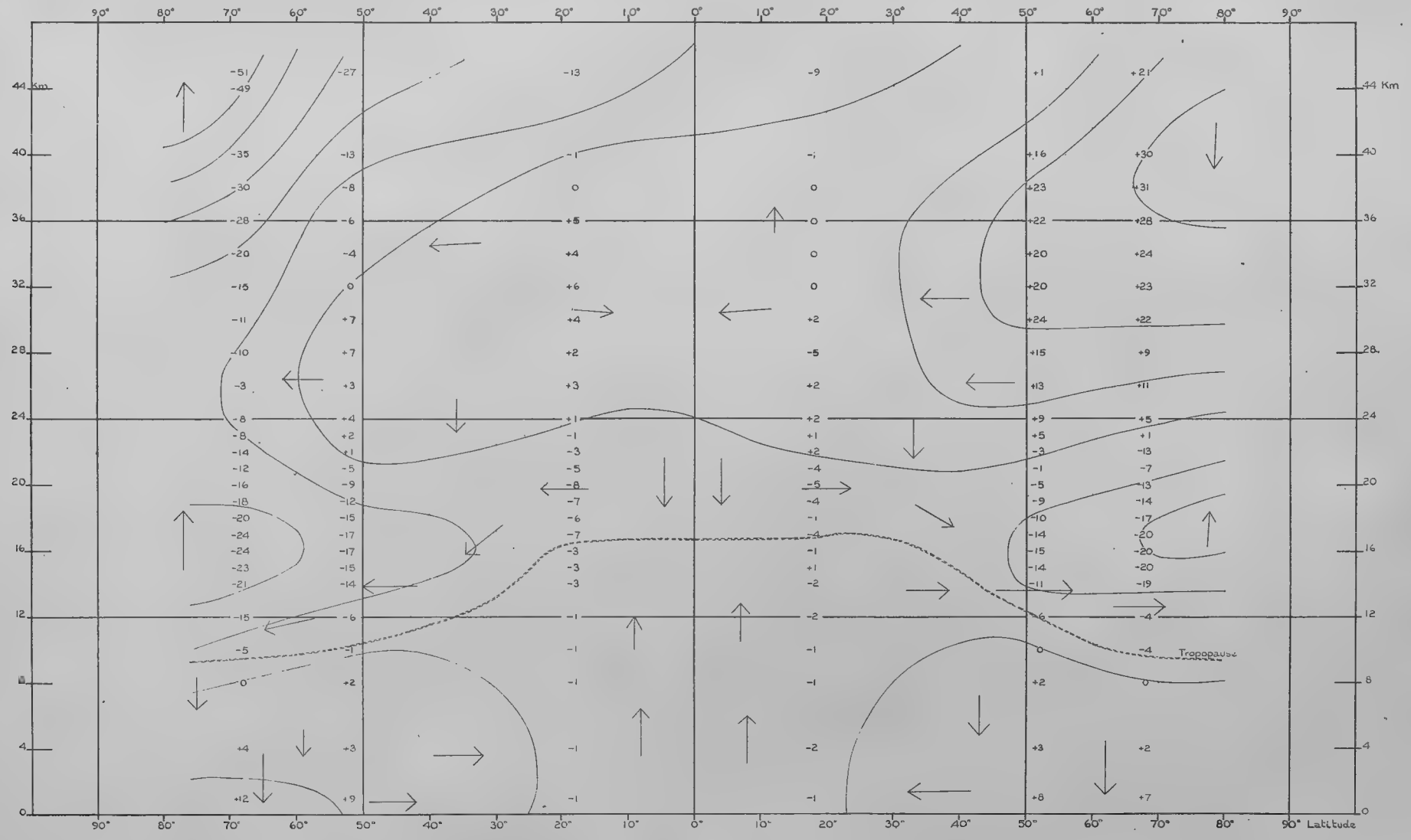
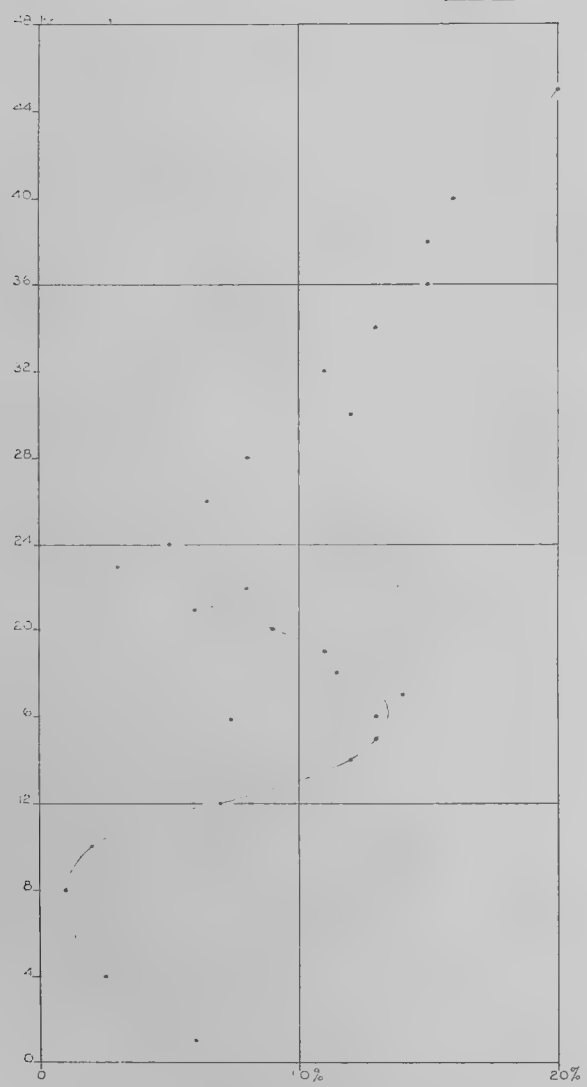


FIG 3 PERCENTAGE DIFFERENCE OF AIR DENSITY FROM DENSITY AT EQUATOR AT SAME LEVEL.

(Arrows show probable resultant circulation in meridian plane)



Mean % departure of density from density at Equator

FIG 3A LATITUDE VARIABILITY OF DENSITY WITH HEIGHT

1/2537

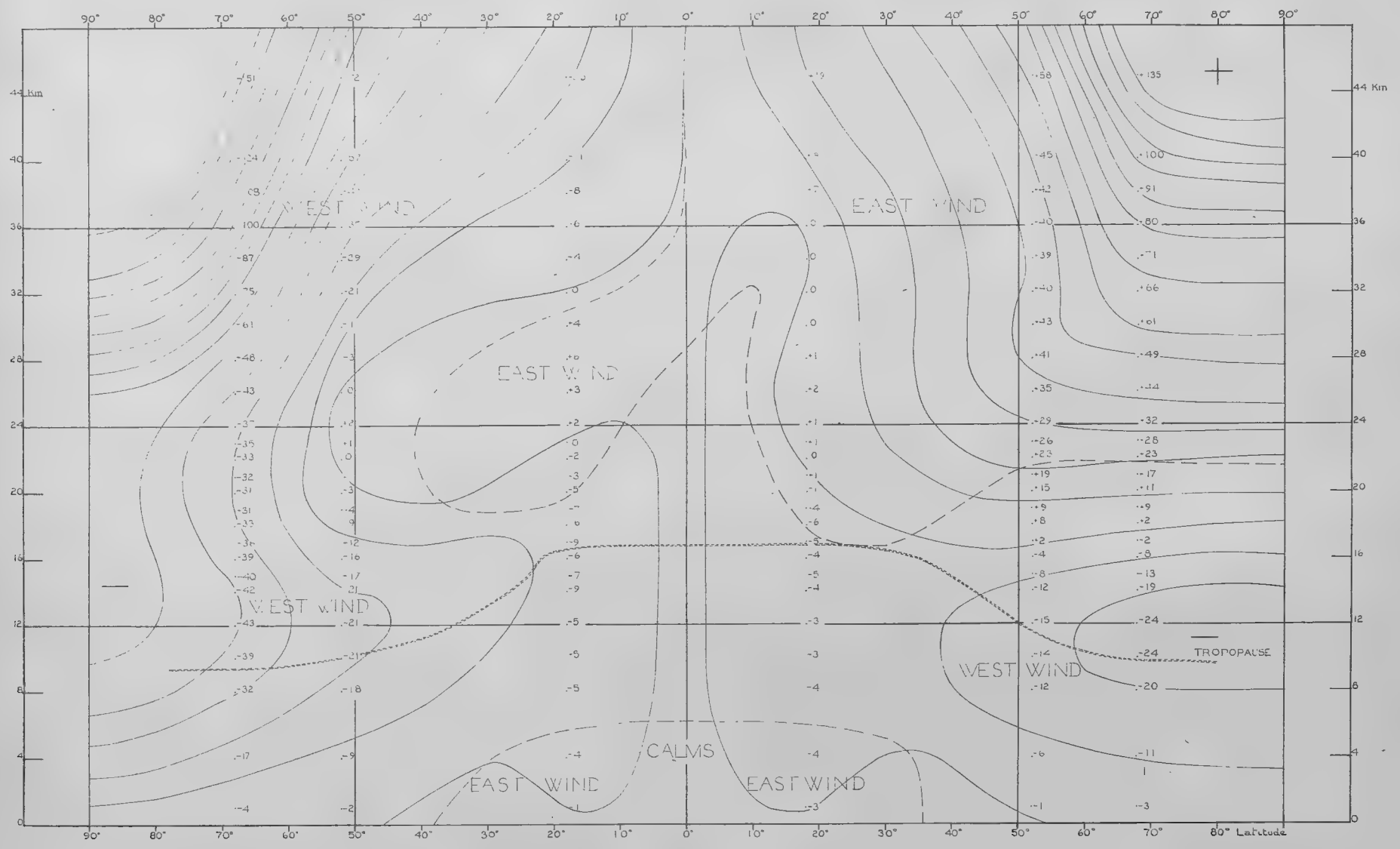


FIG 4. DISTRIBUTION OF $\frac{\Delta p}{p}/\frac{p}{T}$ AND WIND COMPONENTS PERPENDICULAR TO MERIDIAN

(NORTHERN SUMMER)

Broken lines show boundaries between W + E Winds

1/3589

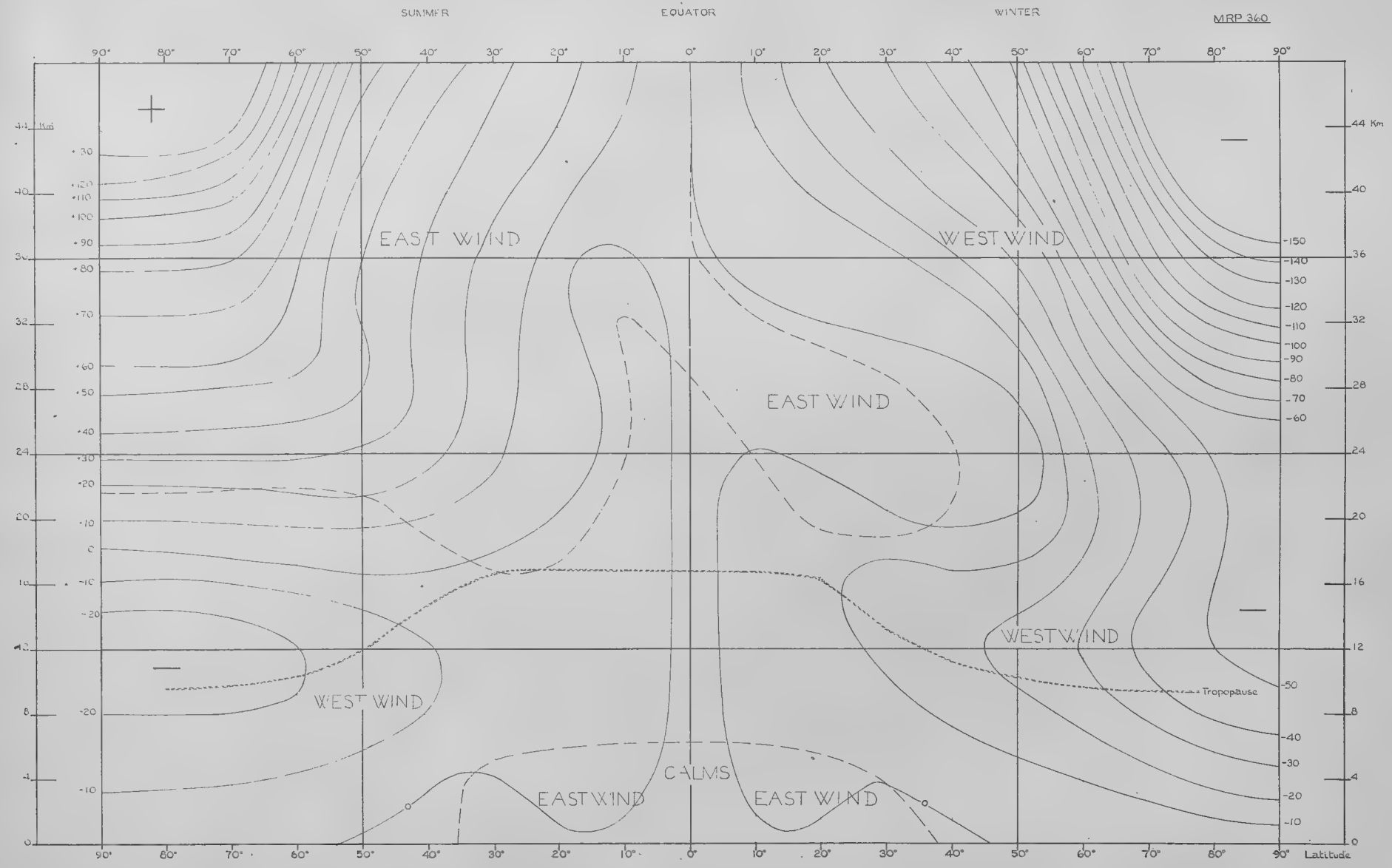


FIG 5 DISTRIBUTION OF $\frac{\Delta p}{p}/\frac{\Delta}{T}$ AND WIND COMPONENTS PERPENDICULAR TO MERIDIAN

(NORTHERN WINTER)

Broken lines show boundaries between W+E winds

13589

WEATHER SERVICE
TECHNICAL REFERENCE FILES

AIR MINISTRY

M.R.P.362
30th September, 1947

METEOROLOGICAL RESEARCH COMMITTEE

A PRELIMINARY STUDY OF THE VERTICAL EXCHANGE OF MOISTURE NEAR THE GROUND

BY F. PASQUILL

SUMMARY

An account is given of a short study which considers the determination of short term evaporation loss from a knowledge of turbulent exchange processes. On the experimental side the investigation comprised the simultaneous observation of the vertical profiles of wind velocity, temperature and vapour pressure in the first 2 metres above a pasture on Gault clay. These observations were carried out over 15 minute periods at hourly intervals, for a total period of one week. At the beginning and end of the whole period the soil moisture profile was measured down to a depth of 3 feet, in order to provide a direct measurement of the total water loss.

A fairly detailed description is given of the laboratory tests on the aspirated thermocouple psychrometer equipment developed specially for the present profile observations. These established that the wet bulb depression was sufficiently independent of likely variations in the rate of aspiration, that the lag coefficients of the dry and wet bulbs were respectively 1.8 and 0.9 minutes, and that the nominal limit of reading the dry bulb or wet bulb temperature differences was 0.02°F., the corresponding limit in vapour pressure being 0.02 mm. Hg. Control observations in the field with the psychrometers arranged to be at a constant height indicated departures from the mean value in the region of 0.05 mm. Hg., though the indication was that such departures might be consistent enough to warrant the future use of control observations as a correction to observed profiles.

A suitable selection of the observations was made in order to eliminate possible spurious effects and observations possessing a low order of accuracy. The main groups of observations remaining are shown to conform to a certain type of identity in the form of the profiles of wind speed, absolute humidity and temperature over the layer 25-100 cm. and a similar identity as regards the absence of effect of atmospheric stability defined in terms of vertical wind shear and temperature gradient. This is put forward, with some qualification, in support of the customary assumption of identity of eddy viscosity and eddy diffusivity and of the concept of virtual 'adiabaticity' in the atmosphere very near the ground, for the main range of conditions encompassed in the present measurements. With these assumptions the eddy diffusivity is calculated from the wind velocity profile in terms of the well known extension of Prandtl's treatment to the atmosphere, and from this and the measured humidity gradient the vertical flux of water vapour is computed. From an integration of these computed values and a measurement of the rainfall the net water loss from the soil is evaluated as 1.25 gm/cm^2. The soil moisture measurements are interpreted to give values of 0.4 and 2.2 gm/cm^2 both of which values are statistically insignificant. Despite the latter feature it is thought justifiable to regard an agreement between the evaporation computed on a meteorological basis and that determined from soil measurements as established in order of magnitude. A brief discussion is finally given on the constancy of the vertical flux of vapour in the first few metres of the atmosphere.

A PRELIMINARY STUDY OF THE VERTICAL EXCHANGE OF MOISTURE NEAR THE GROUND

INTRODUCTION.

1. The short study described in the present paper constitutes a preliminary investigation of the general problem of the exchange of water between ground and air. Besides possessing obvious meteorological importance this problem has a fundamental connection with a number of aspects of agricultural science, a notable example of which is the question of moisture movement and drainage in soils, and it was primarily in this connection that the present study was initiated.

2. The estimation of the amount of water lost by evaporation at the earth's surface may be attempted along three main lines. There is the method of gravimetric estimation employed, for example, in the 'lysimeter' or 'percolation gauge', which involves essentially the isolation of a block of soil in order that a study may be made of one or more of the natural processes which subscribe to the final 'water balance'. An application of the 'water balance' principle is also made by water engineers in estimating evaporation from catchment areas when, for sufficiently long periods, the evaporation may be defined as the difference between precipitation and run-off. The 'energy balance' method, first used by Angstrom (1920) in a discussion of evaporation from lakes, evaluates the latent heat appropriate to the water loss by measuring or estimating the remaining terms in the equation prescribing the balance of heat at the earth's surface. A number of workers, notably Cummings (1940), have developed this method. The principal remaining method, which considers evaporation as a problem of turbulent exchange in the atmosphere, reduces ultimately to the measurement of the vertical gradient of water vapour content in the air and the evaluation of the vertical component of the 'eddy diffusivity', i.e. the virtual diffusion coefficient which replaces the classical coefficient of molecular diffusion. Of these three methods the 'water balance' method is probably incapable in general of giving more that the total evaporation loss over long periods. For an examination of the evaporation during periods of the order of one hour, say, the most promise is considered to lie in the 'energy balance' and hydrodynamical methods, and in the present considerations attention is restricted to the latter method.

3. The discussion of the effects of turbulent motion on the spacial distribution of heat, matter and momentum in the atmosphere, in the absence of information on the mechanics of turbulent flow, have relied upon somewhat intuitive and semiempirical formulations of expressions for the virtual diffusion coefficients of the type mentioned above. One method of approach is the application to the atmosphere of the laws of turbulent flow put forward on a partly theoretical basis by Prandtl and von Kármán (see Brunt, 1939, p. 244 et seq.) and established in laboratory experiments by Nikuradse (1932), Schlichting (1936) and Dryden (1936) on flow through pipes and over flat plates. Notable among such applications are those of Sverdrup (1936, 1937, 1946) and Montgomery (1940) to evaporation from the ocean, Thornthwaite and Holzman (1939) to evaporation from land surfaces and Calder (1947) in a recent consideration of the problem of diffusion of smoke and vapour. An alternative approach is Sutton's (1934, 1947) extension of Taylor's theory of diffusion by continuous movements, which was shown to give excellent agreement with the laboratory data for the turbulent transport of momentum, vapour and heat (the last two by the present writer 1943) above aerodynamically smooth surfaces. In the application of Sutton's treatment to the atmosphere certain difficulties arise owing to the fact that in general the earth's surface is aerodynamically rough, but despite this the treatment has provided for a number of years the only satisfactory interpretation of a number of practical problems in atmospheric diffusion near the ground.

/4.

4. In the previous considerations of the evaporation of water from the earth's surface the evaporating area is considered to be infinite in extent, stationary conditions are assumed to exist and thus the vertical flux of water vapour is assumed to be constant in space. This is, of course, a parallel to the customary assumption of constancy of horizontal turbulent shearing stress in the lower atmosphere. It is then necessary to postulate that the transfer of water vapour and of momentum follow the same laws, i.e that the eddy diffusivity and eddy viscosity are identical and the eddy viscosity is then derived from the wind profile according to the extension of Prandtl's work. The only application, so far, to land evaporation (Thornthwaite and Holzman, loc. cit.) takes no account of the effects of thermal stability and instability on the processes of turbulent exchange, and thus assumes the eddy viscosity under all conditions to be identical with that for isothermal or 'adiabatic' conditions. Finally it has not been possible so far to provide any critical test of the local rates of evaporation so computed.

5. The present study was designed to explore simultaneously the vertical distribution of wind speed, water vapour and temperature in the very lowest layer of the atmosphere, and a considerable part of this paper is of necessity devoted to the instrumental aspects involved. In conjunction with this investigation, which was continued over a period of approximately one week, data were obtained on the soil moisture profiles obtaining at the beginning and end of the period, in the ground over which the profile observations were conducted. It was hoped that this coordinated investigation of so many of the factors which are relevant to the physical processes of moisture exchange would permit some critical appraisal of the assumptions outlined above, and that the soil moisture data would provide a final and conclusive test on the rates of evaporation ultimately computed. Although in the actual course of events the results of the investigation fall somewhat short of these aims it may be stated here that some contribution is made to the justification of the underlying assumptions and that, in particular, the validity of the computed rates of evaporation is established in order of magnitude.

EXPERIMENTAL AND ANALYTICAL DETAILS

The Site.

6. The site chosen for the present investigation is a pasture near the northern extremity of the University Farm at Cambridge. The soil is Gault clay, with a cover of grass and clover, the latter of which was still only in the leaf stage during the course of the experiment. From the observation point there is at least 150 yards of level uniform terrain in all directions, a distance which, in the absence of major obstacles upwind is thought to be more than adequate for the adjustment to a representative boundary layer of depth 2 m. The nearest variation in level occurs at about 150 yards to the S. and S.E. where the ground then slopes gently into a distinct valley. Between S.E. and N.N.W. the nearest and most noticeable obstacle is Bull Close, which is 250 by 80 yards in extent, about 50 ft. high, at a distance of 450 yards. On the N.E. boundary of the pasture, however, which at the nearest point is only 150 yards from the nominal observation site, there is a belt of small trees about 20 - 30 ft. high and extending over a width of about 15 yards and it is accepted that as far as possible the taking of observations in winds between N. and E. should be avoided.

Instrumental Equipment.

7. For the measurement of wind speed at four heights, 200, 100, 50, 25 cm. above ground level, sensitive cup anemometers were employed. These anemometers are a development of a type described by Sheppard (1940) and the particular instruments used here were capable of registering wind speeds down to 1 ft./sec. A light vane (a development of the instrument used by Best, 1935) was used at a height of 200 cm for the purpose of providing records of wind direction. For the measurements of temperature and humidity gradient required here, there exists no stock instrumental equipment and it was necessary to make up and test the necessary apparatus in the laboratory. In the interests of simplicity of construction and convenience in registration at a distance single junction wet and dry bulb thermocouples were

/adopted

adopted for the purpose and made up into psychrometers on the Assmann principle for mounting at 200, 100, 50, 25, 12½ and 6¼ cm. above ground level on a common aspiration mast, the thermoelectric currents being registered by suitable portable galvanometers.

8. Horizontal and vertical sections showing the essential features of the psychrometers are reproduced in Fig. 1 while some external aspects of the instrument may be seen in the photograph. Each 'thermometer' consisted of a single junction of 28 swg. copper - constantan wires braided and contained in flexible plastic tubing, and inserted in a Tufnol tube of outside diameter ¼" which formed the stem of the thermometer. The junction of the wires was soldered into a 1¼ inch length of copper rod of nominal diameter 3/16 inch which was then cemented a distance of ¼inch into the Tufnol stem and formed the bulb of the element, and as a protective measure stem and bulb were given a coating of thinned Durofix. The only difference between the dry and wet-bulb elements was that the Tufnol stem of the latter was turned down over a distance of 1 inch so as to be flush with the copper bulb, and a close fitting single thickness sheath of linen was drawn over the bulb and the first inch of the stem. Exploratory experiments had shown that the diminution in wet-bulb depression due to conduction of heat down the wires immediately preceding the junction, i.e. the counter part of the emergent stem error in the mercury-in-glass thermometer, was effectively eliminated by extending the wet-bulb covering 1 inch beyond the copper bulb.

9. Each of the six pairs of 'thermometers' was mounted in a double walled Tufnol housing, rectangular in cross-section, with the entrance section divided by a 1/16 inch plate of Tufnol. The wet-bulb feed, consisting of a dozen strands of cotton, passed through a narrow tube into the water chamber attached to the underside of the housing. The side tube on the housing was inserted in a branch tube on the mast, which was made of fabric based Tufnol tube 1 inch inside diameter and 1½ inches outside diameter, and which formed a common aspiration conduit for the six psychrometers. Air was drawn through the housings and thence through the mast by means of a unit consisting of three small fans operated by a total of four 6v. accumulators. The leads from the thermocouples passed into a junction box attached to the mast and from there approximately 25 yards of 14 core cable were lead to a hut containing two switching units and two galvanometers, by means of which the differences between the wet and dry bulb temperatures at various heights and those obtaining at 200 cm. were indicated. To minimise radiation error the housings, mast and leads were painted white. One additional dry bulb element was carried in a separate housing at 200 cm. so as to provide a check on the zero characteristics of the instruments. Another element and a mercury-in-glass thermometer were carried on the mast, the bulbs being wrapped together in copper wire and surrounded by a brass cylinder and then by a Tufnol tube. The latter arrangement gave a reference temperature from which the differential values were converted to absolute values.

10. In designing the above equipment, and in establishing its characteristics, attention was focussed primarily on the temperature sensitivity of the arrangement, the response of the indicating galvanometers, the constancy of the wet bulb depression and the thermal lag of the wet and dry bulb elements. Ultimately of course much of the design was controlled by such factors as the type of galvanometer available and the accumulator resources which could be applied in practice. Furthermore, although the characteristics established in the controlled conditions of the laboratory may well appear to be of somewhat academic significance in relation to the field use of the instrument, it is thought appropriate to give a fairly detailed account of the laboratory work involved.

11. The galvanometers used were of the Tinsley sensitive portable type with nominal resistance 450 ohms, sensitivity 450 mm. per microamp. and critical damping resistance 12,000 ohms. Use of the full damping resistance was out of the question and it was a matter of compromising so as to obtain the highest temperature sensitivity consistent with an acceptable response on the part of the galvanometer. With thirty seconds set as the maximum allowable time for the galvanometer to reach its final reading it was found that an external resistance as low as about 2,000 ohms could be used, giving a temperature sensitivity in

/the

the region of about 0.5 cm. per °F. temperature difference. The final calibration was carried out in well stirred baths, in two parts, firstly a comparative calibration of all the differential circuits for a temperature difference of approximately 5°F. and secondly an absolute calibration of the wet and dry bulb elements used at 200 cm. and the reference element against sensitive mercury-in-glass thermometers. The comparative calibration showed that in each of the two sets of differential circuits individual circuits possessed temperature sensitivities which varied from the average value by less than 0.5%. The results of the absolute calibration carried out, before and after the present series of experiments, at temperatures in the region of 60°F. are reproduced in Fig. 2(b), from which it will be seen that, as anticipated, the Temperature/Deflection relation is sensibly linear over a range of 10°F. Furthermore very little difference in sensitivity is exhibited by the values obtained on the two occasions, but since the later calibration was carried out with new thermometers possessing N.P.L. certificates the calibration factors computed from this were used in the final reduction of the data. These factors were 0.534 and 0.449 cm./°F. for the dry and wet bulb circuits respectively. The galvanometer readings were estimated to 0.01 cm. so that the nominal lower limit of detectable temperature difference was 0.02 °F.

12. The variation of the depression of the wet bulb with air speed past the bulb was investigated by employing two of the psychrometers, arranged to draw common air, in one case the aspiration being maintained at a constant value corresponding to an average of 2.8 m./sec. in the space immediately surrounding the wet bulb, while in the other case the aspiration was varied from 0.7 to 5.9 m./sec. The rate of aspiration was measured by means of an instrument of the gas meter type. It is noteworthy at this stage that the inner rectangular tube of the housing had six 3/16 inch diameter holes in the wall so as to bypass a small proportion of the ventilation through the space between the inner and outer tubes. From the calibrations of a 'hot-bulb' anemometer (Middleton, 1943, p.153) in the position normally occupied by the thermoelement it was deduced that approximately 80% of the air passed through the inner space, from which result it was then possible to compute the average air speed in the space immediately surrounding the wet bulb. The investigation of the wet bulb depression was performed with an air temperature in the region of 60°F. and a depression of 12°F. and the readings taken corresponded to the difference in the two wet bulb temperatures. These readings were then reduced to wet bulb depressions corresponding to a depression of 10°F. at the standard aspiration of 2.8 m./sec. and the values are reproduced in Fig. 2(a).

13. It will be noted that in the region of only 2 m./sec. the wet bulb depression was only 0.6% below the constant value attained above 4 m./sec., and that a 5% variation on either side of the 2 m./sec. resulted in only 0.1% variation in the depression. An exploration of the aspiration in the various psychrometers when mounted on the mast showed that with slight adjustments of the resistances of two of the branch tubes the variation in rate of aspiration with position on the mast was only ± 4%. Furthermore it was found that the use of a total battery voltage of 24 volts gave an average ventilation of 2.2 m./sec. Accordingly for practical purposes 24 volts was adopted as the normal working voltage, so that, theoretically, as far as the effects of ventilation were concerned, the wet bulb depressions should be constant within 0.1%.

14. The response of the dry and wet bulb elements to changes of temperature and humidity are naturally of fundamental importance in measurements of the present nature. In view of the response of the indicating instruments and the number of temperature differences to be measured, it was not possible to take a reading of any one difference, say that between the wet bulb temperatures at 200 cm. and 25 cm., more frequently than once every three minutes. It was necessary therefore that each reading should be as near as possible to the mean value actually corresponding to the previous and succeeding 90 secs. The thermoelement bulbs were accordingly constructed so as to have an appreciable thermal inertia and thus be insensitive to the rapid fluctuations which, though a fundamental feature of the whole process of atmospheric flow, are merely a source of confusion where mean values over the order of several minutes are required. On the other hand it was appreciated that this lack of response would be a source of error on occasions when the temperature exhibited a sustained rise or fall. In order to permit corrections to be made on such occasions the responses of the wet and dry bulb elements were examined in the laboratory.

/15.

15. The temperature of the air drawn into one psychrometer housing was raised approximately 10°F by means of a small heating coil carried in an extension tube attached temporarily to the housing, the normal ventilation rate of 2.2 m./sec. being imposed. Influx of the normal air stream was initiated by withdrawing this extension tube, and the indicated excess of temperature (measured against a psychrometer permanently drawing the normal air stream) was noted at subsequent intervals. A similar procedure was carried out with an increase in humidity corresponding to an increase in wet bulb temperature of 3°F., dry bulb temperature remaining constant, and finally with an increase of temperature (10°F.) of the dry bulb element only, as distinct from an increase in temperature of the air. Throughout these measurements the dry and wet bulb temperatures of the normal air stream were approximately 65 and 53°F respectively. The results are shown graphically in Fig. 2(c), ΔT_o being the initial departure from the normal dry or wet bulb temperature, and ΔT the departure at subsequent intervals t. It will be noted that in all cases the results conform fairly closely to the familiar exponential law (see Middleton, loc. cit., p. 61) of the form

$$\Delta T = \Delta T_o \exp. \left(- \frac{t}{\alpha} \right),$$

where α is a 'lag coefficient' with the dimensions of time. Instead of attempting to fit the best straight line to each set of values the value of t at which $\Delta T / \Delta T_o$ falls to $1/e$ has been interpolated and is adopted as the value of α. The following table summarises the results.

Element changed	Value of α in minutes	
	Dry bulb	Wet bulb
Air temperature	1.8	1.05
Absolute Humidity		0.8
Element temperature	1.5	

The outstanding features are the difference in the dry and wet bulb lag coefficients for the changes in air temperature, an effect which is consistent with Spilhaus's (1936) experimental and theoretical treatment, and the difference in the wet bulb lag according as the temperature or absolute humidity is changed. It seemed that a possible explanation of this effect lay in the fact that in a change of air temperature some contribution to the total lag would be made by the housing, whereas with a change of humidity only the temperature of the wet bulb would be affected. This is borne out by the smaller lag coefficient obtained when the temperature of the dry bulb element only is changed, suggesting that in practice some 10% of the lag according to air temperature will be contributed by the housing. It may be shown (see for example Bilham, 1935) that with lag coefficients of the above magnitude the thermoelements should be practically unresponsive to all temperature fluctuations with periods of one minute or less.

16. One other feature of the instruments was investigated in the early stages of development, namely the errors due to the housing and outer covering of the thermocouple leads being at a temperature different from that of the air. The effect of the former was found to be of a higher order than the latter, and with the normal aspiration was found to amount to approximately 0.2°F. for a difference of 10°F. between the temperature of the outside of the housing and that of the air. This probably represents a very artificial reproduction of radiation error, since in practice the irregularities of air movement around the housing will be of some importance, but the test served to establish that as far as differential work was concerned (where individual radiation errors would compensate) the resultant error was likely to be of a very small order.

Observational Procedure.

17. The mast carrying the thermocouple psychrometers was set up at a distance of approximately 25 yards from the hut housing the registering instruments, in one of two positions depending on the direction of the wind. The aim was to

/avoid

avoid the downwind effect of the hut, whatever the wind direction, though for other reasons it will be recalled that winds in the N.-E. quarter were not desirable. The mast was set in a stout iron pipe driven some 16 inches into the ground, the use of guy ropes thereby being avoided, and was rotated so that the housing entrances were on the upwind side of the mast assembly and also so that the sun did not shine into the housing entrance. This requirement presented some difficulty when sun and wind were in the same direction and when the wind direction was variable and it was necessary to maintain fairly constant watch on the mast in order to ensure the greatest freedom from such sources of error. The anemometers and vane were set up in a line across wind with the psychrometer mast, the whole line of instruments extending about 10 yards. The aspirating unit, which was connected to the psychrometer mast by heavy rubber hose pipe of $\frac{3}{4}$" bore was positioned downwind of the line of masts at a distance of about 10 ft. The assembly is shown in the photograph.

18. Each 'observation period' occupied 15 minutes, during which five equally spaced readings were taken of the galvanometer indications corresponding to each of the six dry bulb and six wet bulb temperature differences and of the thermometer mounted with the reference element on the mast. The anemometers were operated for the whole period and two 3 - minute vane records were obtained. Aspiration of the psychrometers was started 5 minutes before the readings began and the galvanometer zeros were read immediately before and immediately after each o observation period. A fairly detailed account was kept of the state of the ground and sky and a Meteorological Office pattern 5-inch rain gauge was read at 0900 hours each morning. The whole series of observations was maintained at hourly intervals from 0800 G.M.T. 7.5.47 until 1400 G.M.T. 13.5.47. As frequently as possible, and in practice this meant every three to four hours, anemometer 'control runs' were carried out in order to ensure an adequate comparative performance. The procedure was to set up the four anemometers 2 feet apart at a height of 2 metres in line across wind, and to operate them for the customary 15 minutes. Such observations ultimately provided correction curves by means of which instrumental variations were subsequently eliminated, a procedure which is essential in profile observations of the present type.

19. Provision was also made for mounting the psychrometer mast in a horizontal position at a height of 2 metres and so carrying out 'control observations' designed to provide a check on the differential performance of the thermocouple elements. The carrying out of this procedure in between the normal profile observations proved to be impracticable however and the consideration of this aspect had to rely upon the few control observations carried out before and after the whole series of observations. It will be recalled that the nominal limit of reading temperature difference was 0.02°F. and it may easily be shewn that in the relevant conditions of temperature and humidity the corresponding limit in vapour pressure difference was 0.02 mm. Hg. The control observations carried out in the field, however, yielded vapour pressure values departing from the mean by as much as 0.1 mm. Hg., though the most sustained series of control measurement, carried out in a gusty wind, [redacted] with dry bulb and wet bulb temperatures in the region of 77 and 60°F respectively, gave average departures up to only 0.05 mm. Hg. There was some indication that these departures were repeated sufficiently to warrant the application of corrections to profile measurements and the introduction of regular and frequent control observations among the profile observations would therefore be desirable in future measurements.

20. Although the psychrometer mast was set at one of two fixed positions, variations of wind direction made it necessary to re-orientate the mast from time to time and also to move bodily the anemometer and vane masts. On these occasions heights of the various instruments were adjusted, the datum level being the surface of the soil, and as far as possible the instruments, particularly the lower ones, were arranged to be over surfaces of similar texture. It will be appreciated, however, that with a surface possessing small scale variations in level and variations in vegetation height and density, the constant changes in wind direction and consequent adjustments in the positions of the masts would introduce a rather unpredictable degree of variation in the profile characteristics. This feature may be expected to increase as the surface is approached.

/Reduction

Reduction of Data.

21. For each 15 minute period the deflections and readings corresponding to the various items were averaged and wet and dry bulb temperature differences were computed by using the factors described in para. 11. It will be recalled that these factors were obtained at temperatures in the region of 60°F and since in practice the departures from this temperature were mainly within ± 10°F it was considered unnecessary to attempt any correction for the variation of thermo-electromotive force with temperature. Wind speeds were read off the calibration charts provided, to 0.01 ft. per sec., and reduced so as to correspond to the anemometer chosen as standard by means of the correction factors described in para. 18. The next stage in the analysis was the correction of the wet and dry bulb temperature differences for thermal lag. In this procedure it was necessary to idealise the situation by assuming that the overall change of temperature difference, i.e. that corresponding to the difference between the first and last readings, occurred linearly over the whole period. Then, if this linear variation of temperature difference be designated by β°F per minute, and the lag coefficient as defined in para. 15 is α minutes, it may be shown that the thermometers lag behind the actual temperature difference to the extent of $\alpha\beta$°F . The values of α adopted for this purpose were 1.8 for the dry bulb and 0.9 (i.e. the mean of the values obtained for separately changing temperature and humidity) for the wet bulb. Minor corrections were applied to the wet bulb lag coefficient for variations in the wet bulb temperatures from the standard value of 53°F, on the basis of the treatment given by Spilhaus (loc. cit). The values of β frequently reached 0.1 °F per minute or more. It was noticeable, however, that in general the corrections only amended the vertical profiles occurring during the development or breakdown of a steep lapse or inversion, at which times the rate of change of temperature showed a marked variation with height.

22. From the corrected temperature differences (i.e. the differences between the temperatures at 200 cm. and those at the remaining heights) and the temperatures of the reference element, the actual wet and dry bulb temperatures corresponding to the six heights were obtained. The wet and dry bulb temperatures were then employed to determine the vapour pressure, and as existing psychrometric tables were too coarse for the present considerations, the appropriate values were calculated from the customary formula as given in the Smithsonian Meteorological Tables, the psychrometric constant being corrected for broad changes in barometric pressure and wet bulb temperature.

General Features of Ground and Meteorological Conditions.

23. At the beginning of the experiment the ground was uniformly covered with a growth of young clover 3 - 5 cm. in height with fairly dense grass of about the same height and occasional sparse tufts of grass with individual stalks rising to 10 - 20 cm. Bare patches of clay occupied 5 - 10% of the area in the vicinity of the observation point. By midday on the 11th the clover growth had risen to 4 - 10 cm., and although the vegetation cover had been superficially wet for a good proportion of the time, owing to dew and the slight showers on the 8th and 10th, the soil surface appeared to be harder and dryer than originally and showed distinct cracks. At the termination of the observations at 1400 G.M.T. on the 13th these soil cracks had developed, though the vegetation cover was still green and fresh, the clover having risen slightly to 5 - 10 cm. and occasional individual grass stalks to 25 cm.

24. The general character of the weather would be described in synoptic terms as mainly cloudy, though the cloud was frequently sufficiently high and broken to permit considerable sunshine and a well marked diurnal variation of temperature and wind. Synoptic analysis indicated a number of frontal systems in the area, but of these only two were active enough to give measureable rainfall. This occurred in the form of slight showers on the 8th and 10th and totalled 1.25 mm. Fig. 3 shows the vapour pressure at the 950 and 900 mb. levels as given by Downham Market, the wind direction at the 950 mb. level from the same source and the surface wind direction as observed in the present investigation. It will be noted that

/the

the surface and upper winds exhibit a fair degree of correspondence, the only notable departure being in the early part of 12.5.47. when the surface wind veered and the upper wind backed. The vapour pressures in the upper air showed a fairly sustained rise until early on the 10th, an appreciable fall during the rest of that day, and then a resumption of the original steady rise. The only clear indication of a change in air mass character occurs on the 10th, where the above fall in vapour pressure followed a marked veer of surface wind.

25. In order to give a broad picture of the variations of vapour pressure near the ground the hourly values corresponding to the 200 and 6¼ cm. levels are reproduced in Fig. 4. A well marked diurnal variation of both values and of the difference between them is immediately evident. It is interesting to note at this point that while a pronounced diurnal variation of relative humidity is a familiar feature, the diurnal variation of absolute humidity or vapour pressure is frequently disregarded as being of a very small order. The present results show that, on the contrary, the diurnal variation may be appreciable, the amplitude in the present case being in the region of 5mm. Hg. with the mean value about 10 mm. Hg. Turning to the difference between the vapour pressures at the two levels, it is seen that, in the usual terminology, a lapse of vapour pressure (i.e. vapour pressure decreasing with height) was built up during the day, reaching a maximum about midday. Thereafter it diminished, reaching a very low value or even being reversed in the nocturnal hours. On the present occasions it may be mentioned that the 'inversions' of vapour pressure were invariably associated with the formation of dew, and the extent and duration of the vapour inversion seemed to bear some relation to the amount and persistence of dew.

DISCUSSION OF THE VERTICAL PROFILES OF VAPOUR PRESSURE, WIND SPEED AND TEMPERATURE.

Grouping of Results.

26. Previous work on the forms of the profiles of wind and temperature has been largely concerned with the effects of wind speed, the vertical gradient of temperature and the roughness characteristics of the ground surface, and a discussion on similar lines has been made by Sheppard and Firesah (1947) in connection with a vapour profile investigation. In the latter case the idea of a universal form of profile has been developed. Furthermore previous examination of wind and temperature profile characteristics (e.g. Best, loc. cit.) has frequently pointed to the interdependent effects of wind speed and vertical temperature gradient. It is clear therefore, that a clear appraisal of the importance of wind speed and temperature gradient must either separate the effects of these two factors by a close grouping of results, or view the results in terms of a composite factor suitably incorporating both wind speed and temperature gradient. In the present case, since the variations of the two factors are so considerable and the number of observations is insufficient to permit a satisfactory separation, the second approach is adopted.

27. On physical grounds an inter-relation of wind speed and temperature gradient in determining the so-called stability of the atmosphere is to be expected. The magnitude of the temperature gradient clearly determines the tendency to modification of the flow by buoyancy effects, while the strength of the wind and the associated shearing forces constitute an opposing tendency and may be expected to control the extent of the modification. These ideas have been developed by a number of workers (see Brunt, 1939, p.242) and the general belief now is that the stability of the atmosphere is determined by the following number, usually termed the Richardson Number.

$$\frac{\frac{g}{T}\left(\frac{\partial T}{\partial z} + \Gamma\right)}{\left(\frac{\partial u}{\partial z}\right)^2}$$

$\frac{\partial T}{\partial z}$ being the vertical temperature gradient,
z being measured upwards,

/the

$\frac{\partial u}{\partial z}$ the vertical gradient of wind speed,

T the absolute temperature,

g the acceleration due to gravity,

Γ the adiabatic lapse rate.

In considering the grouping of the present results according to atmospheric stability a convenient practical expression of the Richardson Number was finally employed in the form

$$\frac{T_{200} - T_{25}}{(u_{200} - u_{25})^2}$$

The numerical values of this parameter are given in Fig. 4, for each hour except where in very low wind speeds anemometers did not perform satisfactorily and the denominator of the above expression could not be computed. The numerical factor incorporated in the graphical reproduction of this parameter is due to the fact that values were originally calculated for wind speeds in units of ft./sec. whereas for consistency units of $^{o}F.\ sec.^2/m.^2$ have been inserted on the scale. The sign of the parameter is of course determined entirely by the vertical temperature difference and the familiar diurnal variation of the sign of the latter is apparent.

28. A number of additional practical features controlled the final grouping. To begin with a number of the observations were made in wind directions between N. and E., in which directions the effects of upwind obstacles are thought to be important. In the main stage of the present analysis these observations (amounting to about 30) were discarded. Again, it was considered undesirable to attach significance to the forms of the profiles in which the vapour pressure gradients were of a relatively low order, and in this respect no account was taken of observations when the difference in vapour pressure at 200 and 100 cm. was less than 0.12 mm. Hg. On two occasions observations were discarded because the profile was clearly near the stage of change over from lapse to inversion of vapour. Finally, as will be seen from Fig. 4, only a relatively small proportion of the observations show an inversion of vapour, and furthermore these are usually associated with incomplete wind data. Such observations have also been segregated. The discarding and segregating of the above observations left about 60 observations representing some 40% of the total, for the principal considerations of the profiles. After some inspection it was decided that, in the interests of eliminating apparently random variations a division into more than three categories would be undesirable, and an approximately equal division into three groups was found to be given by the following limits of the stability parameter adopted; the units being $^{o}F.\ sec.^2/m.^2$

'Unstable' conditions	$\leqslant$	$-\ 0.22$		
'Near-adiabatic' conditions	$>$	$-\ 0.22$	$<$	$+\ 0.22$
'Stable' conditions	$\geqslant$	$+\ 0.22$		

the corresponding broad descriptions being adopted henceforward. No attempt was made to examine the effects of the changes of surface roughness occurring over the duration of the experiment.

The Form of the Profiles.

29. After division into the above groups the observations of wind speed, vapour pressure and temperature were considered together with the object of choosing a common method of expressing the vertical profiles of the three different elements. Elimination of the different boundary conditions and the different systems of units was achieved by expressing the differences over the various layers in terms of the corresponding difference over one standard layer, and the parameter originally adopted was

$$\frac{s_{200} - s_z}{s_{200} - s_{100}}$$

s_z being the temperature, wind speed or vapour pressure at any height and the numerical subscripts referring to the heights in cm. Values of this parameter were evaluated for each individual 15-minute observation and mean values were then obtained for each group of observations. This procedure was adopted so as to avoid attaching inappropriate weight to extreme absolute values of s. After a preliminary examination these mean values were reduced to the form

$$\frac{s_{25} - s_z}{s_{25} - s_{50}}$$

as it was clear that a certain feature of the profiles (see below) would be the more clearly illustrated. The essential features of this stage of the analysis are summarised in the attached Table 1. and the profiles are shown graphically in Fig. 5 where the values of the above parameter are plotted against height on a logarithmic scale. In addition to the three main groups the values corresponding to the occasions when an inversion of vapour existed are given and some of those when, with 'unstable' conditions, the wind direction was between N. and E. have been included for interest. Furthermore, to facilitate use of the above data in alternative methods of analysis, values of s_{200} and $s_{25} - s_{50}$ have been meaned for the appropriate groups and are reproduced in Table 2. From Tables 1 and 2 equivalent mean absolute values of wind speed, temperature and vapour pressure may be derived as necessary.

30. Considering first the three main groups it is immediately apparent that all profiles approximate to a logarithmic form. Taking the wind profile under near adiabatic conditions as a reference basis, it is found that in this case the best fit to the points is given by an equation

$$\frac{s_{25} - s_z}{s_{25} - s_{50}} = \frac{\log (25 - d) - \log (z - d)}{\log (25 - d) - \log (50 - d)}$$

with d having the value 3.5 cm. The physical significance of this variation will be noted at a later stage and at the moment it is merely to be looked upon as the equation of a curve against which the remaining profiles will be examined. With this object the curve appropriate to this equation has been drawn in over each set of points. From previous measurements (e.g. Best, 1935) a variation of the form of the wind profile with vertical temperature gradient is to be expected, and in the attached table the variations of the quantity

$$\frac{u_{200} - u_{100}}{u_{200}}$$

are in the anticipated direction. However, examining the present logarithmic representation of the wind profile for effects of atmospheric stability as defined in para. 27 the only systematic change which can be detected is restricted to the 200 cm. value, where in the 'unstable' case this value would imply a greater concavity (upwards) of the curve best fitting the points, and a reverse effect in the 'stable' case. As far as the 100 cm. point is concerned there is no suggestion of any effect of this 'stability' and indeed this is most strikingly shown by the tabulated values, where the value of the quantity

$$\frac{u_{25} - u_{100}}{u_{25} - u_{50}}$$

is 1.89, 1.89 and 1.87 for the 'unstable', 'near adiabatic' and 'stable' cases respectively. Turning now to the temperature and vapour profiles it is seen

/that

that in the range 25-200 cm. no systematic departure from the reference curve is exhibited, except apparently in the 'near adiabatic' case of the vapour profile. However the reference curves have been drawn in so as to give an almost exact fit at the 25 and 50 cm. levels and it is clear that only a slight displacement of the curve away from the 50 cm. point would provide a fit to the whole profile commensurate with that demonstrated in the other cases. In other words it would appear that except for a systematic though slight effect on the wind profile in the region of 200 cm., the wind, temperature and vapour profiles in the region 25-200 cm., in the range of the present main groups of observations, can all be represented by a single equation, for the range of temperature gradient and wind speed involved in the present groups of observations.

31. For the observations gathered in Group 4 under the heading "Stable - Vapour Inversion" the incompleteness of wind speed data prohibited the numerical evaluation of the stability parameter. There seems little doubt, however, that the values would have been, if anything, higher than those defining the 'stable' category, and these observations may safely be regarded as appropriate to well established "stability". Even here, however, there is no firm indication that the temperature and vapour profiles differ from the reference profile. The final group, classified as "Unstable - Obstacles Upwind" gives a vapour profile which is in remarkable agreement with the reference profile, but the group contains only four observations and it is doubtful whether any reliable inference can be drawn as to the effect of nearby obstacles. Owing to the absence of wind speed data below 25 cm. a discussion on similar terms of the profiles below 25 cm is not possible. The position here is somewhat obscure, for while the two temperature profiles and the vapour pressure profile for the 'unstable' case conform to a fairly natural extrapolation of the reference curve, the same is by no means true of this section of the vapour profile in the 'near adiabatic' and 'stable' cases. If the experimental data is accepted then it would mean that some departure from universality in the profile relationship occurs as the ground is approached.

EVALUATION OF THE EDDY DIFFUSIVITY AND VERTICAL FLUX OF WATER VAPOUR AND CONSIDERATION OF WATER BALANCE.

32. In the following considerations a knowledge of the wind velocity profile is fundamentally necessary and it is now proposed to deal only with the layer 25 - 200 cm. in which this knowledge is directly available. For all but the top of this layer the foregoing examination of the wind and vapour profiles detects no substantial difference in the profiles according as wind speed, temperature or vapour content are being considered and no systematic effect of changes of atmospheric stability as defined here. The significance of this may be seen as follows. The profile equation given in para 30 leads to

$$\frac{\partial s}{\partial z} \propto \frac{1}{z - d}$$

and the present analysis suggests that to a good approximation d is the same for momentum, temperature and absolute humidity and is unaffected by substantial variations in the quantitative criterion of atmospheric stability adopted here. With the customary assumption of constancy with height of the vertical turbulent flux (and the validity of this with respect to vapour transfer will be reconsidered in the final paragraph of this paper) it follows that the variation with height of the eddy diffusion coefficient is given by

$$K \propto \frac{1}{\partial s/\partial z} \propto (z - d)$$

It therefore follows that as far as the variation with height is concerned eddy viscosity, conductivity and diffusivity are identical, though no evidence is thereby provided for the relation between the absolute magnitudes of these quantities. With this limitation the present analysis provides some support for the customary assumption that the vertical exchange of momentum and matter are determined by

/the

the same coefficient of diffusion. It also provides support for the idea of negligible effect of atmospheric stability on the processes of turbulent transfer in a shallow layer near the ground, an idea which has been discussed previously by Sverdrup (1938) for the case of stable conditions over a snow surface.

33. Assuming now that the eddy viscosity and eddy diffusivity are identical both formally and numerically and that in the range of the present observations neither is affected by changes in atmospheric stability, the vertical flux of vapour, which with the assumed constancy of flux with height is of course identical with rate of evaporation from the ground, is given by the product of the eddy viscosity in 'adiabatic' conditions and the vertical gradient of water vapour content at a specified height. The latter is given directly by the present measurements while it is proposed to calculate the eddy viscosity from the wind profile by the method mentioned briefly in the introduction. In order to avoid the difficulties imposed by surface roughness in the application of Sutton's method, the direct extension of Prandtl's method and the laboratory studies of turbulent boundary layer flow will be adopted. From this we have the expressions which are now familiar in problems of the present type (e.g. Brunt, p.247), i.e.

$$u = \sqrt{\left(\frac{\tau_o}{\rho}\right)} \frac{1}{k} \log_e \frac{z - d}{z_o}$$

$$K = \frac{u_1 k^2 (z - d)}{\log_e \left(\frac{z_1 - d}{z_o}\right)}$$

where

u represents the wind velocity at a height z,
u_1 represents the wind velocity at a height z_1,
τ_o is the horizontal turbulent shearing stress,
ρ the density of the air,
k the so-called von Kármán's constant,
K the eddy viscosity,
d is a 'zero displacement',
z_o is called the 'roughness parameter' of the surface.

34. The 'zero displacement' is introduced when, as is usual, the height z is measured from a surface on which there are disposed more or less solid elements which constitute the roughness of the surface. For obvious reasons this quantity is more specifically defined in the case of the uniform artificial roughness elements occurring in the laboratory study of the flow over rough surfaces than in the case of natural ground surfaces. In the latter case there seems to be no universal rule for estimating the magnitude of d from the appearance of the surfaces. Fortunately it is readily derived from wind speed observations at three or more heights and indeed it has already been given in the present case as 3.5 cm. The 3.5 cm. is thus to be regarded roughly as the amount by which the effective boundary surface is raised by the vegetation cover and it will be noted that the numerical value is on these terms consistent with the previous description of the vegetation cover.

35. It is perhaps noteworthy that the solution of the above wind profile equation to give the value of d is carried out graphically, having first computed the value of

$$\frac{u_1 - u_2}{u_1 - u_3} = \frac{\log (z_1 - d) - \log (z_2 - d)}{\log (z_1 - d) - \log (z_3 - d)}$$

for a range of values of d. Knowing d the value of $\sqrt{\left(\frac{\tau_o}{\rho}\right)} \cdot \frac{1}{k}$ and thence that of the roughness parameter z_o may be obtained directly from the wind velocities at two heights. The remaining quantity, k,

/which

which is required for evaluation of the eddy diffusivity is given by laboratory experiment as 0.4 (Nikuradse, loc. cit.) and experimental evidence for the validity of adopting this value in the lower atmosphere has been discussed by Sheppard (1947). The following values of d, z_o and K (the latter at a height of 50 cm.) have been computed for two cases, in the first of which, (a), the whole 'near adiabatic' wind profile is employed, while in the second, (b), the profile in the range 25-100 cm. only is used.

	(a)	(b)
d (cm.)	3.5	6.8
z_o (cm.)	0.80	0.46
K_{50} (cm.^{2}/sec.)	1.83 x U_{50}	1.52 x U_{50}

It will be noted that although the values of 'd' and 'z_o' are substantially changed, and that the second value of 'd' seems to be less in keeping with the physical appearance of the surface as described previously, the resulting value of the eddy viscosity is not greatly altered.

36. For the present purposes a rounded mean of the two values of the eddy viscosity has been employed, and the upward vertical turbulent flux of vapour is thus given, in c.g.s. units. as

$$- 1.65 \times \left(u \frac{dq}{dz} \right)_{50}$$

q being the absolute humidity. Strictly speaking the vertical flux should be expressed in terms of the gradient of specific humidity and the present expression then follows by neglecting the variation of air density with height. Values of the humidity gradient have been evaluated graphically from the observations of the vapour pressure profiles corresponding to each hour, variations of absolute temperature being neglected and a standard value of 288°A. being adopted as sufficiently accurate for the present purposes. The hour by hour variation of these values is shown graphically in Fig. 6 together with the associated values of the wind speed at a height of 50 cm., and the resulting magnitudes of the vapour flux. The diurnal variations of wind speed and humidity gradient are seen to be well marked and in phase, with the result that the vapour flux, or rate of evaporation, exhibits a most striking diurnal variation, with a maximum in the middle of the day and a very small positive or negative value (the latter associated with dew formation) during the night.

37. In order to provide a check on the computed values of the rate of evaporation, the water content of the soil down to a depth of 3 feet was measured at the beginning and at the end of the present series of observations. Since there is no water table in the Gault clay on which this work was carried out then the 'water balance' is expressed by the equation

Rainfall + Dew - Evaporation = Gain of Soil Moisture

and theoretically it should therefore be possible to verify the computed values of dew and evaporation integrated over a sufficiently long period. The soil moisture measurements, undertaken by the staff of the School of Agriculture, comprised on each occasion the taking of samples with an 'undisturbed core' sampler and also with an auger. This sampling removed soil 1" at a time down to 1 ft. and then 3" at a time down to 3 ft. and this procedure was carried out at six positions on each occasion. It should perhaps be explained that the 'undisturbed core' sampler was specially designed and used in order that ultimately the heat capacity per unit volume could be evaluated for use in other considerations as well as for the primary purpose of estimating moisture content per unit volume of soil. The final results of the soild moisture measurements expressed in the form of the water loss from various layers over a horizontal area of 15.9 sq. cm., i.e. the cross section of the sampler, are given below.

/Soil

Soil layer (inches)	Water loss in gm. from 15.9 sq. cm.
0 - 1	0.885
1 - 2	- 0.430
2 - 3	1.216
3 - 4	0.540
4 - 5	0.567
5 - 6	0.643
6 - 7	1.722
7 - 8	- 0.421
8 - 9	0.367
9 - 10	0.063
10 - 11	2.245
11 - 12	- 0.640
12 - 15	- 0.528
15 - 18	0.532
18 - 21	- 2.99
21 - 24	- 4.73
24 - 27	- 7.02
27 - 30	- 5.04
30 - 33	- 5.35
33 - 36	- 1.24

38. From previous knowledge of the moisture profile in soil of the present type the changes in moisture content below about the 18" level are viewed with some suspicion. It is thought that they are probably at least partly due to a soil heterogeneity, unsuspected before sampling began, causing point to point variation beyond the capacity of the randomisation technique. The total water loss has therefore been worked out in two cases, in the first of which all changes below 18" are disregarded while in the second the changes between 18 and 36 inches have been accepted as an increase in the 'datum level' of soil moisture of approximately 4.5 gm. per 3" depth. The two methods give total water losses of

0.43 and 2.16 gm./sq. cm.

respectively, neither of which is statistically significant. Integrated values of the evaporation and condensation as given in Fig.6 over the whole period of the trial, and the total rainfall, are as follows:

Evaporation	1.40 gm./sq. cm.
Condensation	0.02 gm./sq. cm.
Rainfall	0.13 gm./sq. cm.
Net loss of water	1.25 gm./sq. cm.

The above computed net loss of water is to be compared with the loss given by the soil sampling and on first sight it is apparent that a very reasonable measure of agreement holds. However, as already mentioned, the change in moisture content given by the soil sampling is statistically insignificant, so that strictly speaking the agreement lies in the fact that both the computed and measured water losses are insignificant in terms of the accuracy of the soil sampling method. The test cannot therefore be regarded as final and conclusive though there is thought to be some justification for regarding an agreement in order of magnitude as established. It is clear, of course, that the computed water loss is dominated by the daytime values of the vapour flux and any agreement which is established is concerned largely with the conditions then obtaining.

39. Having confirmed the order of magnitude of the vertical flux of vapour it is of interest now to consider how far the previous assumption of constancy of

/flux

flux with height is justified. Assuming no processes of advection the equation representing the diffusion of water vapour is

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial q}{\partial z} \right)$$

This merely expresses that the rate of change in moisture content of the air in a certain horizontal layer of the atmosphere is equal to the difference in the values of the vertical flux at the upper and lower boundaries of the layer. Examining the temporal variations of vapour pressure shown in Fig. 4 it is seen that during the present experiment the maximum rate of change of vapour pressure was of the order of 1mm. Hg. per hour. In terms of a layer of the atmosphere up to 2m. this would correspond to a change in vapour flux of the order of $5.\ 10^{-8}$gm./sq. cm. sec., which is of the same order as the minimum vapour flux computed i.e. 10^{-7}gm./sq.cm.sec. From Fig. 6 it is readily seen that over the majority of the period of this experiment the computed vapour flux is greatly in excess of these values and in the middle of the day it normally attains values of the order of 100 times greater. In general, therefore, it would seem that the constancy of the vertical flux of vapour is justified as far as the first few metres of the atmosphere is concerned, and it is only on the occasions of almost zero vapour gradient or at the time of dew formation that anyrsubstantial departure from this is to be expected. This point is of some interest apart from its justification of a customary assumption, since on it may be measured the possibility of obtaining a numerical solution of the diffusion equation. Theoretically such a solution is possible by adopting some functional variation of K with height, and computing the relative magnitudes of the vapour flux at three levels, the differences being equated to the changes in moisture content of the air between the levels. Clearly the possibility of accomplishing this must decrease the more closely the constancy of flux holds, and an examination of the present data on these grounds has so far offered little hope of success in this direction.

G 171909

Stability classification	No. of obs.	Stability parameter (i) Range (Mean) σ_T^2/m^2 sec	u_2 Ra (M m/
1 Unstable	19	-0.24 to -1.94 (-0.62)	2.
2 Near adiabatic	20	-0.17 to +0.18 (-0.01)	2.
3 Stable	18	+0.24 to 3.50 (+1.24)	1.
4 Stable - vapour inversion	9		0.
5 Unstable - obstacles upwind	4	-0.22 to -0.94 (-0.51)	3.

Notes (i) See para.
(ii) Subscript
(iii) Temperatu
(iv) Wind prof of the

G.171909 (d)

36.
type
some
soil
varia
loss
bala
have
4.5

resp
of t
the

The above computed net loss of water is to be compared with the loss given by the soil sampling and on first sight it is apparent that a very reasonable measure of agreement holds. However, as already mentioned, the change in moisture content given by the soil sampling is statistically insignificant, so that strictly speaking the agreement lies in the fact that both the computed and measured water losses are insignificant in terms of the accuracy of the soil sampling method. The test cannot therefore be regarded as final and conclusive though there is thought to be some justification for regarding an agreement in order of magnitude as established. It is clear, of course, that the computed water loss is dominated by the daytime values of the vapour flux and any agreement which is established is concerned largely with the conditions then obtaining.

39. Having confirmed the order of magnitude of the vertical flux of vapour it is of interest now to consider how far the previous assumption of constancy of

/flux

INDEX

	Paragraph
Introduction	1 - 5
Experimental and Analytical Aspects	
The Site	6
Instrumental Equipment	7 - 16
Observational Procedure	17 - 20
Reduction of Data	21 - 22
General Features of Ground and Meteorological Conditions	23 - 25
Discussion of the Vertical Profiles of Vapour Pressure, Wind Speed and Temperature	
Grouping of Results	26 - 28
The form of the Profiles	29 - 31
Evaluation of the Eddy Diffusivity and Vertical Flux of Water Vapour and Water Balance Considerations	32 - 39

References

Tabular Summaries of Profile Data

Diagrams

1. Sectional Representation of Thermocouple Psychrometer.
2. Characteristics of Thermocouple Psychrometer.
3. Variation of Upper Air Vapour Pressures and Wind Direction and of Surface Wind Direction.
4. Vapour Pressures at Heights of 200 and $6\frac{1}{4}$ cm. and Stability Parameter.
5. Universal Representation of Profiles.
6. Humidity Gradient, Wind Velocity, and Vertical Flux of Water Vapour at a height of 50 cm.

Photograph

G.171909(a)

Stability classification	No. of obs.	Stability parameter (i) Range (Mean) σ_T sec^2/m^2	u_{20} Rar (Me m/s
1 Unstable	19	-0.24 to -1.94 (-0.62)	2.((
2 Near adiabatic	20	-0.17 to +0.18 (-0.01)	2.((
3 Stable	18	+0.24 to 3.50 (+1.24)	1.((
4 Stable- vapour inversion	9		0.((
5 Unstable- obstacles upwind	4	-0.22 to -0.94 (-0.51)	3.((

Notes (i) See para.
(ii) Subscripti
(iii) Temperatu
(iv) Wind prof
of the

G.171909 (d)

38.
type
som
soil
vari
less
bela
have
4.5

resp
of t
the

The above computed net loss of water is to be compared with the loss given by the soil sampling and on first sight it is apparent that a very reasonable measure of agreement holds. However, as already mentioned, the change in moisture content given by the soil sampling is statistically insignificant, so that strictly speaking the agreement lies in the fact that both the computed and measured water losses are insignificant in terms of the accuracy of the soil sampling method. The test cannot therefore be regarded as final and conclusive though there is thought to be some justification for regarding an agreement in order of magnitude as established. It is clear, of course, that the computed water loss is dominated by the daytime values of the vapour flux and any agreement which is established is concerned largely with the conditions then obtaining.

39. Having confirmed the order of magnitude of the vertical flux of vapour it is of interest now to consider how far the previous assumption of constancy of

/flux

TABLE 1

SUMMARY OF PROFILE DATA

Stability classification	No. of obs.	Stability parameter (i) Range (Mean) $^{o}F\ sec^2/m^2$	u_{200} Range (Mean) m/sec.	$\frac{u_{200}-u_{100}}{u_{200}}$ (ii)	$\frac{u_{25}-u_z}{u_{25}-u_{50}}$ for heights 200	100	$T_{200}-T_{100}$ ^{o}F	$\frac{T_{25}-T_z}{T_{25}-T_{50}}$ for heights 200	100	$12\frac{1}{2}$	$6\frac{1}{4}$	$e_{200}-e_{100}$ mm.Hg	$\frac{e_{25}-e_z}{e_{25}-e_{50}}$ for heights 200	100	$12\frac{1}{2}$	$6\frac{1}{4}$
1 Unstable	19	-0.24 to -1.94 (-0.62)	2.2 - 6.0 (3.7)	0.119	2.75	1.89	-0.44	2.87	1.77	-1.20	-2.33	-0.41	2.75	1.88	-0.90	-2.06
2 Near adiabatic	20	-0.17 to +0.18 (-0.01)	2.5 - 8.6 (4.9)	0.129	2.80	1.89	(iii)					-0.33	2.61	1.70	-0.73	-1.48
3 Stable	18	+0.24 to 3.50 (+1.24)	1.5 - 4.8 (2.9)	0.144	2.85	1.87	+0.39	2.85	2.05	-0.97	-2.16	-0.25	2.87	1.77	-0.74	-1.50
4 Stable-vapour inversion	9		0.5 - 1.7 (1.0)	(iv)			+1.60	3.03	2.10	-0.73	-1.17	+0.14	2.74	2.09	-0.58	-1.38
5. Unstable-obstacles upwind	4	-0.22 to -0.94 (-0.51)	3.7 - 4.5 (4.1)	0.121	2.60	1.82	-0.46	2.84	1.81	-1.31	-2.33	-0.37	2.79	1.91	-1.18	-2.81

Notes (i) See para. 27.
(ii) Subscripts refer to heights in centimetres.
(iii) Temperature gradients too small to be used in this connection.
(iv) Wind profile data incomplete due to low wind speeds and failure of one or more of the anemometers.

TABLE 2

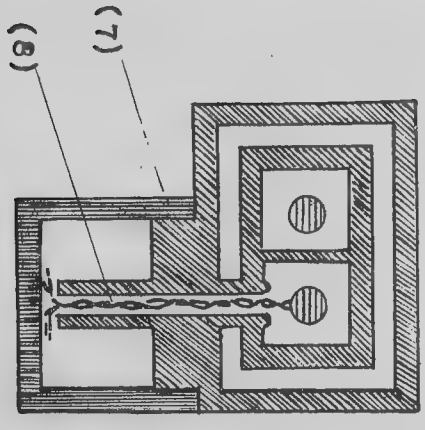
Values of S_{200} and $S_{25} - S_{50}$

Stability Classification	U_{200} Range (Mean) m/sec.	$U_{25} - U_{50}$ m/sec.	T_{200} Range (Mean) °F	$T_{25} - T_{50}$ °F	C_{200} Range (Mean) mm. Hg.	$C_{25} - C_{50}$ mm. Hg.
1. Unstable	2.2 - 6.0 (3.7)	-0.51	52.9 - 74.3 (64.4)	+0.40	7.37 - 13.36 (9.27)	+0.47
2. Near Adiabatic	2.5 - 8.6. (4.9)	-0.69			6.93 - 13.23 (9.93)	+0.36
3. Stable	1.5 - 4.8 (2.9)	-0.43	55.3 - 71.2 (63.3)	-0.49	6.91 - 11.20 (9.47)	+0.23
4. Stable - Vapour Inversion	0.5 - 1.7 (1.0)		46.3 - 56.6 (51.2)	-1.72	7.58 - 9.03 (8.15)	-0.22
5. Unstable - Obstacles Upwind	3.7 - 4.5 (4.1)	-0.64	64.7 - 67.5 (66.5)	+0.45	8.41 - 8.76 (8.61)	+0.42

REFERENCES

Angstrom, A. 1920, Geog. Ann. 2:, 237 - 252.
Best, A.C., 1935, Meteorological Office Geophysical Mmeoir No.65.
Bilham, E.G., 1935, Q.J.R.Met.Soc., 61, 159 - 166.
Brunt, D., 1939, Physical and Dynamical Meteorology, 2nd Edn., Cambridge.
Calder, K.L., 1947, Unpublished.
Cummings, M.W., 1940, Amer.Geophys.Union Trans., 21, 512 - 522.
Dryden, H.L., 1936, Rep.Nat.Adv.Comm.Aero., Washington, No. 562.
Montgomery, R.B., 1940, Pap.Phys.Oceanogr.Met., Mass.Inst. Tech., 7, 4.
Nikuradse, J., 1932, Forschungsh.Ver.dtsch.Ing., No. 356.
Middleton, W.E.Knowles, 1943, Meteorological Instruments, 2nd Edn., University of Toronto Press.
Pasquill, F., 1943, Proc.Roy.Soc.A, 182, 75.
Schlichting, H. 1936, Ingenieur.Archiv., 7, 1.
Sutton, O.G., 1934, Proc.Roy.Soc. A, 146, 701.
1947, in course of publication in Q.J.R.Met.Soc.
Sheppard, P.A., 1940, J.Sci.Inst., 17, 218.
1947, Proc.Roy.Soc., A, 188, 208.
Sheppard, P.A. and Firesah, 1947, unpublished.
Spilhaus, A.F., 1936, Trans.Roy.Soc. S.Africa, 24, 185.
Sverdrup, H.U., 1936, Ann. d. Hydrogr., 64, 2.
1937, J. Marine Research, 1.
1938, 5th. Inter.Cong.App.Mech., 369.
1946, Journ. of Met., 3, No.1.
Thornthwaite, C.W., and Holzman, B., 1939, U.S.Dept.Agric.Tech.Bull. No. 817.

G.171909(b)



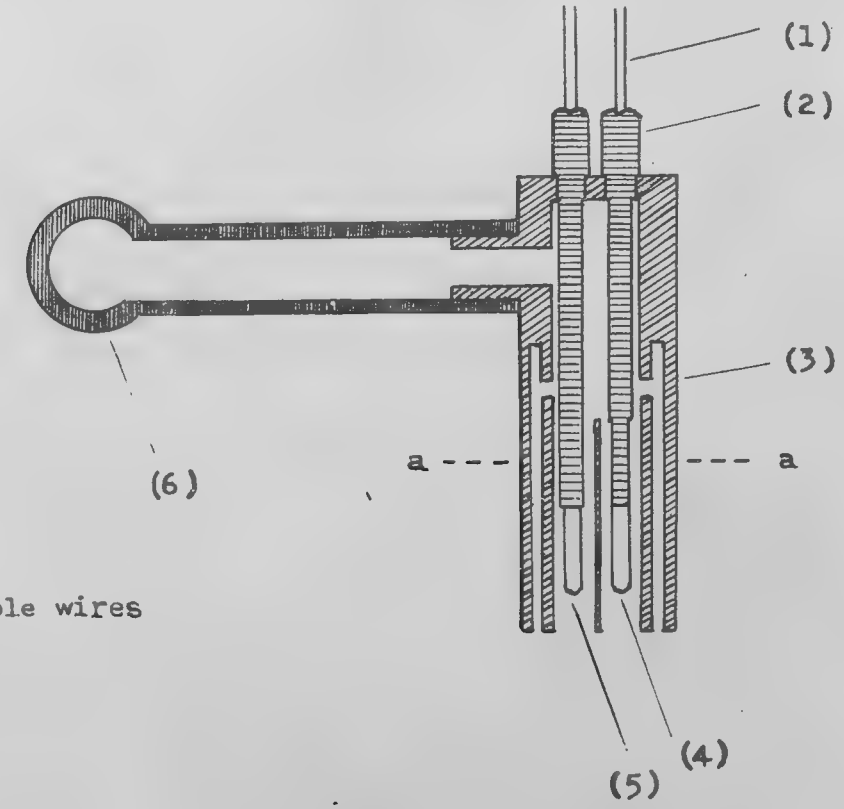
Vertical section at a....a (approx. full scale)

M.R.P. 362

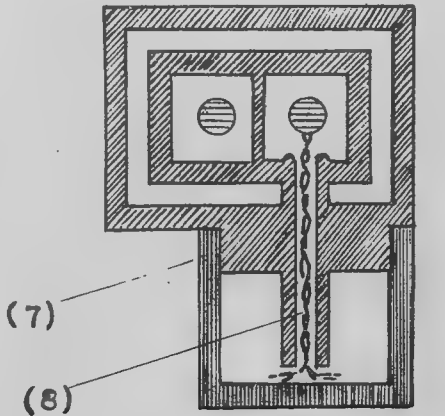
1/3963

KEY

(1) Tubes carrying thermocouple wires
(2) Stem of thermo-elements
(3) Double walled housing
(4) Wet bulb
(5) Dry bulb
(6) Common Aspiration Mast
(7) Water Chamber
(8) Cotton feed

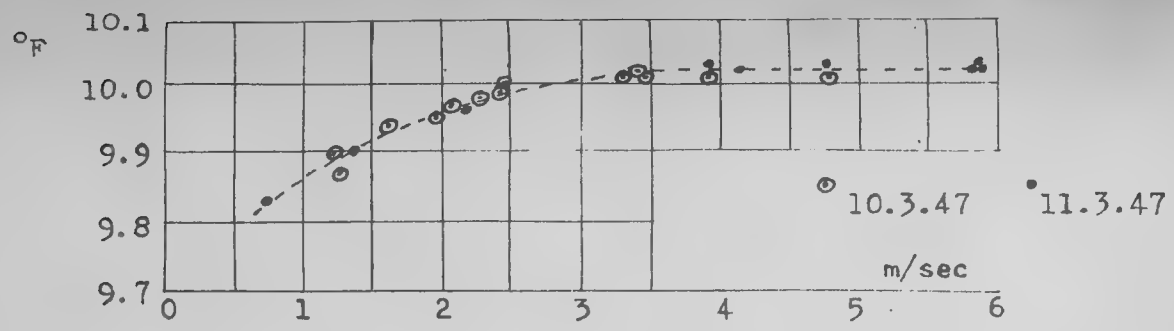


Horizontal section
(approx. half scale)

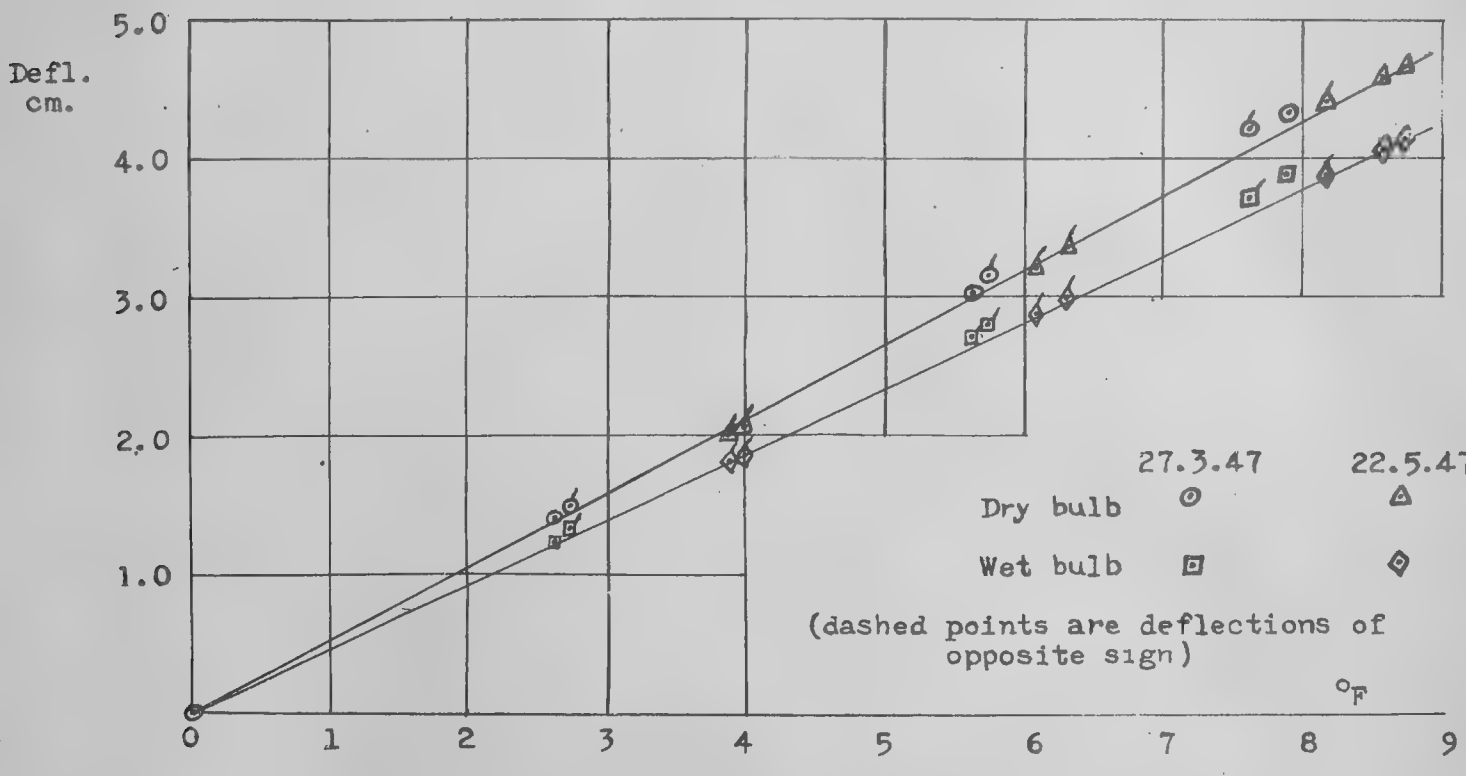


Vertical section
at a.....a
(approx. full scale)

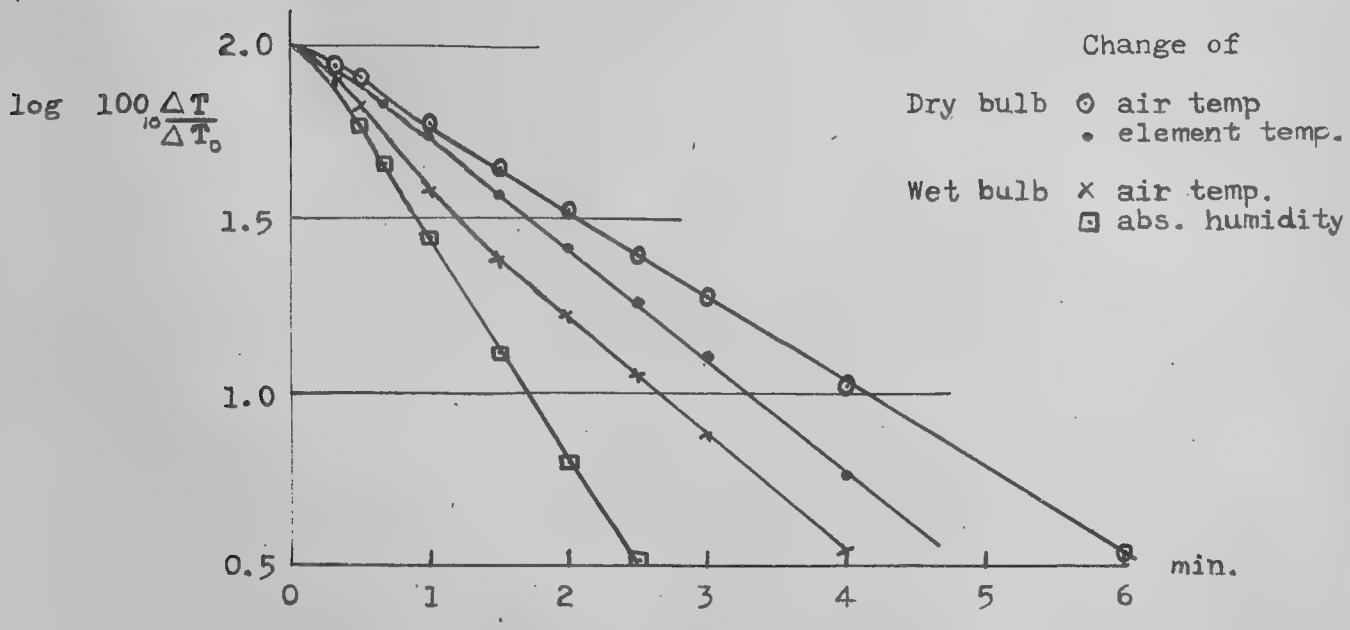
FIG. 1. SECTIONAL REPRESENTATION OF THERMOCOUPLE PSYCHROMETER



(a) Variation of Wet Bulb Depression with Aspiration



(b) Differential Temperature Sensitivity



(c) Lag characteristics (see para. 15)

FIG. 2. CHARACTERISTICS OF THERMOCOUPLE PSYCHROMETER

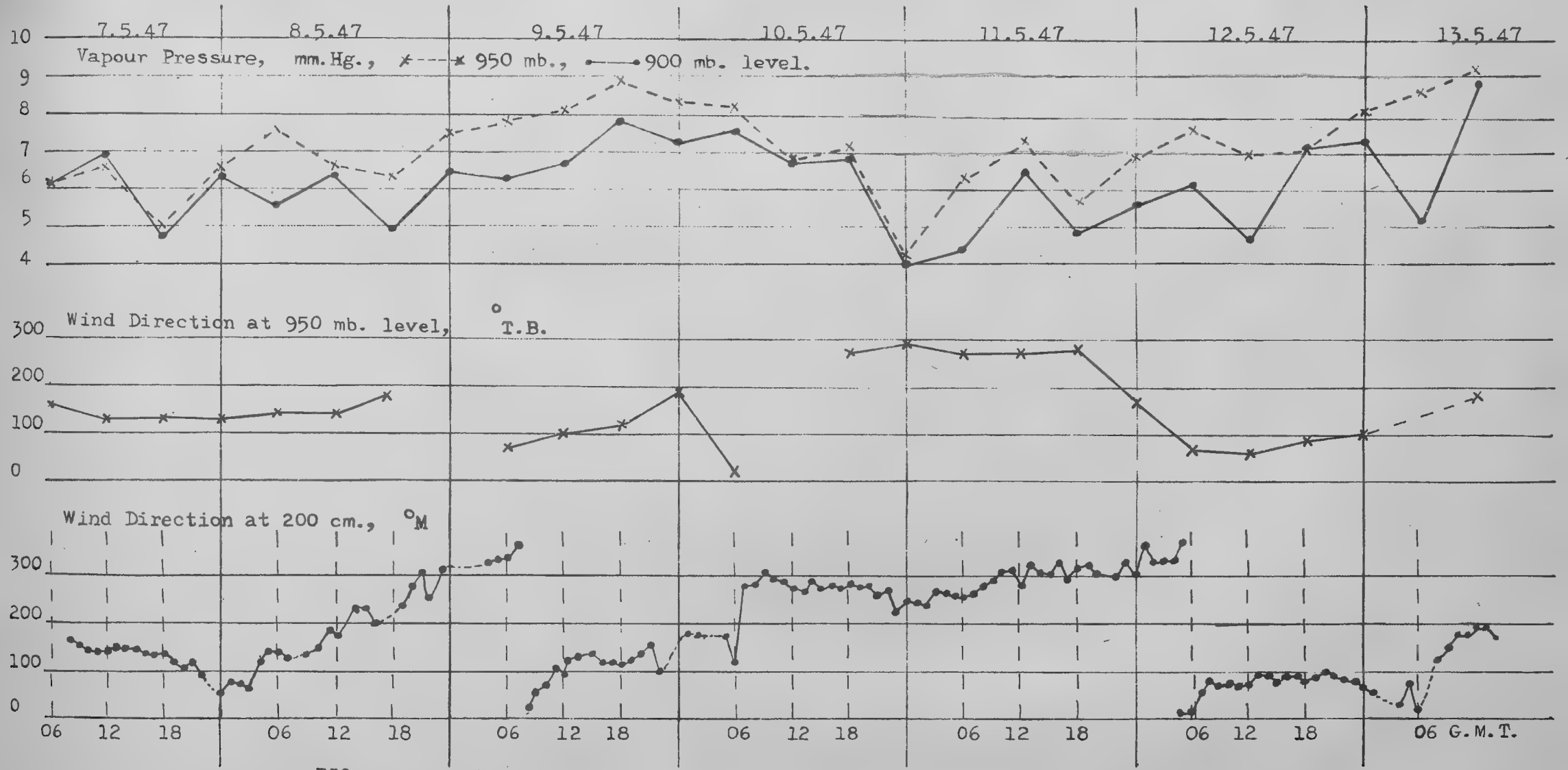


FIG. 3. VARIATION OF UPPER AIR VAPOUR PRESSURES AND WIND DIRECTION AND OF SURFACE WIND DIRECTION.

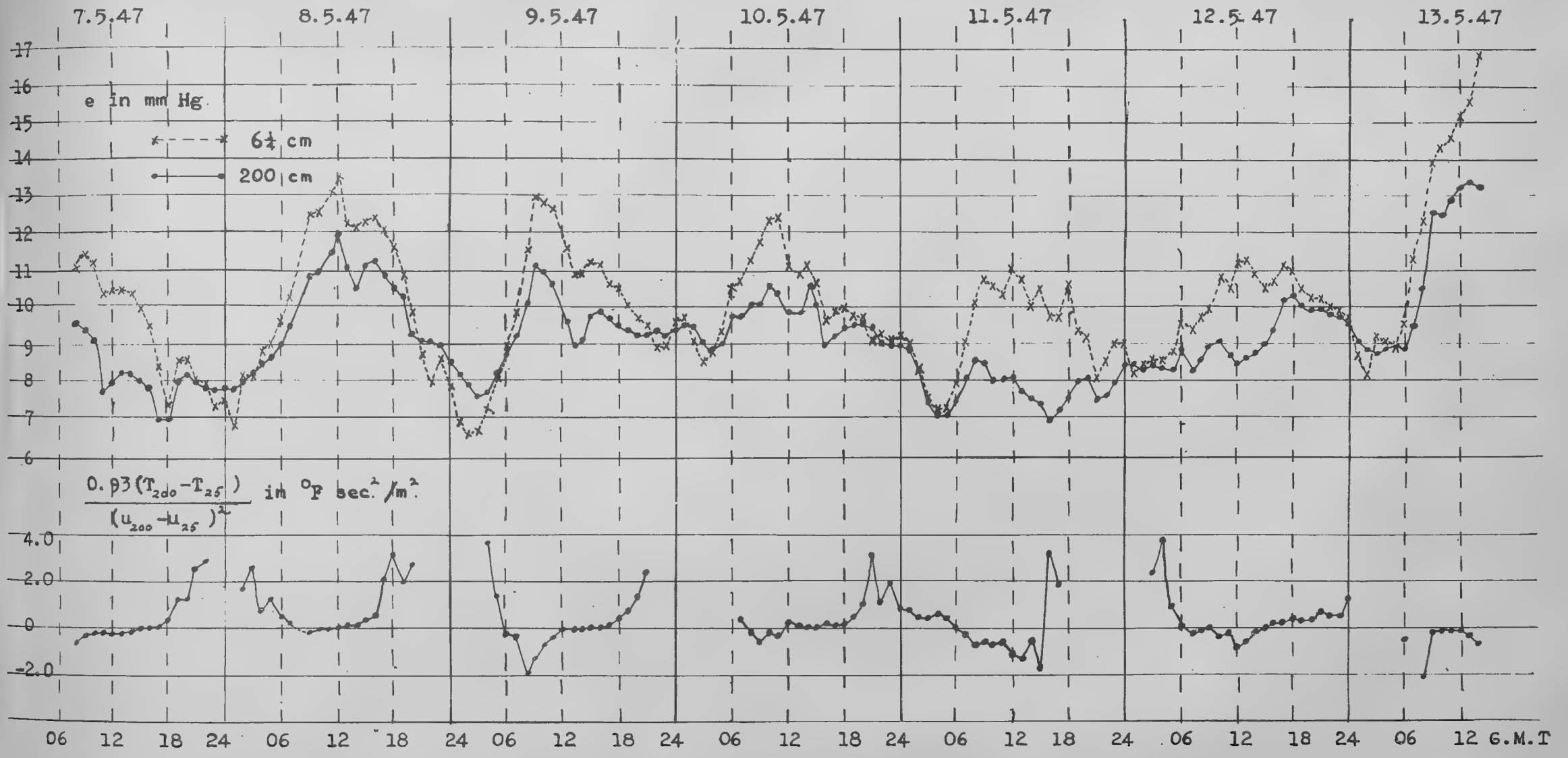


FIG. 4. VAPOUR PRESSURES AT 200 & 6¼ cm. AND STABILITY PARAMETER.

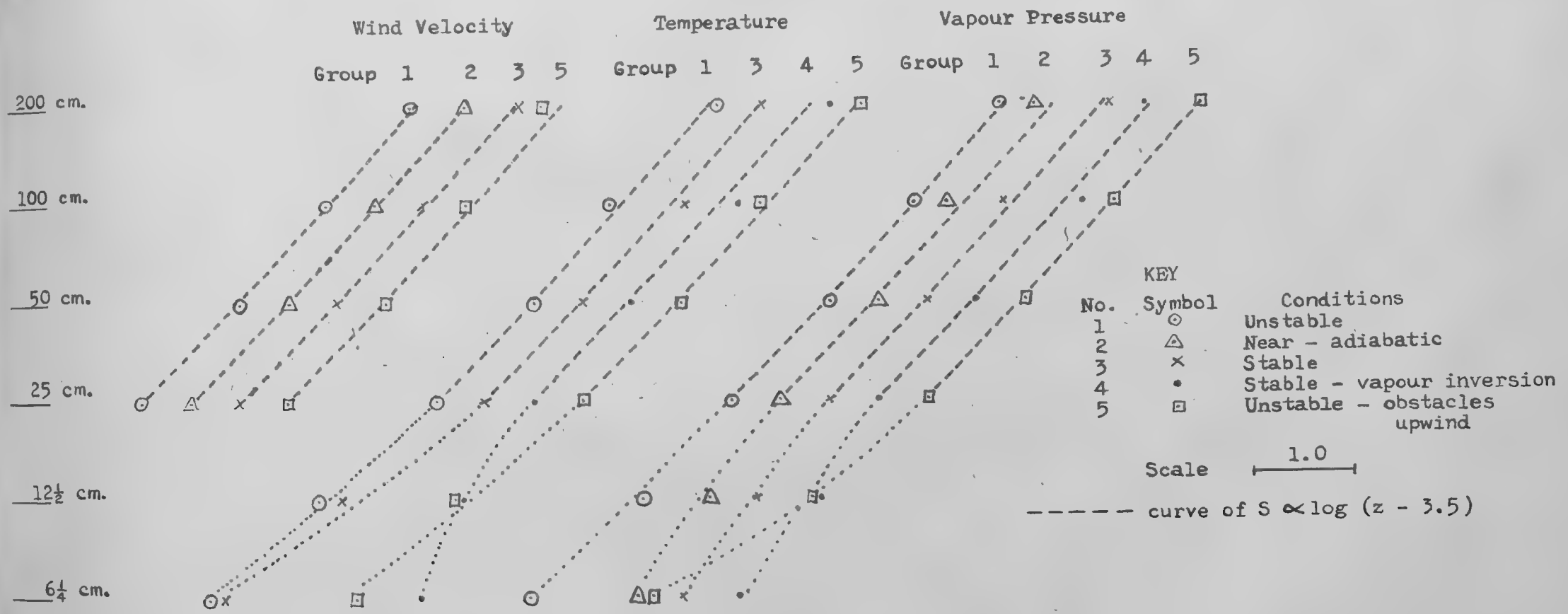


FIG. 5. UNIVERSAL REPRESENTATION OF PROFILES
(plot of $S_{25} - S_z / S_{25} - S_{6\cdot0}$ against logarithm of height, where S is wind velocity, temperature or vapour pressure)

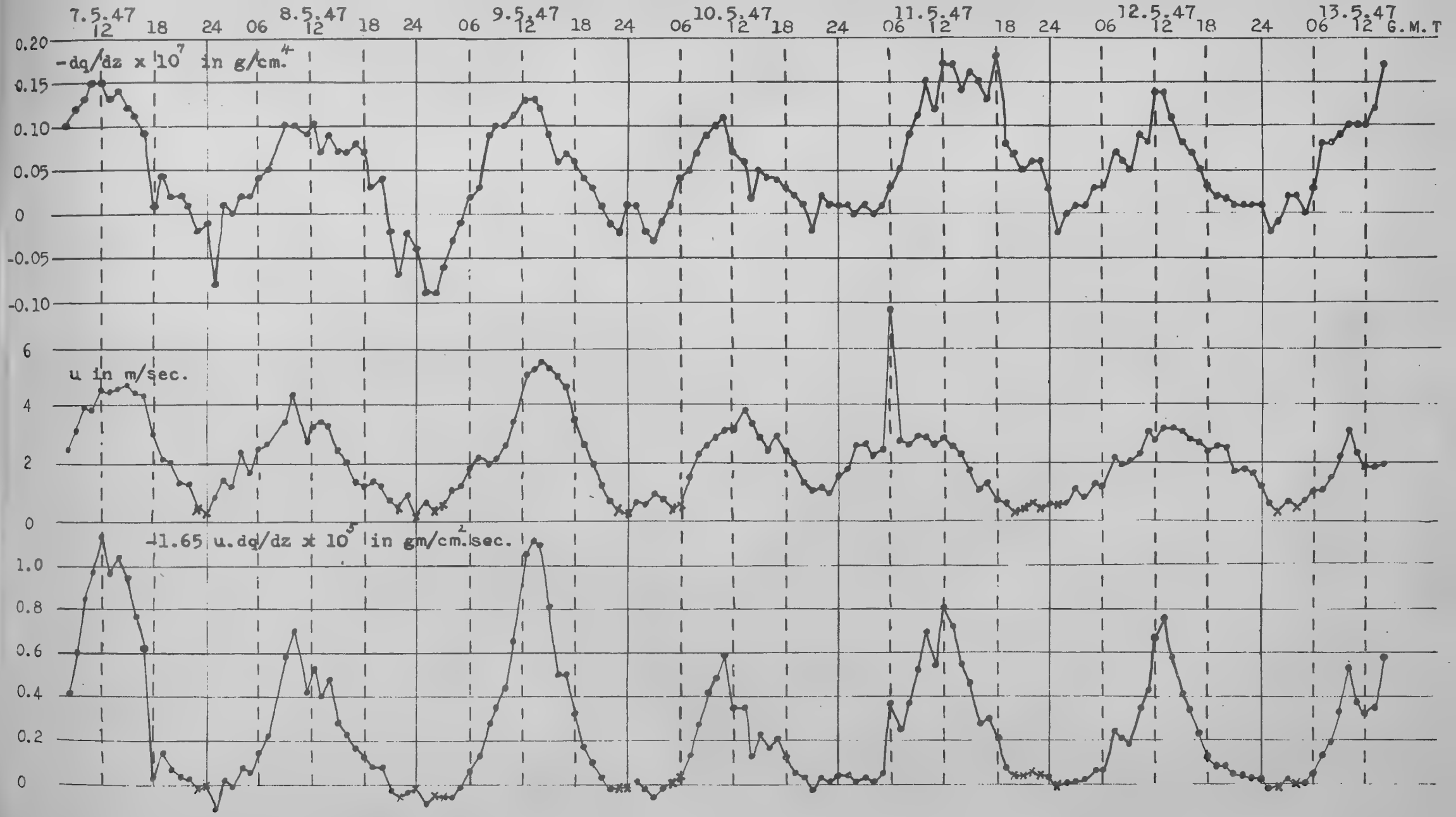
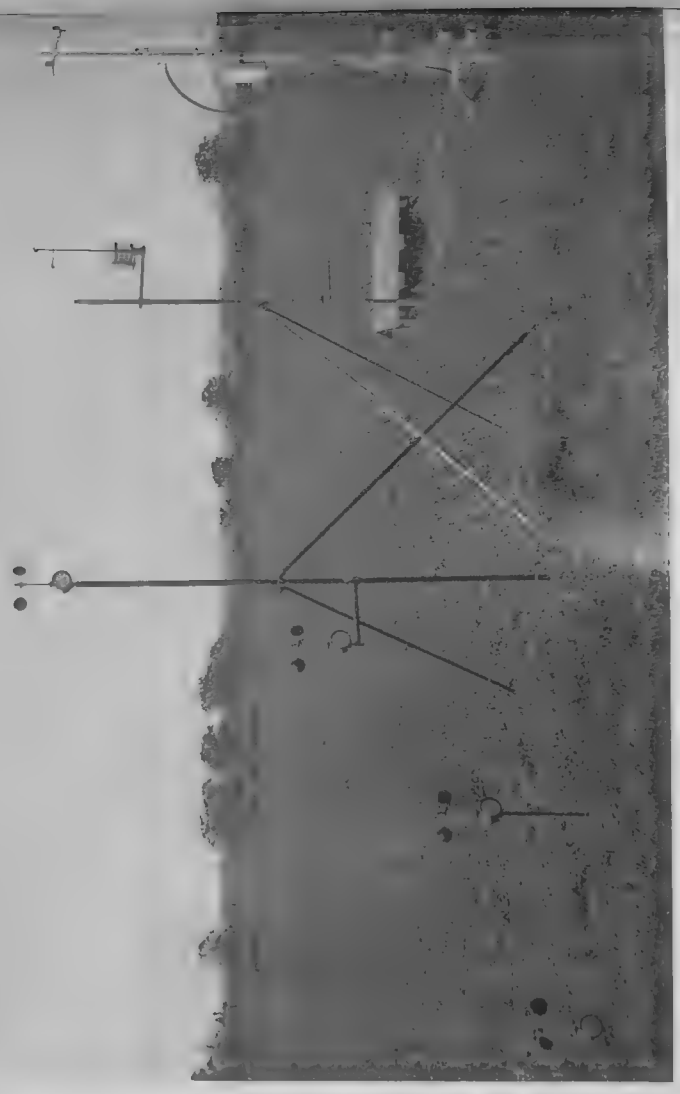


FIG. 6. HUMIDITY GRADIENT, WIND VELOCITY & VERTICAL FLUX OF WATER VAPOUR AT A HEIGHT OF 50 cm.

M.R.P. 363
30th September, 1947



AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

Data on the vertical profile of [illegible] above a clay-land [illegible] terms of humidity [illegible] edately above the grass cover.

as a convenient basis for interpretation of the [illegible] expression connecting the vapour profile and the rate of evaporation in [illegible] heights, only the humidity [illegible] former and the [illegible] value of the [illegible] a guide to the [illegible] the evaporation values corresponding [illegible]

[illegible] as of the method and its applications to [illegible]

[illegible] AND THEORETICAL ASPECTS.

[illegible] ficial flux of water vapour with height above ground [illegible] computed from the measured humidity gradient [illegible] integrated over a period of approximately an [illegible] confirmed in order of magnitude by measurements of [illegible] and be extended to a variety of surfaces. Subject [illegible] ich the measure[illegible] of the present treatment to ad[illegible]

This report is published with the permission of the Controller, H.M. Stationery Office, but the contents must not be quoted.

METEOROLOGICAL RESEARCH COMMITTEE

A TENTATIVE EVALUATION OF EVAPORATION FROM GRASSLAND IN TERMS OF TEMPERATURE MEASUREMENTS NEAR THE SURFACE.

BY F. PASQUILL

SUMMARY

Data on the vertical profiles of absolute humidity in the first metre above a clayland pasture have been discussed in an earlier paper. These are here reconsidered with the aim of deriving short term rates of evaporation in terms of humidity at one height and measurements of temperature in and immediately above the grass cover.

As a convenient basis for interpretation of the data an expression is derived on the lines of Sverdrup's treatment of evaporation from the sea. This expression prescribes the vapour profile and the rate of evaporation in terms of absolute humidity at the surface and at one height, eddy diffusivity as prescribed previously and the thickness of the laminar layer which is assumed to exist immediately above the surface.

In the absence of further data the surface value of the absolute humidity is identified, with some qualification, with the saturation values corresponding to the temperatures mentioned above. Furthermore, owing to the difficulty of defining the thickness of the laminar layer, an empirical approach is adopted from this point, and the above expression is shown to give encouraging agreement with the observed vapour profiles by adopting a laminar layer thickness which is consistent with a previous independent estimate. Similar agreement is also demonstrated between the rates of evaporation computed from the expression and those computed previously from the humidity gradient at some height above the ground surface. The limitations of the method and its applications to certain immediate requirements and to more general assessments of evaporation are briefly discussed.

INTRODUCTION AND THEORETICAL ASPECTS.

1. In a recent analysis of wind and humidity profile data (i) obtained over a clayland pasture some support was provided for the customary assumption of identity of eddy viscosity and eddy diffusivity, as well as for the concept of virtual 'adiabaticity' in the atmosphere near the ground and for the assumption of constancy of the vertical flux of water vapour with height above ground. Hourly rates of evaporation, computed from the measured humidity gradient and an eddy diffusivity based on the extension of Prandtl's wind profile treatment to the lower atmosphere, and integrated over a period of approximately one week, were regarded as confirmed in order of magnitude by measurements of the change in moisture content of the soil over the same period. It is clear that a closer confirmation of the computed rates of evaporation is desirable, particularly with regard to the considerable diurnal variation implied, and that the investigation should be extended to a variety of surfaces. Subject to these features, however, it seems likely that evaporation rates can be adequately prescribed by a knowledge of the vertical gradients of water vapour and of wind speed near the ground. Of these two factors the former is the more difficult to measure. Furthermore, it has not been investigated sufficiently widely to permit in practice an approximate evaluation in terms of more commonly or easily measured meteorological quantities. In the treatment of evaporation from the sea an attempt was made to overcome this difficulty by semi-theoretical treatments, in terms of which the measurement of humidity gradient in the air was replaced by measurement of sea surface temperature and of humidity at one specified height. It is the object of the present treatment to adapt similar methods to some aspects of land evaporation.

2. The theoretical approach adopted here as a convenient basis is a simple adaptation of the method first used by Sverdrup (1936) and developed by Millar (1937), Montgomery (1940) and again by Sverdrup (1946). This method postulates

/the

the existence of a thin layer next to the sea surface, in which turbulent exchange is absent, and above which turbulence is operative. In the case of an aerodynamically rough sea surface it is necessary to assume that the turbulent layer consists first of an 'intermediate' layer, in which the conditions of flow are appropriate to an aerodynamically smooth surface, and at the top of which the conditions change suddenly to those appropriate to aerodynamically rough flow. On physical grounds it seems reasonable to suppose that over any rough surface the vertical transport of vapour changes from that appropriate to molecular action at the surface to a fully developed turbulent exchange at some height at which the full effects of the roughness elements have come into play both as regards friction and the emission of water vapour. The above assumptions attempt to represent this transition in terms of specific layers.

3. Over a natural land surface the conditions actually obtaining in immediate proximity to the surface are somewhat obscure and for the present purposes it is thought preferable to postulate only the simplest of conditions. These are listed below.

(a) Near the surface, but well outside the region of roughness elements, the eddy diffusivity is identical with the eddy viscosity and is prescribed by the application to the atmosphere of the laws of turbulent flow established for flow through pipes and over flat plates.

(b) The height of the effective horizontal surface is identified with the zero point displacement occurring in the wind profile, and within a distance Δ of this surface the transport of water vapour is assumed to be controlled by the molecular diffusivity D.

(c) The vertical flux of water vapour is constant with height above the surface and is thus equal to the local rate of evaporation, E.

4. The water vapour content of the air will be denoted by the absolute humidity, q, and for simplicity the air density will be assumed independent of height. In the laminar layer, with z measured vertically upward from the effective surface,

$$E = -D\frac{dq}{dz} \qquad \ldots\ldots (1)$$

from which, by integration over the layer, with q_o as the surface value,

$$q_o - q_\Delta = \frac{E\Delta}{D} \qquad \ldots\ldots (2)$$

In the turbulent layer,

$$E = -K\frac{dq}{dz} = -\frac{K_1}{z_1}z\frac{dq}{dz} \qquad \ldots\ldots (3)$$

where K_1 is the eddy diffusivity at height z_1. Integrating (3) from the top of the laminar layer,

$$q_\Delta - q = \frac{Ez_1}{K_1}\log_e\frac{z}{\Delta}, \qquad \ldots\ldots (4)$$

and eliminating q_Δ from (2) and (4),

$$q_o - q = E\left(\frac{\Delta}{D} + \frac{z_1}{K_1}\log_e\frac{z}{\Delta}\right) \qquad \ldots\ldots (5)$$

A convenient form of (5), in which E is eliminated, and in which the

/Absolute

absolute humidity is replaced by vapour pressure, e , variation of absolute temperature being neglected, may be written in the form

$$\frac{e_o - e_2}{e_o - e_3} = \frac{\frac{\Delta}{D} + \frac{z_1}{K_1}\log_e \frac{z_2}{\Delta}}{\frac{\Delta}{D} + \frac{z_1}{K_1}\log_e \frac{z_3}{\Delta}} \qquad \ldots\ldots (6)$$

Equations (5) and (6) completely prescribe the vapour profile and the local rate of evaporation providing K_1 , q at one height, q_o the surface value and Δ are known. The obvious obscurity in the definition of q_o and Δ prohibits further development on a priori grounds and the succeeding interpretation is entirely empirical.

INTERPRETATION OF OBSERVED HUMIDITY PROFILES.

5. The vapour profile observations provide hourly values of the vapour pressure at a number of heights up to 2 metres, over a period of approximately one week. Of these the values at 25 and 100cm. will be considered here since previous discussion (i) was concerned largely with this layer. Again, in the previous analysis the indication was that the outward flux of vapour occurred predominantly in the daytime hours, the nocturnal values being small or even reversed with the appearance of dew at the surface. For this reason the present considerations are restricted to the hours from 0500 to 2000 GMT. inclusive. Mean hourly values of the vapour pressures at 25 and 100 cm. above the soil surface, for the period 0800 GMT., 7.5.47 to 0700 GMT., 13.5.47 are shown graphically in Fig. 1. Corresponding values of the wind speed at a height of 50 cm. above the soil surface are reproduced in Fig. 2.

6. For the specification of q_o , the ' surface ' value of the absolute humidity, the simplest possible assumption would be that saturation conditions exist at the surface. Geiger (1927) quotes observations which illustrate the very high values of relative humidity which occur among grass and fern leaves, though he suggests that complete saturation is seldom attained. On botanical grounds there seems to be evidence that, in the most favourable conditions, with the stomata fully open, the vapour pressure on the outer surface of the leaf may be substantially below saturation, though of course it would rise rapidly to the saturation value inside the leaf. However, even if these approximations are accepted and saturation at the surface is assumed, there remains the difficulty of defining the temperature at the surface. In the recent profile investigation over a pasture the closest approach to such a temperature is provided by the readings of a mercury - -in-glass thermometer lying on the ground with its bulb among the grass and clover leaves and also by the dry bulb readings of the lowest psychrometer which, being set at $6\frac{1}{4}$ cm. above soil level, was only a matter of 1 or 2 cm. above the vegetation. Apart from any other factors, the physical form of the vegetation cover and the varying angle of incidence of incoming radiation must lead to a complex temperature structure in and immediately above a vegetation cover, and data such as the above can only be regarded as a very rough indication of the temperature of the leafy surfaces. The above temperature data are employed here with this limitation very definitely in mind. Values of the saturation vapour pressures corresponding in one case to the readings of the mercury-in-glass thermometer and in the other case to the means of the readings of the mercury-in-glass thermometer and the lowest dry bulb element are reproduced in Fig. 1.

7. Values of the parameter $(e_o - e_{25})/(e_o - e_{100})$ evaluated from the above experimental data, are tabulated below, the value under (a) corresponding to the temperature as given by the mercury-in-glass thermometer and those under (b) to the mean of the temperatures given by the mercury-in-glass thermometer and the lowest psychrometer.

/GMT.

GMT.	$(e_o - e_{25})/(e_o - e_{100})$ (a)	(b)
0500	0.93	0.91
0600	0.86	0.86
0700	0.84	0.85
0800	0.81	0.83
0900	0.84	0.85
1000	0.88	0.88
1100	0.91	0.91
1200	0.91	0.90
1300	0.92	0.91
1400	0.91	0.91
1500	0.91	0.90
1600	0.89	0.89
1700	0.89	0.89
1800	0.89	0.89
1900	0.89	0.89
2000	0.90	0.89

It will be noted that the two sets of values show very close agreement and that apart from the distinct drop during the hours 0600 to 0900 inclusive there is little departure from a constant value of approximately 0.90. In the previous analysis of this experimental data it appeared that in general the most marked instability occurred during the forenoon hours, and it is conceivable that the low values of the above parameter are genuinely associated with this instability. On the other hand these variations in the parameter could be merely a reflection of inadequacy in the specification of the ' surface ' value of the vapour pressure. Without more direct estimates of the temperature and humidity on and in the vegetation cover a clarification of this point would seem to be impossible.

8. It is now proposed to examine the experimental values of the vapour profile parameter in terms of equation (6). As mentioned previously an empirical approach will be adopted, and by not imposing conditions of continuity in the diffusivity at the hypothetical junction of the laminar and turbulent boundary layers some freedom in the choice of the magnitude of Δ is implied. On aerodynamic grounds (Brunt, p.257) the thickness of the laminar boundary layer is expected to be inversely proportional to the wind speed at some height removed from the surface. A preliminary evaluation of the parameter from equation (6) , using a value of the eddy diffusivity as deduced in the previous analysis (i) , i.e.,

$$K_{50} = 1.65\, u_{50} \quad \text{c.g.s.} ,$$

shows that satisfactory agreement with the main range of experimental values, which are associated with wind speeds in the region of 3 m./sec., is given by

$$\Delta = 0.2 \text{ cm.}$$

On this basis the value

$$\Delta = 60 / u_{50}$$

is suggested. The close agreement of the numerical factor with that given by Brunt is somewhat remarkable, though in view of the approximations involved in the present analysis it can only be regarded as fortuitious.

/9.

9. Applying the suggested value of Δ generally it is found that for the range of values of D and u_{50} involved, the values of the vapour profile parameter computed from equation (6) are constant at 0.90 throughout the period 0500 to 2000 GMT. Apart from the disparity in the experimental and computed values during the hours 0600 to 0900, it thus appears that the vapour profile is represented remarkably well by equation (6), using the eddy diffusivity previously deduced and a laminar boundary layer thickness which is functionally and numerically consistent with previous estimates. To satisfy the main purpose of the present analysis, however, it is necessary to examine how far this agreement is reproduced in the rates of evaporation. These have accordingly been computed from equation (5) employing values of q_o, q_{100}, Δ, and K_{50} appropriate to the above considerations, and remembering that in this equation (as in equation (6) also) the height zero is fixed by the ' zero displacement ' (of magnitude 3.5 cm. in the present case) discussed in the previous analysis. The values are tabulated below as mean values for each hour, together with the corresponding values as deduced previously from the humidity gradient and eddy diffusivity at a height of 50 cm.

Computed Rates of Evaporation.

(a) Computed from measured humidity gradient and deduced eddy diffusivity at a height of 50cm.

(b) Computed from equation (5), identifying the 'surface' value of the absolute humidity with the saturation value corresponding to the temperature given by a thermometer on the ground with its bulb in the grass cover.

(c) As in (b), but using a mean of the above temperature and the temperature indicated by a psychrometer just above the grass cover.

	Rate of Evaporation in gm./sq.cm.sec. x 10^5		
GMT.	(a)	(b)	(c)
0500	0.03	0.05	0.04
0600	0.12	0.08	0.08
0700	0.18	0.13	0.14
0800	0.31	0.17	0.19
0900	0.44	0.28	0.31
1000	0.59	0.49	0.49
1100	0.59	0.65	0.63
1200	0.75	0.91	0.83
1300	0.72	0.91	0.83
1400	0.64	0.76	0.71
1500	0.52	0.58	0.57
1600	0.36	0.34	0.36
1700	0.34	0.38	0.39
1800	0.16	0.22	0.23
1900	0.10	0.13	0.13
2000	0.07	0.08	0.08

The above values are also reproduced graphically in Fig. 3.

10. From the above table and the graph in Fig. 3. it is seen that there is a fair degree of correspondence in the three sets of computed values, the discrepancies showing up in the form of an enhanced diurnal variation in the case of the values computed from equation (6), the discrepancy being greater when the temperature given by the mercury-in-glass ground thermometer is employed alone. It is also evident that the discrepancies are primarily a reflection of the departures of the experimental values of the vapour profile parameter in para. 7 from the constant value given by equation (5). Nevertheless the degree of consistency is thought to be promising, and this is all

/the

the more evident if the data are employed to evaluate the total evaporation loss over the six days. Disregarding the evaporation and condensation occurring outside the above hours, and integrating approximately by summing the above values, the corresponding total evaporation losses are

(a) 1.28 gm./sq.cm.
(b) 1.33 " "
(c) 1.30 " "

In comparing the figure given for (a) with the corresponding figure given previously, where evaporation minus condensation amounted to 1.38 gm./sq.cm., it should be remembered that the latter value includes the hours from 0800 to 1400 GMT. on 13.5.47, which are omitted in the present mean diurnal variation.

APPLICATIONS AND LIMITATIONS.

11. The numerical data discussed at the end of the preceding section demonstrates, in effect, the existence of a very good empirical relation between the vertical gradient of absolute humidity in the first metre above a live grass surface, and the difference between the absolute humidity at some convenient height and the saturation value corresponding to a temperature approximating to that of the grass cover. It suggests that, in an investigation in which, as previously, it is required to evaluate the evaporation loss in terms of meteorological factors, the detailed and continuous exploration of the vapour profile could be considerably reduced and replaced by a measurement of absolute humidity at one height and of a 'surface' temperature. For temperature and humidity ranges similar to those previously involved it should be possible to obtain equivalent values of $e_o - e_{100}$ within $\pm$ 10% using a hygrograph at 1 metre and a mercury in steel recorder with bulbs at 1 metre and in the grass cover.

12. The legitimacy of attaching some fundamental significance to equation (5), and of applying it to a wider range of conditions of ground and weather, is of course somewhat problematical. In any assessment of this possibility the following features need to be emphasised.

(a) A thin laminar boundary layer of acceptable thickness and dependence on wind speed is assumed. However, a satisfactory empirical interpretation of the present data requires that the actual boundary layer thickness should be such that continuity of diffusivity is not satisfied at the junction of the laminar and turbulent boundary layers. Such continuity actually requires values of Δ approximately 1/10 of those employed and these lead to an increase of approximately 50% in the evaporation rates computed from equation (5). This discontinuity in diffusivity could be regarded as a reflection of the inadequacy of representing a presumably smooth transition in diffusivity and vapour flux by two distinct layers.

(b) Even if consideration is restricted to a vegetation cover which is alive the specification of the 'boundary' value of the humidity is complicated by the geometrical form of such surfaces and by the scanty knowledge which seems to exist with regard to the temperatures and humidities obtaining there.

13. Granting the need for clarification of the points raised in the last paragraph it is of interest to speculate on the wider application of the present method to the estimation of evaporation as a climatological factor. At present this estimation seems to depend almost exclusively on the conventional form of evaporimeter, the indications of which are of somewhat debatable significance. The addition of regular readings of a ground thermometer to the normal routine of a climatological station would provide data for an approximate evaluation of equation (5), which could of course be reduced to a simpler form for the purpose. Such computed values, viewed with appropriate reservation, might be of considerable value in appraising the distribution of natural evaporation with locality and season.

/References:

References :

Brunt, D., Physical and Dynamical Meteorology, 2nd.Edn. 1939
Geiger, R., 1927, Das Klima der Bodennahen Luftschicht.
Millar, F.G., 1937, Canadian Met.Mem., 1, 39-65.
Montgomery, R.B., Mass.Inst.Tech., Pap.Phys. Oceanogr. Met., 7. No. 4.
Sverdrup, H.U., 1936, Ann.der Hydrogr., 64. 2. 1946. Journ. of Met., 3, No. 1.
Pasquill, F., ' A Preliminary Study of the Exchange of Moisture Near the Ground', M.R.P. 362.

.T.

G.172019

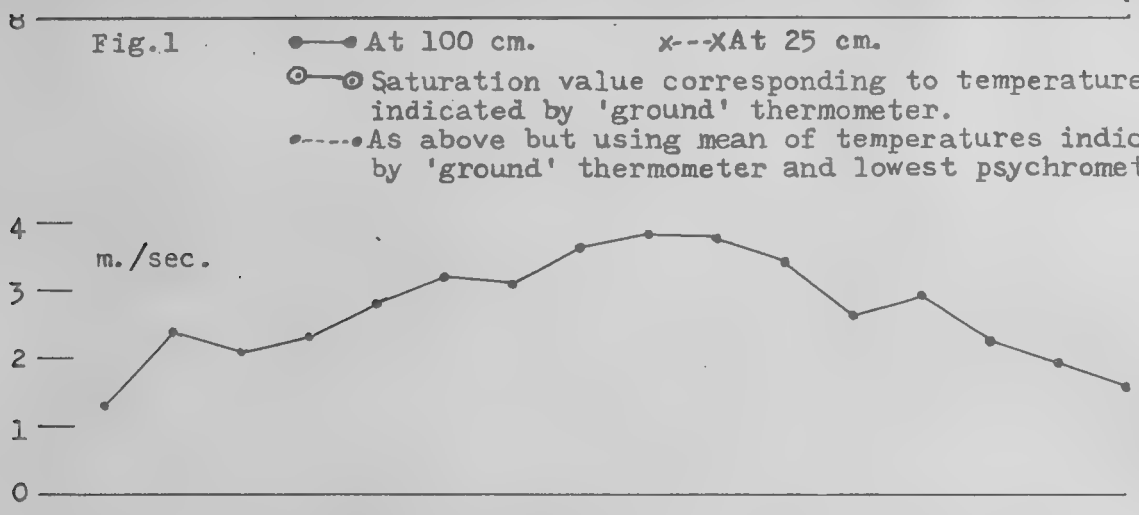


Fig.1 ●—● At 100 cm. x--x At 25 cm.
⊙—⊙ Saturation value corresponding to temperature indicated by 'ground' thermometer.
●----● As above but using mean of temperatures indicated by 'ground' thermometer and lowest psychrometer.

Fig. 2. Wind Speed at 50 cm.

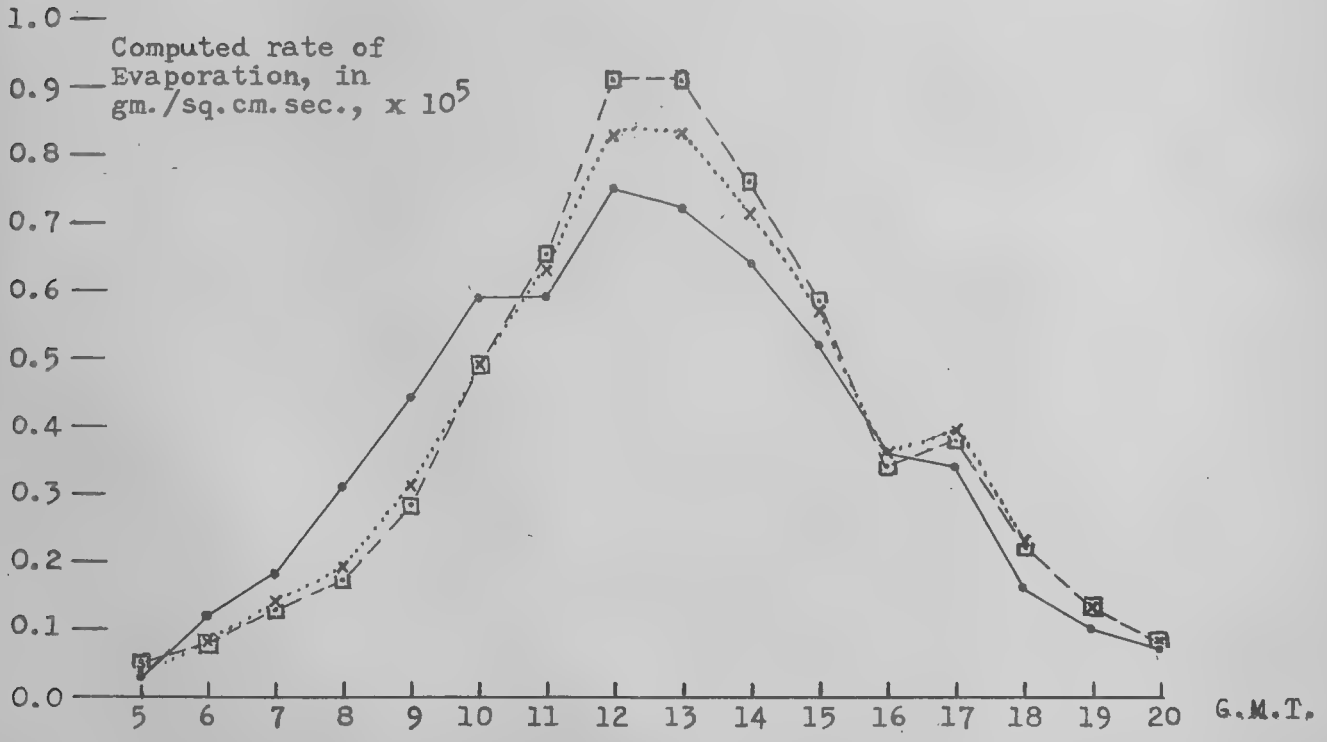


Fig. 3. • Computed from humidity gradient and eddy diffusivity at 50 cm.
⊡ From eqn. (5) using temperature indicated by 'ground' thermometer.
x From eqn. (5) using mean of temperatures indicated by 'ground' thermometer and lowest psychrometer.

the mor
over th
side th
the cor

In compa
previous
it shoul
1400 GMT

11. The
in effec
gradient
the diff
saturation value corresponding to a temperature approximating to that of the grass cover. It suggests that, in an investigation in which, as previously, it is required to evaluate the evaporation loss in terms of meteorological factors, the detailed and continuous exploration of the vapour profile could be considerably reduced and replaced by a measurement of absolute humidity at one height and of a 'surface' temperature. For temperature and humidity ranges similar to those previously involved it should be possible to obtain equivalent values of $e_o - e_{100}$ within $\pm$ 10% using a hygrograph at 1 metre and a mercury in steel recorder with bulbs at 1 metre and in the grass cover.

12. The legitimacy of attaching some fundamental significance to equation (5), and of applying it to a wider range of conditions of ground and weather, is of course somewhat problematical. In any assessment of this possibility the following features need to be emphasised.

(a) A thin laminar boundary layer of acceptable thickness and dependence on wind speed is assumed. However, a satisfactory empirical interpretation of the present data requires that the actual boundary layer thickness should be such that continuity of diffusivity is not satisfied at the junction of the laminar and turbulent boundary layers. Such continuity actually requires values of Δ approximately 1/10 of those employed and these lead to an increase of approximately 50% in the evaporation rates computed from equation (5). This discontinuity in diffusivity could be regarded as a reflection of the inadequacy of representing a presumably smooth transition in diffusivity and vapour flux by two distinct layers.

(b) Even if consideration is restricted to a vegetation cover which is alive the specification of the 'boundary' value of the humidity is complicated by the geometrical form of such surfaces and by the scanty knowledge which seems to exist with regard to the temperatures and humidities obtaining there.

13. Granting the need for clarification of the points raised in the last paragraph it is of interest to speculate on the wider application of the present method to the estimation of evaporation as a climatological factor. At present this estimation seems to depend almost exclusively on the conventional form of evaporimeter, the indications of which are of somewhat debatable significance. The addition of regular readings of a ground thermometer to the normal routine of a climatological station would provide data for an approximate evaluation of equation (5), which could of course be reduced to a simpler form for the purpose. Such computed values, viewed with appropriate reservation, might be of considerable value in appraising the distribution of natural evaporation with locality and season.

/References:

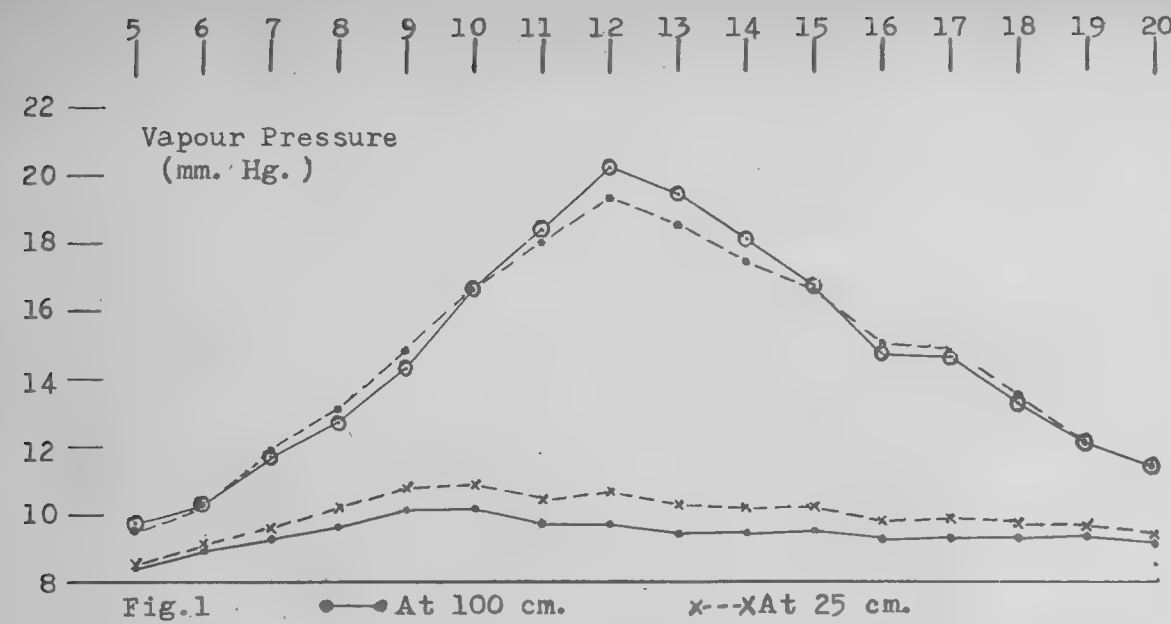


Fig.1 —●— At 100 cm. x--x At 25 cm.
—⊙— Saturation value corresponding to temperature indicated by 'ground' thermometer.
----● As above but using mean of temperatures indicated by 'ground' thermometer and lowest psychrometer.

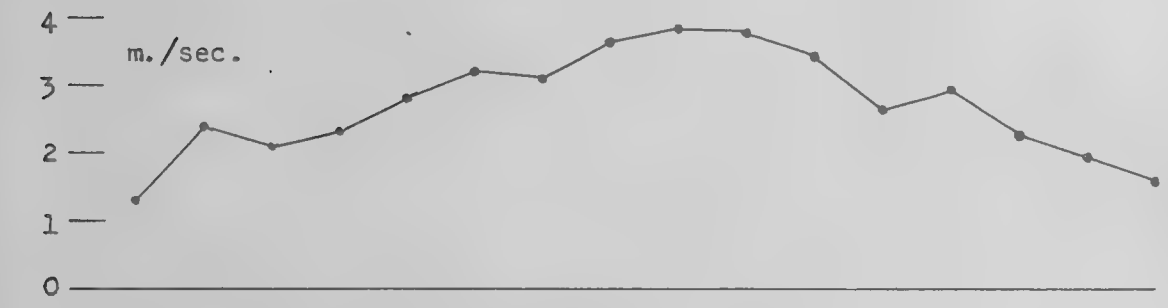


Fig. 2. Wind Speed at 50 cm.

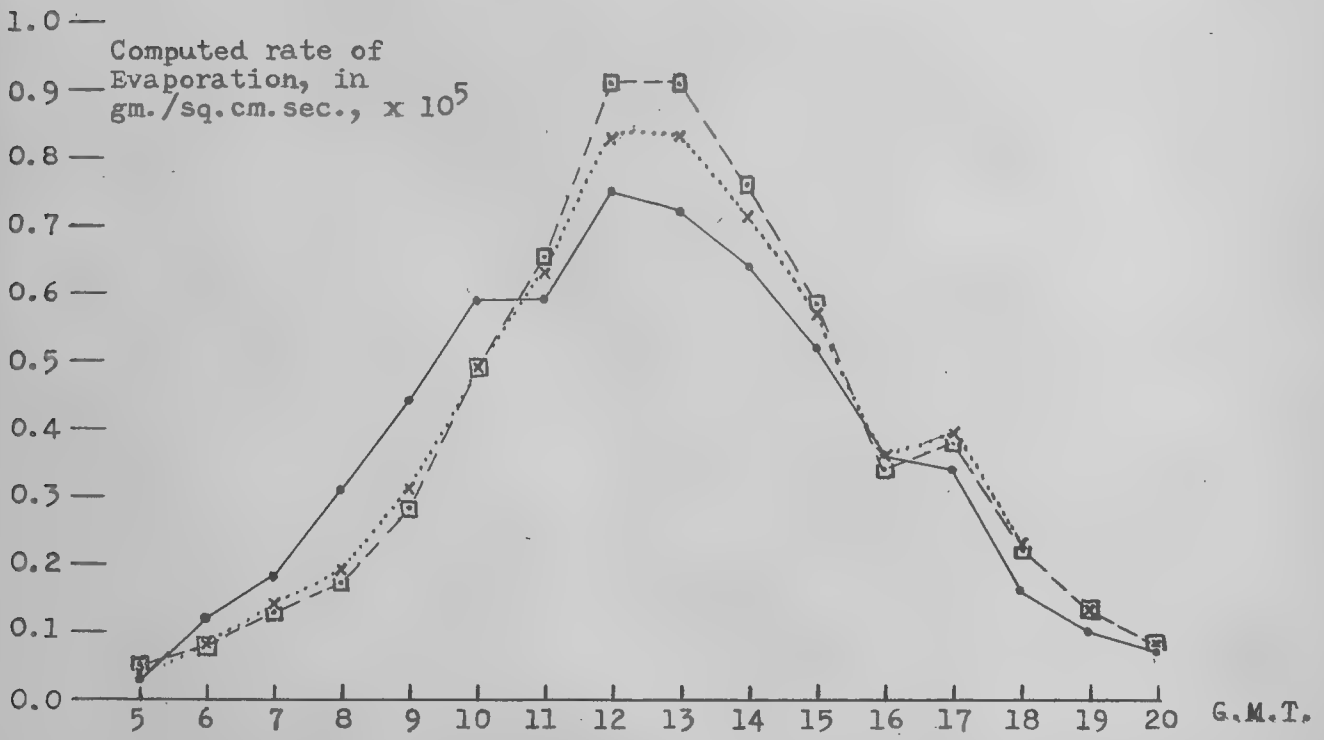


Fig. 3. • Computed from humidity gradient and eddy diffusivity at 50 cm.
⊡ From eqn. (5) using temperature indicated by 'ground' thermometer.
x From eqn. (5) using mean of temperatures indicated by 'ground' thermometer and lowest psychrometer.

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

Comparison of G.I. III.B. and S.C.R.658

by

A.F. Ayre

The purpose of the present report is to compare the performance of the [illegible] (658 for convenience hereafter) and the G.I. III b (G.I. hereafter) used for windfinding in the British network of Upper Air stations. In particular the report refers entirely to performance of the 658 at Stornoway and G.I. at Aldergrove. The following points should be borne in mind:-

a high degree of [illegible] was required.

The 658 was [illegible] exclusively for providing Meteorological information [illegible] utility use.

[illegible] the normal required for tracking, a pulse of radio [illegible] only is thus passive.

[illegible] was a power unit and the 658 itself. The [illegible] ...e at 110 volts 50cs/sec and has a rated output [illegible] consists of the stacked directional aerial array.

[illegible] must be run off a different power supply [illegible] found as standard viz. P.E. 214 A was unreliable [illegible] ation at Stornoway was by means of 230 volt [illegible] off the same supplies through transformer

10th October, 1947

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

Comparison of G.L. III.B. and S.C.R.658 Wind Finding Equipment

by

R.F. Apps

The purpose of the present report is to compare the performance of the SCR 658-T1 (658 for convenience hereafter) and the G.L.III B (G.L. hereafter) as used for windfinding in the British network of Upper Air stations. In particular the report refers entirely to performance of the 658 at Stornoway and the G.L. at Aldergrove. The following points should be borne in mind:-

1. The G.L. was designed primarily for gunlaying with A.A. artillery and a high degree of accuracy was required.

2. The 658 was designed exclusively for providing Meteorological information for field artillery use.

3. The G.L. measures all the parameters required for windfinding from a single observing post.

4. The 658 measures only azimuth and elevation, and the length parameter required for windfinding must be determined from pressure measurements.

5. The G.L. originates the signal required for tracking, a pulse of radio-frequency energy being reflected from a suitable balloon borne target. The airborne assembly is thus passive.

6. The signal provided for tracking with the 658 is obtained from a small transmitter on the airborne assembly. Pressure determinations are obtained either by means of a calibrated pressure unit (baroswitch) which interrupts the signal at certain pressures as indicated by its calibration or from the indications of a radiosonde attached to the same balloon.

General description of the 658.

The complete station comprises a power unit and the 658 itself. The power unit provides A.C. voltages at 110 volts 60cs/sec and has a rated output of 300 watts. The 658 itself consists of the stacked directional aerial array, receiver, wave generator and ionic switching assembly; the 658 will operate off either 110 volt 60cs/sec or 220-230 volt 50-70 cs/sec supplies but all accessories are designed for 110 volt operation. As the load of the instrument itself is 295 watts at 110 volts accessories must be run off a different power supply than the set. Again the power unit issued as standard viz. P.E.214 A was unreliable and easily overloaded. All operation at Stornoway was by means of 230 volt 50 cs/sec mains and accessories run off the same supplies through transformers. The load of 658 and accessories on commercial wiring is within the normal rating of power cable. The stacked aerial system consists of four bays each of eight half wave elements: the output from each bay can be fed either directly or through a phase shifting cable to the receiver. By suitable combinations of the routing of the bay outputs the beam can be switched to the four cardinal positions. (See G.L. descriptions). Split beam tracking thus results. As the receiver operates on 397 mcs/sec. the half wave elements are cut for this frequency. To simplify construction, the R/F stage of the receiver is not tuneable but has a wide band width and all transmitters are set to 397 mcs/sec before flight. Variations in R/F in flight are taken care of by the R/F band

/width

width and oscillator tuning. Receiver design is conventional except for the R/F stage which is replaceable as a unit and for which spares are normally held. Switching is accomplished by means of firing or extinguishing small gas filled triodes in the ionic switching system which, by altering the impedances of the direct and cable paths from the bays to the receiver, determine which path shall be used. The striking voltages for these tubes are generated in the wave generator which also generates the complex time base voltage for the display tube enabling the two elevation and the two azimuth breaks to be displayed in pairs on one cathode ray tube. The circuits involved in the wave generator are simple, and both receiver and wave generator are replaceable as units very easily and a spare of each is held. The expected accuracy of the 658 is given as:-

1. At elevations 13° - 75° elevation 0.16° and azimuth 0.20° (Report on test of SCR 658 at Fort Bragg 1944 File No.413.44)

2. At elevations above 15° elevation 0.13° and azimuth 0.17° (Engineering Memorandum No. C-1 1944)

In both these cases the instrument was not enclosed in a shelter.

3. At elevations above 15° with instrument inside a shelter elevation 0.17° and azimuth 0.20°. (Engineering Memo. No. C-1 1944.) (See Note 1.)

The complete 658 can be loaded on to a standard United States Army 1 ton cargo trailer with an overload of 400 lbs. It is readily broken into smaller loads (as each section has its own carrying case) the heaviest of which weighs 405 lbs; the towing vehicle need not therefore be large and the equipment can be deployed in relatively inaccessible locations.

General description of G.L.

The complete station comprises the Lister generator which supplies power for the G.L. and accessories and the G.L. cabin itself. The latter contains the transmitter, receiver, aerial system and display units and the all up weight of the cabin is 9½ tons. A heavy duty towing vehicle is required and the apparatus is mobile and can be deployed in easily accessible locations. It cannot be dismantled without the use of heavy duty cranes and even then the parts would weigh in the order of tons. The transmitter transmits pulses of energy at a radio frequency of approx. 3,000 mcs/sec with a pulse duration of 1 microsecond at a repetition frequency of 420 cs/sec. This is reflected from the airborne target and the time taken for its to and from journey enables the range of the target to be found. (This range is the slant range and further references to range should be interpreted as slant range.) The reflected signal is received on an aerial system rotating at 105 cs/sec in the focal plane of a parabolic reflector. Thus four pulses are received for each revolution of the aerial system and the circuits are so arranged that these are received at the four cardinal points. These are the positions at extreme left and extreme right and extreme top and bottom when the paraboloids are at zero elevation and their axis of rotation is vertical; they are fixed reference positions. The expected accuracy of the instrument is 0.17° in azimuth and elevation with elevations greater than 10°. At lower elevations ground errors are introduced and the expected accuracy is 0.5° in azimuth and elevation. (See Note 1.) It should be noted that this equipment cannot be run off commercial mains as the normal cables would not take the load and the frequency and voltage are not sufficiently steady. Range readings are correct to 25 yards.

Departure from design conditions for windfinding with G.L.III.

There are only two outstanding modifications to the equipment as used for windfinding, viz. fitting of a range doubler to extend the range to 66,000 yards and the provision of high speed magslips in place of the early warning dials to enable azimuth and elevation to be read to the nearest 1/10th degree. The latter were usually carried at the predictor. These modifications are not thought to alter the accuracy of the instrument.

Departure from design conditions for windfinding with 658.

Two alterations from the design conditions were present at Stornoway on the 658. These were:-

1. The housing of the instrument in a shelter. This has no effect on the performance of the SCR.658 when the shelter is dry (Engineering Report No.C-1) but the effect of a wet shelter is not known. There appeared to be no effect but the matter was not investigated quantitatively. It is of interest that the fact that the standard of observation of the protected personnel would probably be better than that of unprotected staff was recognised in the above report and acted upon by the provision of a shelter at static sites.

2. The replacement of the original Vistac (liquid dielectric) filled ionic switch by one executed in coaxial cable. The one fitted was the prototype design and originated by the Area Maintenance man (Sgt. DeHaan). Its effect on directivity pattern was thus never measured. It was suggested that the beam width might be slightly less and the switching angle slightly less than the original installation but this was never proved. These changes would, however, allow a slightly lower elevation before running into ground errors.

Lobe switching, ground errors and siting

The theory of split beam tracking demands either a symmetrical directivity pattern or the displacement of the electrical axis from the optical axis in a way such that equal signals are received when the instrument dials record the true bearing of the target. Anything which causes distortion of the directivity pattern will cause errors in the bearings in either case. The main source of distortion is reflected energy from other targets and the main source of these reflections are topographical features at low elevations and the ground itself. In the case of the 658 these reflections introduce appreciable errors at a theoretical angle of $16\frac{1}{2}^{o}$ on a level site ($9\frac{1}{2}^{o}$beam width and 7^{o} displacement) at search but the practical limit is taken at 15^{o}above the elevation of the highest terrain in the direction of search in practical cases. (Electronics Information.) Siting requirements for 658 requires a level site on fairly high ground and no objects of metal etc. within 200 yards. It follows that any distant high ground should have the lowest possible elevation from the 658. The G.L. runs into these ground errors at a lower elevation viz. 10^{o} because of its smaller beam width and smaller search displacement angle. Siting requires the minimum of ground clutter i.e. reflections from topographical features at 5^{o} elevation and a clear site for about 200 yards radius. There is one point of interest and that is that, whereas the 658 always searches at the cardinal points, the G.L. only searches at these points when the target range is 13,000 yards. This is because the transmitter fires when the receiving aerial is displaced by an angle of 3^{o} from the cardinal points and moving into the cardinal position. The speed of rotation is such that the aerial moves exactly into position if the range of the target is 13,000 yards. Thus, at ranges of less than 13,000 yards the transmitter fires relatively too soon and at greater ranges relatively too late. At 66,000 yards the search is being made at 12^{o} from the cardinal points. This would appear, at a superficial glance, to increase the interaction between azimuth and elevation measurements. (See Note 2.)

Interaction between azimuth and elevation measurements

Taking the figures given in S.D.T.M. 64 as correct there is less interaction between azimuth and elevation measurements with the 658 than with the G.L. Figures for the latter are $x/3^{o}$ for x^{o} error in the deliberately displaced bearing: the corresponding figure for the 658 is 2/15.x for x^{o} error in the deliberately displaced bearing.

/Limitations

Limitations of the G.L. for windfinding

The range to which the target can be followed with the G.L. is limited by the reflected signal strength and the noise level of the receiver. The present limit is 66,000 yards range, but tracking at this range with 3 ft. passive targets is difficult, and readable signals only obtained when a favourable aspect of the target is presented to the transmitted beam. This results in a "flashing" signal and tracking with one operator for elevation and azimuth is difficult on the separated tubes. The limits to which the elevation may drop without the introduction of ground errors is 10° there being no upper limit as the angle between the plane of polarisation of the transmitted beam and the receiver aerial is unchanged with elevation. It is possible that there is an appreciable error introduced into the azimuth readings at elevations above 10° at long ranges where the asymmetry in search at a displacement from the cardinal point is occurring. Assuming a lower limiting elevation of 10° and a normal type of run in which the elevation rises from release and then falls it should be possible to obtain winds to at least 34,000 feet with the G.L. However, at low elevations (less than about 15°) there seems to be some attenuation of the signal strength. In conditions of high surface winds and high upper winds it is possible that the elevation may never rise above 10°; in this case the 658 is also in the region of its ground errors; in fact even more so.

Limitations on 658 for windfinding

The only limitation on the range of the 658 is the signal/noise ratio of the received signal. The transmitter is computed to have an adequate S/N ratio at distances up to 80 miles and this gives 15° elevation as the limiting elevation before ground errors occur would enable winds to be measured up to at least 70,000 feet. There is a similarity between the type of errors to which both instruments are liable at low elevations and the above figure assumes a normal type of ascent. In addition the 658 is difficult to follow at elevations above 75° owing to changing aspects of the transmitter aerial and the absence of radiation "off the end" of the transmitter aerial. Also the plane of the aerial system is becoming horizontal and that of the transmitted radiation is still more or less vertical. With care, tracking is possible up to elevations of 85°. This is facilitated in part by the fact that the two displays are on the same cathode ray tube and "pip" heights when the signal flashes up are taken in at a glance.

Reduction of observations for windfinding

The reduction of G.L. III observations for windfinding is straightforward. Readings of range, elevation and azimuth are read at minute intervals from the moment of release of the balloon. The horizontal distance and height above observing station are then computed on a standard M.O. pilot balloon slide rule; horizontal distance is plotted against azimuth on a chart of scale 1:50,000. (The height progression serves as a rough check on the accuracy of the range and elevation observations, assuming that they are not required for Pilar results.) The track is smoothed for the removal of casual errors in the same way as the plots for D/F winds and measurement for each required pressure or height level made over the most nearly correct two or three minute interval. Using this system the % of cases in the accuracy groups defined in M.O.464 were A.92%, B.7%, C.1% and D.0% (see Note 3) in the period December 1946 to March 1947, at Aldergrove and the present practical limit with this equipment is 64,000 yards (5/4/47).

The reduction of the 658 observations for horizontal distance is done in precisely the same way after the length parameter for each minute has been determined. The process of estimating the vector errors for these winds from the smoothed trace tacitly ignores the systematic errors in the length parameter determinations but removes the random errors in this measurement along with the other random errors. (Strictly, this should read attempts to remove the random errors.) As the systematic errors in the length measurement will be changing slowly with height in the interval in height (approx. 4,000 feet) used

/for

for wind determination at a particular level the remaining errors due to errors in height measurement are not likely to be large. Wind speeds and directions as determined by the G.L. are independent of R/S errors although when the Prawt code is used these velocities may be assigned to an incorrect pressure level. Winds from the 658 are in theory subject to R/S errors although as just mentioned the resultant errors will be small. In the case of 658 used with a baroswitch and standard tables for pressure to height conversion, the errors introduced may be large. It was because the standard tables supplied by the Americans were drawn up for R/S ascents made over the continent of America that the nearest ascent in time and space was used for Praw work with the 658 by British operators at Stornoway. The accuracy classification for winds at Stornoway using the above system with the 658 during the period June 1945 to May 1946 were 77%, 17%, 5% and 1%. In a letter from Stornoway dated 8th August, 1945, information of a direct comparison of winds deduced by simultaneous 658 and theodolite observations on a balloon were given. These figures give the probable vector error distribution for this ascent as 62%, 23%, 15% and 0%. This was made on a very small number of observations. (See Note 4.)

Figures for the approach to the upper limit of range obtained with the 658 in the period June 1945 to May 1946 is given as 55,000 yards ± 19,000 yards and for July 1945 as 52,600 yards ± 15,000 yards. These are in fair agreement. Exceptional ranges of 105,000, 155,000, 144,000, and 142,000 yards were obtained in periods of high winds and at low elevations and are subject to ground errors in the elevation readings from which they were determined but probably represent the fact that transmitter performance is the predominant factor in the range limit of the 658. In the period June 1945 to May 1946, 60,000 yards range was exceeded on 15% of all occasions and 100,000 yards on 4% of all occasions.

Maintenance

The 658 is from this point of view the simpler of the equipments. All vital parts are replaceable as units in a short time and spares of each are held as a unit. It is thus a matter of removing the faulty part, replacing by the spare to be in operation again. The fault can be remedied at leisure. All circuits are relatively simple and the complex R/F stage in the receiver is changed as a unit. Faults in the wave generator are easily traced as each stage of the generator can be monitored on the display C.R.T. by means of an incorporated switching system. Main troubles occur through cables becoming slack, gas tubes in the ionic switch becoming u/s, cable deterioration and gradual deterioration in performance; the latter is hard to distinguish and it is best to interchange units periodically as a check. Valve ageing is the usual cause. Mechanical maintenance is straightforward and consists of oiling, level checking and removal of backlash in the elevation and azimuth drives. This can be kept to better than 0.1° in elevation and 0.2° in azimuth. Maintenance of the petrol generator was never done as it was never in use. Accessories used in conjunction with the 658 gave little trouble except the small charger used for charging the flight batteries. This was exposed to acid action owing to the method of battery filling and needed to be cleaned of corrosion regularly.

The G.L. requires specialised knowledge at all stages of its maintenance and the equipment as a whole needs daily attention. It is however more easy to spot gradual deterioration in performance than in the case of the 658. The main drawback to the equipment is its extreme sensitivity to damp; a slight amount of damp has a marked effect on performance. The 658 was designed for open use and has not this disadvantage.

Adequate spares are usually carried for each equipment although it was found to be impossible to obtain replacements for receiver R/F stages at Stornoway and one such stage was in operation for about two years. It is likely that these sections may still be scarce.

Operation

The 658 is the easier to locate in difficult locations, but there is little to choose between them in the matter of orientation and calibration. Each requires a balloon run for calibration but it is possible to apply corrections to the 658 at various stages of the one balloon run, adjustments to the telescope

setting being the method of correction. (Each instrument has its own telescope attachment). This is not possible with the G.L. as the method of correction is by adjustment of the paraboloids. Once aligned both instruments seem to retain their correct adjustment for long periods and only damage to the aerial and reflector system and phasing cables in the case of the 658 and the aerial and paraboloids in the case of the G.L. affect the alignment. The latter troubles mean a complete realignment and are infrequent. The possibility of cable deterioration makes it relatively more likely in the case of the 658. It should be noted that it is impossible to conduct calibrations on the standard system if the 658 is inside a shelter and indirect calibration must be done.

Operation of the 658 for routine work is simple. The power unit is started and the main switch thrown and the whole instrument allowed to warm up for about half to one hour. If the mains supply is being used it is only necessary to switch on as if an ordinary commercial radio. The patterns for the wave generator are then checked and the signal/noise ratio of the transmitter for flight checked roughly. The transmitter should be about 1,000 yards away for this test to be effective. The main part of the pre-flight operation consists of the preparation of batteries, transmitter and (if necessary) baroswitch. All these operations are simple although requiring care.

The preparation of the G.L. for flight requires the starting of the power supply, tuning of the transmitter, setting up time bases correctly and setting up of the range doubler when fitted. Performance can be checked on a known distant standing break. (The one used at Aldergrove has a range of 60,000 yards.) Performance can be seriously affected by small errors in tuning either of the transmitter or receiver. (Small inaccuracies of the tuning of the 658 transmitter for flight are not serious as the wide R/F band width covers them.) Generally the setting up of the G.L. is a specialised job. Preparation of the target for use with the G.L. requires no special skill.

Commencing the observations starts with the location of the target. Owing to the narrower beam width and the necessity for the target to be in the transmitter beam before any signal can be detected this is more difficult with the G.L. than with the 658. This is specially so in the case of light winds and deviations from the expected direction of movement of the target in the dark or cases of low ceiling. Otherwise, it is simple to set on target visually. The use of a 2 volt lamp overrun from two $1\frac{1}{2}$ volt cells enables location to be made in the dark with a fairly high ceiling, visual tracking for two minutes being all that is required. In cases where visual setting is not possible a search drill must be followed using predicted target positions. In the case of the 658 the transmitter is transmitting from release and a signal is obtained from release. Thus departures from the expected path are not confusing as they are followed from release. The only difficulty is to remain on target with the main lobe of the directivity pattern directed towards the transmitter. If one accidentally starts using a side lobe of the directivity pattern it soon becomes obvious and search must be made for the correct bearings. With both instruments should the target pass directly overhead, the instrument must be rotated through 180° and the target re-located. The G.L. has provision for power slewing whereas the 658 is manually operated. After initial location tracking is simple, being a matter of equalising two pairs of breaks displayed side by side. The aerial system orientation is power driven in the case of the G.L. and hand operated in the 658, but little physical effort is required in the latter case. Azimuth and elevations are taken off dials calibrated to 1/10th° in each case and the range in the case of the G.L. can be estimated to 5 yards. It is impracticable for one operator to track on all three parameters and undesirable that one should track on both azimuth and elevation at long ranges with the G.L. The 658 has provision for operation by two men but it is not necessary because of the better display system.

/Conclusions

Conclusions

1. The 658 is better adapted for use in difficultly accessible positions being more readily reduced to small components.

2. The 658 is simpler in maintenance and operation.

3. The 658 is more adaptable to power supply and has only a relatively light power consumption.

4. Both instruments require good sites, the 658 being more critical in this respect. Both require the immediate vicinity to be clear of electrically reflecting objects.

5. The G.L. is more useful for windfinding as it measures all the necessary parameters and height readings deduced from the observations serve as a rough check on the accuracy of range and elevation measurements.

6. Tracking with the G.L. can be taken to 66,000 yards range at 10° elevation without serious instrumental errors. The 658 can track to elevations of greater than 15° without serious instrumental error and to a theoretical range of 80 miles. At 15° elevation and extreme range, the G.L. would obtain winds to 51,000 feet. The greater potential range of the 658 is thus not a decisive factor for windfinding below this height. In cases where the G.L. range limit is reached before reaching this height i.e. in cases of high winds the 658 will be well outside its permissible elevation limits. Given an average bursting height for the balloon as 60,000 feet the G.L. will reach this, provided the elevation does not drop below $17\frac{1}{2}^{\circ}$.

7. The accuracy figures show the superiority of the G.L. as a practical instrument although the figures for the 658 compare favourably with those obtained for D/F winds.

8. There is a possibility that the replacement supply for the 658 may be a bigger problem than for the G.L.

9. The accessories for the G.L. are matched to the supply (power) as is the case for 658. In the event of the use of the 658 from 220 - 230 volt supply additional transformers will be required for the accessories which only work off 110 volt supplies. This largely offsets the advantage of the voltage range for operating the 658.

G. 172050

performance. transmitter performance.

12.

Accuracy	For elevations > 10°	For elevations 15° - 75° :-
Azimuth	0.17°	0.17° to 0.20°
Elevation	0.17°	0.13° to 0.17°
Range	25 yds. approx.	

13.

Distribution of vector errors in wind measurement	%	%
0 - 3 kt	92	77
3 - 5 kt	7	17
5 - 10 kt	1	5
> 10 kt	0	0

14.

Limitations to accuracy	Ground echoes at elevations up to 10°	Ground echoes at elevation up to 15° above highest terrain.
		Poor signal strength at elevations above 75°

G. 172050

/NOTES

confusing as they are followed from release. The only difficulty is to remain on target with the main lobe of the directivity pattern directed towards the transmitter. If one accidentally starts using a side lobe of the directivity pattern it soon becomes obvious and search must be made for the correct bearings. With both instruments should the target pass directly overhead, the instrument must be rotated through 180° and the target re-located. The G.L. has provision for power slewing whereas the 658 is manually operated. After initial location tracking is simple, being a matter of equalising two pairs of breaks displayed side by side. The aerial system orientation is power driven in the case of the G.L. and hand operated in the 658, but little physical effort is required in the latter case. Azimuth and elevations are taken off dials calibrated to 1/10th° in each case and the range in the case of the G.L. can be estimated to 5 yards. It is impracticable for one operator to track on all three parameters and undesirable that one should track on both azimuth and elevation at long ranges with the G.L. The 658 has provision for operation by two men but it is not necessary because of the better display system.

/Conclusions

SUMMARY

Comparative Features of G.L. III and SCR 658 Wind-finding Equipment

		G.L. III	SCR 658
1.	Mobility	Heavy duty vehicles required.	Breaks down into small sections less than 405 lb.
2.	Siting	Reflecting (e.g. metal) objects must not be within 200 yards or taller than 5° elevation.	Reflecting objects must not be within 200 yards and surrounding terrain should be 15° below site.
3.	Power Supply	Lister generator (mains voltage is normally insufficiently steady due to heavy load (8 kw) imposed).	Generator or mains (110 volts).
4.	Operating frequency	3,000 Mc/s (10 cm)	397 Mc/s (0·75m)
5.	Maintenance	Specialist staff essential; careful daily attention required; equipment sensitive to damp.	Parts replaceable; little technical attention necessary; equipment insensitive to slight damp. Shelter not essential.
6.	Airborne equipment	Passive reflector.	Transmitter (a source of signal failure).
7.	Data provided	All parameters necessary for wind-finding.	No length parameter (pressure levels are indicated).
8.	Pre-flight operations	Warming up, tuning, setting of time bases and range unit.	Warming up; preparation of battery and transmitter.
9.	Initial location of airborne equipment.	Comparatively difficult owing to narrow beam width and necessity to find target before signal is received.	Simpler owing to wide beam and fact that signal is emitted from airborne transmitter.
10.	Number of observers.	At least 2, preferably 3.	At least 1, preferably 2.
11.	Range limit.	Theoretically : 66,000 yds. Practically: slightly less with 3 foot targets dependent on G.L. III performance.	Theoretically : 140,000 yds. Practically: 55,000 yds. (approx.) dependent on SCR 658 and transmitter performance.
12.	Accuracy Azimuth Elevation Range	For elevations $>$ 10° 0.17° 0.17° 25 yds. approx.	For elevations 15° - 75° :- 0.17° to 0.20° 0.13° to 0.17°
13.	Distribution of vector errors in wind measurement	%	%
	0 - 3 kt	92	77
	3 - 5 kt	7	17
	5 - 10 kt	1	5
	$>$ 10 kt	0	0
14.	Limitations to accuracy	Ground echoes at elevations up to 10°	Ground echoes at elevation up to 15° above highest terrain. Poor signal strength at elevations above 75°

/NOTES

N O T E S

1. The figures for expected accuracy of the G.L.III and S.C.R.658 are quoted from the literature describing the respective instruments and were not determined by the author of the report.

2. The setting of the aerial motor phasing in the G.L.III to give optimum performance at a range of 13,000 yards was quite arbitrary, but was a reasonable compromise for a maximum range of 36,000 yards. All G.L.III stations have now been advised to reset the aerial motor stator to give optimum performance at 40,000 yards range. If ranges in excess of 66,000 yards are obtained it may be necessary to modify the G.L.III set to give continuous adjustment of the aerial motor phasing.

It is very doubtful whether the phasing effect does increase the interaction between azimuth and elevation measurements and so reduce the accuracy.

3. This accuracy classification is given for the observations made with each instrument in terms of standard groupings labelled A, B, C and D. These groupings were originally used for the classification of wind determinations using D/F observations and the original method of determining the classification of reported wind is described in M.O.464, page 54. The classifications were meant to be tolerance values for the probable vector error, thus:-

A	0 - 3 Knots
B	3 - 5 "
C	5 - 10 "
D	over 10 Knots

But the limits of the classifications were shown later on by comparison with theodolite and with G.L.III observations to correspond approximately to 75% errors. With the introduction of the G.L.III for wind-finding the same system of classification was applied to observations made with the G.L.III. However, owing to the great accuracy obtainable with the G.L.III the vast majority of the observations are in Class A; only a few at the end of each ascent, when the signal is failing, fall into lower categories. The method of assessment depends entirely on the amount of smoothing of the wind trace which is necessary and is largely a matter of judgement based on the experience of the officer-in-charge of the sounding. There is no comparison with any standard. In an exactly comparable way classification figures were given to the winds determined with the SCR 658.

Thus although the percentage figures quoted are not obtained by comparison with standards, they can be compared with one another to show the relative performances of the two types of equipment, since the method of assessment is identical for each type and Mr. Apps supervised the standards used for each equipment.

4. Though one ascent is hardly a basis for a sweeping statement, it is of interest to see that the figures do resemble the classifications obtained by considerations of the smoothing necessary to the wind trace. It seems quite evident that the accuracy obtainable from the SCR 658 is considerably lower than that obtainable from the G.L.III. The SCR 658 performance is indeed about equal to that obtained by observations from a first class D/F lay-out.

by the author of the report.

2. The setting of the aerial rotor phasing in the S.C.R.111 t[illegible] performance at a range of 13,000 yards was quite arbitrary, b[illegible]

azimuth and elevation measurements and so reduce the accuracy.

A	0 - 3 Knots
B	3 - 5 "
C	5 - 10 "
D	over 10 Knots

[illegible] to the most accuracy obtainable with the S.C.R.111 the vast m[illegible] observations are in C[illegible] and D, [illegible] the end of each [illegible] and is veiling. Fall into four categories. The method [illegible]

[illegible] for each type and Mr. Ayre supervised the standards u[illegible]

[illegible] which are absent is hardly a basis for a sweeping stat[illegible]

CONFIDENTIALLY. Its contents must not be quoted.

1st December, 1947

Acc. 48. S 487

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

SOLAR RADIATION AT KEW OBSERVATORY

By
J.M. STAGG.

1. Introduction

I. Direct solar radiation of normal incidence

2. Instruments used.
3. Standardisation of instruments.
4. Corrections to 1938-45 material.
5. Monthly and annual totals (all days).
6. Daily totals for various catergories of day.
7. Frequency distribution of daily totals.
8. Daily variation of intensity.
9. Intensity as a function of sun's altitude.
10. Components of direct solar radiation of normal incidence.
11. Radiation intensity as a function of air mass.
12. Atmospheric transmission of direct solar radiation.
13. Measures of atmospheric transmission or turbidity.
    (a) The transmission coefficient
    (b) Linke's turbidity factor (T)
    (c) Angström's coefficient of turbidity (β)

II. Direct and diffuse short wave radiation from sun and sky on a horizontal surface

14. Introduction and instruments used.
15. Calibration of the thermopiles.
16. Monthly and daily totals, T and D.
17. Extreme values of T, D and S and the sky conditions which produced them.
18. Variation of T and D on days of highest T and D income.
19. Diffuse solar radiation from a cloudless sky.
20. Diffuse radiation from a cloudy sky.
21. Relation between intensity and daily income of diffuse radiation.
22. Daily variation of T and D.

/Introduction

1. Introduction

Records of direct solar radiation have been continuously maintained at Kew Observatory since July 1932 and intermittently for a few years before then. As these records form one of the few sources of information on solar radiation in the British Isles, and because of their interest to meteorologists as well as to architects, specialists in building research, heating and fuel engineers and agricultural and fishery authorities, it is desirable to summarise the data.

Though the material to be discussed is concerned primarily with direct solar radiation received on a surface maintained normal to the direction of the radiation, measurements of the radiation received on a horizontal surface from sun and sky, and from the sky alone have also been made at Kew Observatory since July1946. As information about these aspects of solar radiation is probably more immediately useful in many practical applications than data concerning direct radiation of normal incidence, some preliminary results from the first year's measurements are given in the second part of the discussion.

I. Direct solar radiation of normal incidence

2. Instruments used.

Except for radical reconditioning in 1945 (see para.3), the instrumental equipment at Kew Observatory and the conditions of exposure have remained the same throughout. A Gorczynski pyrheliograph which receives the radiation consists of a Moll large-surface thermopile (80 thermocouple junctions) on an equatorial mounting, driven by a robust pendulum clock. With slight changes in elevation each day to keep pace with changing solar declination the heliostat ensures that the thermopile surface is kept normal to the direct radiation from the sun. A wire frame attached to a collar which fits on to the thermopile holder carries three metal diaphragms spaced outwards from the thermopile so that the angular aperture allows only radiation from the sun and a narrow annulus of sky to fall on the thermopile. The thermopile is protected by a glass cover, and its output is led into a (Richard) recording millivoltmeter.

The transmission characteristics of the glass cover are not precisely known: but as the instrument is standardised by an Angström compensation pyrheliometer with unshielded black strips, the radiation intensities tabulated from the Gorczynski records are in effect the intensities incident on an unprotected thermopile surface - provided the solar radiation transmitted by the glass cover bears a constant relation to the total radiation falling on the glass surface. As the limit of transmission of ordinary clear glass a few millimetres thick is usually 2.8 to 3μ the proportion of total radiation cut off is probably very small: furthermore the absorption bands of the main variable absorber in the atmosphere in this region (ϕ, ψ and Ω of water vapour) are within the spectral range transmitted by glass.

Up to now it has not been practicable at Kew Observatory to measure the proportion of red radiation in the total intensity. This is now being done by use of a twin heliostat carrying two thermopiles screened by filters, so that the spectral components of the direct radiation are continuously recorded. Figure 1 shows the arrangement of the Gorczynski pyrheliograph and other radiation equipment on the roof of the Observatory.

3. Standisation of instruments

On suitable days when the sky in the neighbourhood of the sun is free of cloud a measurement of the intensity of direct solar radiation I is made using an Ångström compensation pyrheliometer, No.24 which was calibrated at Upsala and has been compared from time to time with other substandard pyrheliometers. Investigations at the National Physical Laboratory (Guild, 1937) showed that the error of the scale of the Angstrom instrument by which the Kew standardisation measurements were made did not exceed 0.5 per cent, and more recent inter-comparisons with other pyrheliometers have shown that the Kew standard instrument has maintained its constant unchanged. Comparison of the pyrheliometer values of I with the simultaneous values registered by the Gorczynski instrument give a factor (F) for relating the continuous records to the standard scale.

The value of F was originally fixed at 1.00 by adjustment of the resistance in the thermopile circuit: it normally varies only slightly from month to month and shows no progressive drift. Fluctuations greater than 2 or 3% of the long period mean value of F are exceptional, and when both thermopile (with its recorder) and pyrheliometer are functioning satisfactorily, it is legitmate to smooth them out as being attributable to inaccuracies of timing in the standardisation experiments or in the record, or to inaccurate setting in one or other of the two instruments.

From the summer of 1932 until near the end of 1937, F remained practically constant at 1.00: but in 1938 it began to increase (indicating decreasing sensitivity of the thermopile) at first slowly and almost imperceptibly, then more rapidly until, by the latter part of 1944, it had become 1.26. During those years of progressive change in F, its variability also increased to 12-15% of its mean value. Table 1 gives for comparison the F values for 1943 and 1946 by which latter year the thermopile had been repaired. A smooth progressive change in sensitivity of the thermopile could be taken account of by making corresponding changes in the reduction factor of the records: but the serious question remained, was the increased variability in F an indication of real short-period changes of sensitivity superposed on the progressive drift?

Unfortunately the circumstances of the period during which these uncertainties were taking place were against providing an easy answer. Comparison observations between the pyrheliograph and the pyrheliometer had been made as frequently as practicable: but even without the wartime shortage of trained observatory staff and the need for attention to more urgent matters, the normal climate of England ensures that there are seldom more than one or two days a month when conditions are suitable for satisfactory standisation measurements.

4. Corrections to the 1938-1945 material.

When the thermopile was examined in 1945 a number of its junctions were found to have decayed, probably owing to electro-chemical effects arising from deposition of moisture and oxidation. It seemed very likely that spurious contacts had been effected between adjacent elements, thus disrupting the continuous series linkage of the elements and putting a proportion of them in parallel. Now on this evidence it was easy enough to account for long period changes (progressive or discontinuous) in the sensitivity of the pile but the short period variability remained unexplained. Any hypothesis required that some of the defective contacts should right themselves for short intervals (of the order of hours or a few days) and then seriously deteriorate again, as had been indicated by the fluctuations in F on those few occasions when it had been possible to make standardising measurements against the Angström pyrheliometer on successive days. It did not seem practicable that this could have occurred.

As all the hourly values of direct solar radiation had already been tabulated and reduced to energy units assuming a steady factor of 1.00, corrections were made to the tabulations from 1937 onwards: and as there was no satisfactory basis for taking account of the short period fluctuations, the adjustments were applied over periods of a year or part of a year as shown by the adopted values of F given in Table 1(b). At this early stage, the intention was to provide only longterm averages of radiation: if day-to-day changes had actually taken place in the behaviour of the thermopile in the general course of its deterioration they would probably not affect the monthly and annual means to any substantial extent.

As will be explained in paragraph 6, it was subsequently decided to form estimates of the maximum direct radiation likely to be recorded at Kew: this was done using the records from the unexceptionable years 1945 and 1946 after the thermopile had been reconditioned. When the radiation values I for exceptionally clear conditions in those years were compared with the adjusted data for the 1938-44 period, some isolated hours were found in the 1938-44 data with radiation intensities above the "ceiling" values. For example, the ceiling intensity deduced for local mid-day in May was 1.33 cal./cm.^{2}/min., but the intensity as tabulated using the adjusted factor 1.20 for 18th May 1943 was 1.43 cal./cm.^{2}/min. On that occasion and therefore presumably on other occasions it is likely that the mean factor which had been accepted did not truly represent the sensitivity of the thermopile. Reference to Table I(a) shows that May 1943 was in

the period when the daily values of F fluctuated wildly, and that the adoption of the observed F (1.08) instead of the smoothed value (1.20) would have brought the tabulated intensity on 18th May below the ceiling intensity. But how and where did the discontinuity between the 14th May (F = 1.26) and 19th May (1.17) occur and how many similar variations in sensitivity occurred in the period when no standardisations had been possible? Some evidence was collected which showed that while the progressive drift in the series of reduction factors F was perhaps greater in the early winter months, the greatest fluctuations in the day-to-day values occurred in the summer months. This could be explained by varying effects of humidity and temperature on the thermo-electric junctions.

Having no other continuous radiation instrument with which to compare the pyrheliograph records, little could be done to reduce the uncertainties in the material: probably only the summer records were affected and those only on a few hours of isolated days of highest temperature and humidity (and therefore high radiation intensities). Faults in the measurement of I by an instrument like the Gorczynski pyrheliograph almost always lead to recording I intensities lower than the correct values, for example, by the heliostat being slightly off the sun through imperfect correction for sun's declination or hour angle: the inclusion of some days of questionably high values cannot therefore add seriously to the other uncertainties.

In the formation of monthly and annual means and diurnal variations for selected sets of days, it was therefore decided to accept those few days in which hourly I intensities equalled or slightly exceeded the estimated ceiling values.

5. Monthly and annual totals: all days.

Tables 2 and 3 give the mean monthly and annual totals of direct solar radiation at Kew Observatory in cal./cm.^{2} for all days of the fourteen years of complete data, 1933 to 1946. A few of the more salient features of these Tables are:

(1) The total direct solar radiation received at Kew in an average year is about 56,000 cal./cm.^{2}: in the last fourteen years it has ranged between 69,200 (1943) and 47,300 (1936) cal.cm.^{2}. The four months May to August contribute 56% to the year's total, and the four winter months only 10%.

(2) A monthly total radiation of 10,000 cal./cm.^{2} was exceeded only in May, June and July and only once in each of those months: totals below 1,000 occur in each of the four winter months November to February. The lowest recorded monthly total is 465 cal./cm.^{2} (Dec. 1933) and the highest is 12,263 (May 1943): probably the next highest (June, 1940 with 12,141 cal./cm.^{2}) is the more dependable value.

(3) October is the steadiest month of the year: the range between the highest and lowest October values in the series, as a percentage of the mean, is 44%. February on this basis is the most variable month, 144%. Of the summer months, June and August are less variable than July, 60% compared with 91%.

The reason for giving separate means for 1942-46 in Table 3 is that an unexplained discontinuity was found in the 1942 data in the course of a minor enquiry (see paragraph 6) into the relationship between I and sunshine. As the same sunshine recorder was used throughout, the discontinuity had to be attributed to the measures of radiation. But the factor for reducing the radiation records had changed less than 1% between 1941 and 1942, and though there was a considerable scatter in F in both years there was no occasion when a permanent discontinuity could be said to occur. Furthermore the reconditioning of the thermopile in 1945 did not affect the I/S relation: the same value was deduced for 1944 using the modified reduction factors for that year given in Table 1(b) as was deduced from the 1945 data after the repair of the thermopile. The discontinuity could not therefore be reasonably attributed to change in sensitivity through any deterioration which might not have been taken account of by the modified factors: in any event such omission would most probably have resulted in a change in the I/S ratio in the direction opposite to that actually found, viz. from 36.1 for the earlier years to 44.4 for the years after 1942.

As being the only other observational material at Kew with which direct solar radiation might be expected to have a rough relationship sufficient to indicate gross anomalies in radiation measurements, means were formed of the differences (Solar maximum temperature - Screen temperature) for 1940-42. The solar maximum temperature is the highest temperature reached each day in a thermometer with a blackened bulb within an outer evacuated bulb directly exposed to the sky. Figure 1 shows that no useful inferences could be made from the comparison. In the months September to March of each year the radiation values broadly followed the temperature differences, whereas in the summer months the temperature differences remained relatively flat (about 45-55°F), corresponding roughly with 5000 cal./cm.^{2}, while the monthly totals of radiation rose to about 10,000 cal./cm.^{2}. There was no indication of a discontinuity in the sequence of radiation data relative to the temperature data.

No explanation can be offered for the discontinuity. It was therefore decided to split the data into two sets, one set for 1933 to 1941 and the other for the later years: the seasonal and diurnal variations are probably unaffected.

6. Daily totals for various categories of days.

Average daily totals for each month were formed for all days, and, for comparison, for a group of practically cloudless days: as cloud observations are made at Kew Observatory only at 3-hourly intervals, the first selection of cloudless days was made on the basis of values of percentage possible sunshine but the selection was later modified to depend on the daily radiation totals themselves. At this stage it became apparent that the outcome of a comparison between radiation on an average day and that of a cloudless day depended on the degree of cloudlessness. It requires a highly skilled observer to detect the presence of thin cirrus cloud which may just be in process of formation, or cirrus cloud which has formed between times of observation and is in process of dissipating again. On such occasions there may be little difference in the appearance of the sky but the effect on a record of radiation may be substantial. A more restricted set of cloudless days was therefore formed using as basis of selection standards of total radiation higher than in the first category.

An attempt was also made to estimate the maximum or "ceiling" values of radiation likely to be recorded at Kew on exceptionally clear occasions. The reasons for adopting the procedure actually used in forming these values may not be obvious until subsequent paragraphs (particularly paragraphs 11 and 12) have been read, but it is desirable to describe the manner of formation at this stage. A catalogue was formed of peak values of I recorded in the period May 1945 to January 1947, and after computing the sun's altitude h at the time of each value, dot diagrams of I against h were prepared for pairs of consecutive months. It is to be noted that the highest values cannot be selected just by inspection of records, except perhaps on consecutive days at times of the year when the sun's declination is practically stationary. Similar times on the records normally correspond with different values of h and therefore with different optical path lengths. In comparing I values for short periods on different days, it is therefore necessary to use h as the parameter instead of time.

It may also be noted parenthetically that it was only after these dot diagrams had been constructed that the considerable range in I values for similar values of h in apparently equally clear sky conditions was fully appreciated. It was early noticed that on two days of similarly cloudless sky and good visibility at the same time of year, one with an easterly air drift, the other west or north westerly, the whole level of radiation could be altered by 10 to 15%: but it was not clear until the peak values of I were catalogued and compared against the optical paths of the radiation that, superposed on the relatively steady values on such days, there is generally an irregularly fluctuating variation, related presumably to temporary, though large scale, changes in air clarity through a considerable depth of atmosphere.

Fig.3 reproduces the dot diagram from the June-July peak intensities and shows the envelope drawn to represent the peak or ceiling I values for this pair of months: 62° is approximately the maximum h at Kew, hence the break in the run of dots between forenoon and afternoon in the figure.

/As

As no systematic difference was observable between the a.m. and p.m. I intensities read from the envelopes, they were combined to form the six curves (one for each 2-monthly period) shown in Figure 4.

At first sight the surprising feature about Figure 4 is the practical identity of peak radiation intensities for corresponding h's throughout the year. But apart from a seasonal variation in the "solar constant" of radiation (I_o) due to the elliptical form of the earth's orbit, there is no reason why the clearest atmospheric conditions in mid-summer should not yield I intensities, for however short a time, which are approximately the same as in mid-winter for similar path lengths. Actually the December-January peak intensity for $h=15^o$ is 52.0 cals./hour and the corresponding figure for June-July is 49.4. The intensities for all altitudes from 10^o to 40^o are higher for the winter half year than for the summer half year. The difference is in the right direction and of the size to be accountable by the 7% change in I_o from perihelion to aphelion.

To estimate the maximum I which could be received at Kew if the atmosphere were continuously as clear as on the occasions which produced those peak values, the changes in radiation intensity effected by the seasonal variation in solar constant were ignored, and the six curves of Figure 4 were further combined into one curve relating ceiling intensities to h. From this curve, maximum I values at Kew were estimated for each month (mean hourly and daily totals).

The sets of mean daily values for each month derived from the four different categories of days described above are given in lines A to D of Table 4: as the same letters and categories will be used again later in the discussion they are summarised as follows for easy reference:-

Line A. All available days of recorded radiation in each month, 1933 to 1946, i.e. same basis as for Tables 2 and 3.

Line B. Complete days selected from each month for their high total daily radiation. All the days of each month were selected on which the radiation had exceeded a value which was kept constant in all months of the same name but which was varied from month to month to obtain a representative segregation of days. The limiting value in Cal./cm.^{2}/day and the number of days selected were as follows:

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
Radiation	150	250	350	450	600	650	550	500	450	350	230	200
No. of days	30	28	36	43	38	43	44	35	36	24	15	10

This group of high radiation days formed about 8% of the total number of available days.

Line C. Very limited groups of five or six complete days selected for having the highest daily radiation recorded in the 14 Januaries, Februaries, etc: these groups therefore represent only about 1% of all available days.

Line D. Maximum recorded or "ceiling" values of daily radiation at Kew Observatory on exceptionally clear occasions; they were formed as described above, from very short periods.

The notable points about Table 4 (see also Figure 5) are:

(1) The total direct radiation received on the average day at Kew (line A) is only from 11% (January) to 38% (June) of the value recorded on the clearest complete days (line C): the average for the year is under 25%.

/(2)

(2) On the class of days represented by line B, direct radiation reaches 75% of the maximum values at Kew (line D); and on the very limited number of very clear complete days (line C) it amounts to about 82% of the maximum. Had the process of selection been extended to making up days from groups of hours each of exceptionally high radiation value, a closer approximation to the limiting value would have been obtained, though probably not nearer than 88 or 90%, because the ceiling values in line D were derived from the envelope of limiting intensities, derived from short intervals often not longer than a few minutes.

(3) The percentages of theoretically possible bright sunshine S derived from the Campbell-Stokes recorder for the year as a whole in the three classes of days represented by lines A, B and C of Table 4 are 30, 76 and 82%. The direct solar radiation I and the corresponding possible sunshine values S form a linear relation $I = 7.87 (S - 10)$. Without going into questions of the defects of sunshine recorders and the significance of the percentages S in relation to cloudiness, it is enough to say that the maximum recordable sunshine at Kew on cloudless days probably does not exceed 96% of the theoretical length of day in summer and 85% in winter, and that the relationship between S and amount of cloud is not simple. There is therefore little purpose in considering the above relationship between I and S in more detail: it may be noted, however, that it gives the radiation corresponding with 96% of the possible sunshine as about 680 cal./cm.^{2}/min. whereas the I value for the clearest day in line D of Table 4 is 687.

(4) The figures in line E of Table 4, from Dorno (1919) are means from five European observatories (Montpellier, Davos, Vienna, Warsaw and Potsdam): they are remarkably similar month by month to the Kew radiation totals for all days (line A).

Line F gives the computed (Linke 1929) totals of direct radiation from a clear sky in latitude 50° with a mean turbidity factor of 2.25 (see para.13(b)). The similarity with the corresponding Kew data in line D is noteworthy.

7. Frequency distribution of daily totals.

The percentage frequency distribution of daily totals of direct radiation for each month (14 years January - June, 15 years July - December) is given in Table 5, in steps of 50 cal./cm.^{2} to 200 cal./cm.^{2}, thereafter in steps of 100 to 900. The 20th June, 1944 (941 cal./cm.^{2}) and 17th July, 1943 (902 cal./cm.^{2}) were the only two days on which the total tabulated radiation exceeded 900 cal./cm.^{2}, and they are in years when the intensities are somewhat questionable (see paragraph 6). More certain are some of the 21 days when more than 800 cal./cm.^{2} were recorded. They occurred as follows: April one day, May four days, June 10 days, July five days, and August one day. At the opposite extreme, on nearly three quarters of all days in January and December the day's total of radiation was not more than 50 cal./cm.^{2}.

8. Daily variation of intensity.

Up to this stage in the analysis, daily totals have formed the basis of the treatment. But these totals alone would be of little use to an architect or to a heating engineer who sought to estimate the radiation falling on the side of a building facing a particular direction, or to a meteorologist who wished to examine the effect on radiation intensities of the diurnal convective processes in the atmosphere. To meet such needs it would be necessary to estimate the components of radiation over a sequence of hours and this entails knowledge of (a) the azimuth and altitude of the sun for each hour and (b) hourly intensities of total radiation. These latter are given in Table 6, 7 and 8 for the three classes of days already used, viz. (A) all days, (B) days of high radiation totals, and (C) a very restricted class of the highest recorded daily totals. For comparison with these data, Table 9 gives the probable

/maximum

maximum (or ceiling) values of direct radiation at Kew Observatory derived in the manner described in para. 6. The results for March, June and December in Table 6 to 9 are represented diagramatically in Figure 6.

It is to be noted that Tables 6, 7 and 8 are derived from the measurements of I recorded over 60-minute intervals: as the radiation is seldom, if ever, steady throughout an hour, the intensities do not represent the highest momentary intensities reached within the hour.

The main features of the daily variations may be summarised as follows:-

(i) I increases rapidly in the hours immediately following sunrise and decreases equally rapidly just before sunset. In the summer months there is a clear flattening of the curve of variation in the middle of the day. The explanation is that if m is the thickness of atmosphere through which a beam of radiation penetrates to reach the ground and h the height of the sun above the horizon, dm/dh is steep for small values of h up to about 15°, then falls off rapidly above 30°. The following values illustrate this, the unit of m being the optical thickness of atmosphere traversed by radiation from a zenith sun:

h(°)	3.1	5.2	5.9	6.8	7.8	9.3	11.3	14.3	16.4	19.3	23.5	30.0	41.7	60	90
m	15	10	9	8	7	6	5	4	3.5	3	2.5	2	1.5	1.15	1

As will be discussed later (see paragraph 12) the reduction in intensity is not simply proportional to m but it is related to m. In winter the midday flattening is absent because the sun's altitude barely exceeds 15°. The flattening is emphasised in summer by convection processes carrying water vapour, dust, smoke and other particulate aggregates to greater heights in the atmosphere than in winter.

(ii) On the average of all days (Table 6) the highest radiation intensities of the year occur during May and June in the hours just before noon, but even then they do not much exceed 0.42 cal./cm.^{2}/min. On the best radiation days (Table 8) the intensity reaches 1.272 cal/cm.^{2}/min. about midday in June: in this same very limited class of days during the months March to August the average intensity exceeds 1/cal./cm.^{2}/min. about 70 mw./cm.^{2}) over the six-hour interval centred at local noon. The ceiling intensity (Table 9) is 1.347 cal./cm.^{2}/min. in the two hours centred at midday in June. In this connexion it may be observed that very few direct measurements of solar radiation at Kew Observatory in the period 1933 to 1946 have given I values above 1.3 cal./cm.^{2}/min. The greatest intensities measured using the Angström compensation pyrheliometer have been 1.36 cal./cm.^{2}/min. on 7th May, 1942, 1.34 on 5th May 1936 and 29th June, 1938, and 1.33 cal./cm.^{2}/min. on 14th May, 1938 - all for short periods of a few minutes and within half an hour of local noon.

(iii) On the average day between June and September (Table 6) the time of greatest intensity is 10 - 11 hr. local time: but on days of highest radiation values (Table 8) the greatest intensities are more likely to occur in the two-hour period centred at noon. This change in time of incidence is to be expected. On the average summer day the diurnal convective effects which produce cloud and disperse surface dust and water vapour into the atmosphere, increase steadily till the early afternoon so that the period one to two hours before noon has the best chance of strongest direct radiation. The occasions of highest daily totals, on the other hand, are the days of continuously unclouded sky and good visibility associated generally with fresh north-westerly air of Arctic origin, and brought to S.E. England with the eastward movement of a high pressure ridge after a rainy depression: subsidence in the anti-cyclonic circulation reduces the normal convective processes in the middle hours of the day. Hence the peak of radiation intensity on these days is attained near the time of highest solar altitude.

/These

These same diurnal processes also account for other asymmetries in Tables 6 to 8. For example, the afternoon intensities in summer are on the whole lower than the corresponding forenoon values, while in the winter and spring the a.m. and p.m. values equidistant from noon are approximately the same. As explained in para.6 no systematic difference was found between the a.m. and p.m. ceiling values (Table 9) when reduced to the same path lengths.

9. Radiation intensity (I) as a function of sun's altitude.

As will become apparent in subsequent sections it is necessary to express I as a function of sun's altitude instead of local time as in Table 6 to 9. This has been done in Table 10(a) for 'all' days, and in 10(b) for days selected for their high radiation values in the years 1933-46; these two groups of days were described by the same lettering in para.6. The method of derivation was as follows:

(1) The sun's altitude at Kew at each exact local hour was computed for the first and last days of each month.

(2) From curves with solar altitude as ordinate and the local time as abscissa, hourly mean altitudes (estimated in various ways as checks) were read off.

(3) These hourly mean altitudes were plotted against the corresponding intensities I given in Table 6 and 7 and the value of I read off at 5° steps of altitude, the a.m. and p.m. values being noted separately and meaned as an additional check.

Table 11 gives similar data for the ceiling intensities, described as category D in paragraph 6.

The intensities in Tables 10 and 11 have not been corrected for seasonal changes in the solar radiation intensity outside the atmosphere on account of variations in the distance between sun and earth: the correction varies from -3.3% in early January to +3.4% in early July. Even if this correction were applied, the Tables show that I for any one solar altitude h on days free of cloud in winter would remain preponderatingly higher than the corresponding intensities in summer. At $h = 15^{\circ}$ in Table 10(b) for example, no value between April and August is above 0.47 cal./cm.^{2}/min.: all the other monthly values at that altitude, except in February, are above 0.5 cal./cm.^{2}/min. That only November and December are above 0.6 is probably in part an accident of selection. As explained in paragraph 6, Class B days are those whose total radiation exceeded a limiting value set differently for each month so as to give a representative number of contributing days. The limits accepted for November and December (and perhaps to a less degree, October) were a little too high, with the result that the radiation quality of the days selected from those months is probably above the average of the remaining months of the year. This accident is instructive in showing how the selection of days for study of radiation processes can affect the results, and as each observatory has its own criteria of selection, this renders intercomparison of radiation data difficult and uncertain.

Table 11 substantiates the contentions and procedure of para.6, where the formation of the maximum intensity values was described. True 'maximum' intensities would be approximately the same in all months at the same solar altitude except for the effect of seasonal change in the distance of the earth from the sun on the 'constant' value of solar radiation intensity outside the earth's atmosphere.

It is not surprising that the direct solar radiation intensities in Table 10 vary so irregularly from month to month. As details will be considered later (see paragraph 12), the reader need only be reminded here that I at station level depends primarily on three factors; (1) the thickness of atmosphere traversed, (2) its clearness and dryness (turbidity), and (3) seasonal change in the radiation outside the atmosphere. If factor (1) alone were active the greatest intensities would occur in June at the time of highest solar altitudes.

/But

But the off-setting effect of water vapour in absorbing radiation is greatest in summer, while the suspended matter which diffusely reflects and absorbs the radiation may play its greatest part in the winter months when the concentration near the ground is greatest, or in spring and summer when convection distributes it through greater thicknesses of atmosphere. The seasonal variation in I_o would, by itself, give the greatest ground intensities in winter and the smallest intensities in summer. The combined result of these factors varies with locality, and from year to year: it also depends on the incidence of dominating spells of weather, especially if the station is near a large city or industrial area.

10. Components of direct solar radiation of normal incidence.

Before applying the $I = f(h)$ values to the discussion of atmospheric absorption of solar radiation, it is useful to note how they are used for computing the total radiation falling on a surface other than one maintained normal to the direction of incidence. If I denote the total intensity of direct solar radiation as recorded by the Gorczynski apparatus, and if the horizontal, south, west and vertically downward components be denoted by H, X (south to north), W(West to East) and Z. Then

$$\begin{aligned} H &= I \cos h \\ X &= I \cos h \cos A \\ W &= I \cos h \sin A \\ \text{and } Z &= I \sin h. \end{aligned}$$

In these expressions h and A (measured positive from south through west) are the sun's altitude and azimuth, h being derived from $\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t$ and A from $\sin A = \cos \delta \sin t / \cos h$, (where ϕ is latitude and δ declination and t is the sun's hour angle), or from $\cos A = \tan \phi \tan h - \sin \delta / \cos \phi \cos h$. ϕ for Kew Observatory is 51° 28' N.

With those primary components H, X, W and Z computed, it is easy to estimate the radiation which falls on other surfaces inclined to the principal directions. For example, the intensity is (X-W) cos 45 on a surface facing south east and X sin 30 + Z cos 30 on a south-facing surface inclined at 30° to the vertical.

The daily radiation components have been computed for the clearest conditions at Kew and are illustrated in Figure 7 for the three representative months March, June and December. Except possibly for H, their main features could have been inferred without computations from a consideration of the simultaneous changes in I, h and A as functions of local time. As shown in Figure 8 the variation of intensity H, which determines the energy falling on a vertical surface rotated so as to be always perpendicular to the line of the sun's azimuth, shows two maxima in June, at local noon $\pm$ 4¾ hours. In March the maxima are at 10 a.m. and 2 p.m., and in mid-winter they merge into a single midday maximum.

The hour-to-hour values of the radiation components were used to form Table 12 in which are compared the amounts of radiation which fall on differently orientated surfaces at Kew in the clearest conditions, that is, on the days characterised by the maximum or ceiling values as defined earlier. To obtain approximate values for other classes of days, the entries in Table 12 should be adjusted in the ratio A/D, B/D or C/D where A, B, C and D are the daily totals in Table 4. Points of interest in Table 12 are:

(i) a south-facing surface receives less direct solar radiation in June and July than in any other month of the year: at Kew it receives most in March and October.

(ii) in June a horizontal surface receives more than ten times as much direct radiation as in December. It can readily be inferred from the data discussed above that it is less advantageous in these latitudes for a house to have its four walls facing the cardinal directions than for the walls to be built to face NE, SW etc.

/11.

11. Radiation intensity as a function of air mass.

The thickness of atmosphere or, more precisely, the length of the optical path through the atmosphere traversed by a beam of direct solar radiation, depends on the sun's altitude and the sea level height of the place of measurements. The unit of thickness or air mass (m) is the optical length through the atmosphere when the sun is in the zenith. Assuming the atmosphere to be made up of concentric homogeneous strata, m = sec z where z is the sun's zenith distance. Tables of m have been worked out by Bemporad with corrections for refraction and for a standard surface pressure (p_o): a few representative values were quoted in paragraph 8. Values of m from tables should be corrected for pressure (p/p_o) but this refinement has not been introduced in this analysis.

The greatest solar altitude at Kew (62°) corresponds with m = 1.13: the sun spends 64% of its time above the horizon at or below an altitude of 30° (m = 2), and only 20% is spent above 41.7° (m = 1.5).

By interpolating in Tables 10 or 11 which give I computed as a function of solar altitude or by referring directly to the curves from which those tables were derived, values of radiation at integral values of m are obtained. The latter procedure was adopted to obtain the data for B class days (see para.6) in Table 13, except that the monthly radiation - altitude graphs were smoothed. As the higher of the forenoon and afternoon values were accepted in the process of smoothing, some of the entries in the column m = 2 of Table 13 are slightly higher than the corresponding figures in the 30° altitude column of Table 10.

Table 13 confirms the inference made in paragraph 9 that the standard of selection of representative days of high radiation was higher in November and December, and probably also in October, than in the other months of the year. But even allowing for this artificial effect, none of the columns of the first part of Table 13 shows any clear evidence of a seasonal variation in I. In the four columns m = 6 to m = 3 where at least ten months are represented, I for February to March is greater than for any of the summer months. But keeping in mind the comments of paragraph 9 in this connexion, this evidence by itself does not justify accepting a spring maximum as a normal feature of direct radiation intensity at Kew.

12. Atmospheric transmission of direct solar radiation.

The preceding discussion has been concerned with the basic results derived from continuous recording of direct solar radiation incident on a thermopile surface maintained normal to the direction of the radiation. The remaining paragraphs in this part of the paper will deal with the main atmospheric factors which contribute to the variations of this radiation and with various measures of atmospheric turbidity.

In spite of the many discussions of this subject, mainly in German, Scandinavian and American journals, it is difficult to obtain a clear picture of the quantitative contributions made by the various processes which deplete the energy in a direct beam of solar radiation while traversing the atmosphere. The instrumental equipment used, the treatment of the data, the selection for special attention of particular depleting processes and the nature of the transmission co-efficient used to express the degree of depletion vary from country to country and even from one institute to another within the same country. For ordinary purposes the intensity of direct radiation (I_o) outside the atmosphere may be taken as 1.94 cal./cm.^{2}/min. with a simple correction for earth's distance from the sun. If the intensity I at a sea-level station on a day in mid-June when the sun's altitude is 30°, is measured as 1.050 cal./cm.^{2}/min., then the transmission co-efficient q, or fractional radiation transmitted through unit thickness of atmosphere, is obtained from $I = I_o q^2$. In this hypothetical case q is 0.736. In this simple way, transmission co-efficients can be deduced immediately from such information as that provided in Tables 10 and 11. In default of an internationally acceptable procedure some important meteorological institutions have indeed adopted the hybrid transmission factor q as used above in discussing their data. But except to provide a rough numerical index for comparing the state of the atmosphere in regard to its depleting effect on solar radiation at one

/instant

instant in one locality with its state at another instant in the same locality, q has very limited value and little physical significance. For the relation between I and I_o using q, or its alternative in terms of an absorption co-efficient $I = I_o e^{-am}$ in which $a = -\log_e q$, is valid only for monochromatic radiation and only for particular physical processes.

Nevertheless, if measures of total direct radiation, as distinct from sinultaneous measures of two or more of the partial spectral components, are the only recorded material at a station, there is pratically no alternative to the adoption of the above procedure or a variant of it. Various investigators (e.g. Linke, Angstrom, Gorczynski) have proposed and used turbidity factors, co-efficients of turbidity, etc.: but all of them postulate an integration over a range of wave lengths, and some even over a variety of depleting phenomena which should not be grouped together.

Before examining transmission phenomena further, it is desirable to consider the main physical processes which go to reduce the intensity of a direct solar beam. They are:-

A. Scattering, by molecules and particles of size less than the wave length of the radiation.

B. Selective absorption by (a) atmospheric gases
(b) ozone
(c) water vapour

C. Diffuse reflexion, by molecular aggregates and particulate matter of size greater than in A.

D. Combination of scattering (or diffuse reflexion) with absorption.

E. Reflexion from, and absorption by, cloud.

Molecular (or fine particle) scattering varies with the wave length λ^{-4} and as the distribution of energy in the spectrum alters with the thickness of the atmosphere penetrated (i.e. with the sun's altitude h), the amount of scattering also changes with h. There can therefore be no simple transmission (or extinction) co-efficient to take account of molecular scattering which is valid throughout the whole range of wave lengths in a solar beam and over all values of h. By estimating the energy in the radiation outside the atmosphere in successive spectral intervals (with a correction for the extreme ultra violet) and by careful measurement of the radiation received at the earth's surface in the same intervals in the clearest atmospheric conditions, confirmation of the computed loss of energy by Rayleigh scattering has been obtained (Fowle, 1917: Feussner and Dubois, 1930) and expressed in terms of a single transmission co-efficient which for practical estimates can be identified with q in Bouguer's law $I = I_o q^m$. Values of this co-efficient for air mass thicknesses m = 1 to 8 and the corresponding values of the transmitted radiation (taking I_o = 1.940 cal./cm.^{2}/min.) are given in Table 14.

From this Table we see that molecular scattering alone accounts for the loss of 16% of the initial intensity when the sun's height is 30°, and 31% at 9° (m = 6). At local noon at Kew at the summer solstice (m = 1.13) the loss is 7%.

As regards item B in the above list, although the absorption of solar radiation by oxygen is of fundamental importance for some phenomena in the atmosphere, the contributions to total depletion made by oxygen absorption and by other atmospheric gases except ozone, are negligible for our purpose. The effect of ozone is subject to large variations, of place, and time, and synoptic situation: but, taking three millimetres of ozone as an average quantity, Cabannes and Dufay (1927) estimated that absorption in the range below 0.4 μ is about 5% of the initial intensity, 0.5% between 0.4 and 0.8 μ, and 0.1% above 0.8 μ. Taking 8, 56 and 34% as the percentages of total energy in those three ranges in a solar beam outside the atmosphere, ozone absorption accounts for about 0.81% of the total, i.e. only about 0.016 cal./cm.^{2}/min.: it is therefore negligible in relation to the inaccuracies and uncertainties associated with the other agencies.

/The

The item B(c) has been studied principally by Abbot and Fowle. Measuring the radiation energy in specific groups of wave lengths and simultaneously estimating the amount of water vapour in the atmosphere by the broadening and intensification of the absorption bands in the red and near infra red, Fowle (1918), has given the following empirical relation for the partial reduction (Δ I) of energy due to water vapour distributed in an air mass m corresponding with a mean value e(mm) of vapour pressure at the surface:

$$\Delta I = 0.10 + .0054\ me$$

Average values of e for Kew Observatory for representative months give the figures in Table 15 for air masses 2 and 4. It is particularly to be noted that Fowle's expression connecting Δ I and e is valid only for the averages of many occasions. For individual cases it is necessary to use more direct measures of the amount of water vapour in the atmosphere obtained, for example, by the spectrometer or by estimation from radio sonde observations.

The amount of diffuse reflexion by suspended matter of size greater than the radiation wave length, and the reduction in energy due to absorption after scattering or reflexion cannot be directly estimated even approximately. There is no known method of obtaining an independent measure of the suspended matter (smoke, dust, spores, nuclei, ions, etc.) in a column of the atmosphere and even if there were, the range of size and nature of the particles would preclude application of any physical law to the calculation of their effect on radiation. All that can be done with this group of agencies is to attribute to it the residual depletion after account has been taken of the other factors.

Reflexion from, and absorption by, cloud sheets or particles is normally excluded from this kind of inquiry by restricting the measurements of incoming radiation to occasions of no visible cloud between the sun and station. But unless the sky is under constant observation this procedure can be only partially successful. It is frequently noted in such work that comparatively large changes in recorded radiation take place within a minute with an apparently clear sky in the neighbourhood of the sun. These changes are due in many cases to the movement across the line of sight of the pyrheliometer of pockets of atmosphere where cloud is just dissipating or forming though the cloud density is not sufficient to make it discernible at ground level.

The discussion in the foregoing paragraphs is illustrated by Tables 16 and 17. Table 16 uses the maximum (or ceiling) radiation intensities, and Table 17 uses the intensities from days of high total radiation though not necessarily completely free of cloud: in both tables, section (a) deals with radiation through air mass 2 and section (b) through air mass 4. The entries in the first line of each table are derived from the appropriate columns of Table 13, using values of I_o corrected for earth's distance at the middle of the period. The figures which purport to give the depletion due to ozone absorption were derived from the considerations outlined in the preceding paragraph and are included in the table only to illustrate the probable size of the ozone contribution relative to the other agents. A correction (which may have doubtful validity) was applied to take approximate account of the seasonal change in amount of ozone in the latitude of Kew. The effect due to water vapour absorption was derived using Fowle's relationship from normals of vapour pressure at Kew, not from estimates of actual amount of precipitable water on the days concerned, as is desirable.

At air mass 2, according to Table 16, the most important factor which reduces radiation on exceptionally clear days is molecular scattering (41%) with diffuse reflexion and absorption by suspended matter (33%) and water vapour (24%) next in order. Ozone accounts for 1-2% in the total depletion. For air mass 4 there is no substantial change; the percentages are, for scattering 46%, for suspended matter 30% and for water vapour 23%. A feature of note in Table 16(b) which includes data from all months of the year is that the effect of suspended material is greatest in February-March and least in the mid-summer months.

On the group of days represented in Table 17 (76% of possible sunshine), the depletion with air mass 2 due to dust and other particulate matter (54%) is greater than all the other effects together: scattering contributes 28% and

water vapour (17%). At lower solar altitudes (air mass 4) these figures become 43, 36 and 18%. Table 17 also suggests that suspended matter tends to have its greatest effect in reducing radiation intensity in the spring.

13. Measures of atmospheric transmission or turbidity.

As mentioned at an earlier stage in this discussion, the equipment for recording radiation at Kew Observatory has recently been extended to include a second actinograph carrying two thermopiles covered by coloured glass filters mounted on a heliostat. After preliminary standardisations and comparisons now in progress, each thermopile will record a particular range of the solar spectrum. There will therefore be available measurements of two spectral components of direct solar radiation as well as the total radiation, and from these it will be practicable to compute indices of atmospheric turbidity of greater reliability than can be derived from readings of total radiation alone. Meanwhile it is desirable to indicate the nature of the various indices that have been proposed and to use them for comparing the Kew Observatory data with those from other localities.

(a) The transmission co-efficient (q')

This is the simplest and crudest measure of the state of the atmosphere as regards its depletion of solar radiation: it is derived from $I = I_0 q'^m$ applied throughout the range of the spectrum and without attempt to distinguish it from the true q as applied to monochromatic radiation and to particular depleting agencies. As can be seen from Table 18 where q' values are given for air masses 2 and 4 for maximum intensities and for the days of high radiation at Kew, q' is not independent of the air mass m.

The q' values for Potsdam in Table 18 are of interest. They are based on figures given by Marten (1926), and, judging by that author's description of the sky conditions when the measurements were made, they are to be related to the "maximum intensity" data at Kew rather than to any of the other groups of days used in this discussion. The mean q' of the six pairs of months for Potsdam is the same as for Kew, viz. 0.806; but whereas Potsdam has a clear maximum in winter and minimum in mid-summer, the seasonal range at Kew is negligible.

It should be added that the value used throughout this section for I_0 (1.94 cal./cm.^{2}/min.) has been corrected for earth's distance.

(b) Linke's turbidity factor (T)

Linke's turbidity factor is derived from an extension of the same basic formula, expressed in the form $I = I_0 e^{-am}$, where a is an absorption co-efficient ($-a = \log_e q$). Using the term extinction to describe the total depletion in a beam of solar radiation whether attributable to true scattering to absorption, or to diffuse reflexion or combinations of these processes, Linke (1933) put the co-efficient of total extinction $a = a_g + wa_w + sa_s$ where a_g is the contribution by Rayleigh scattering, a_w the absorption co-efficient for water vapour and a_s the co-efficient describing the effects produced by suspended molecular aggregates and particles: w and s are measures of the content of water vapour and suspended matter (not water vapour) in the beam. Linke then expressed his turbidity factor T in terms of the contribution due to true molecular scattering a_g by putting $T = (a_g + wa_w + sa_s)/a_g$ so that the extinction law becomes $I_m = I_0 e^{-Tag m}$ from which T could be computed as $\frac{1}{m\, a_g \log e} \times \log I_0/I$

The usefulness of the factor T derived in this way depends on the assumption that the separate depletions of solar radiation by water vapour and by particulate material both vary with wave length in the same way as the depletion by true molecular scattering. This is hardly justified. Scattering is inversely proportional to λ^{-4}, while the exponent for diffuse reflexion by larger aggregates may be anywhere between 0.5 and 4, and absorption follows a quite different law. So long as T includes a two-fold integration (over λ and m) and also attempts to provide for a variety of different physical processes, it cannot be a satisfactory, much less a unique, index. Notwithstanding these and other objections, Linke's factor has been fairly widely used, the values (which are usually between 1.5 for mountain stations and 5 or 6 for stations in

/industrial

industrial localities) being interpreted as the equivalent number of dry and clean atmospheres which together would produce the same degree of depletion as the actual atmosphere through which the beam reaches the station.

Judged by the representative values of T given in Table 19 the turbidity factor at Kew Observatory is probably seldom less than 2.3, and is more usually 5 or higher. The precise value depends so much on how the radiation data used in the evaluation have been selected (or, in the case of stations without continuous records of direct radiation, on the criteria of sky conditions adopted for making direct measurements over a few minutes by pyrheliometer or actinometer), that there is little profit to be derived from comparison with T values at other stations. As most comparable with the Kew conditions for maximum radiation intensities the following values for low level stations are, however, of interest (Linke, 1929).

Potsdam	Winter	1.99	:	Frankfurt on Main	Winter	3.08
	Summer	2.72	:		Summer	3.79

In support of Linke's contention that T does not in fact vary substantially with air mass when other variables remain the same, the maximum radiation data in Table 18 give a mean value of T = 2.83 for both m = 2 and m = 4: the corresponding figures for good radiation days are 4.75 and 4.53.

As the basic data used in section (a) of Table 19 were not direct measurements and as the values in section (b) are affected by selection of the days, particularly in the last two or three months of the year, a further set of independent values of the turbidity factor T were computed. These were derived from the highest recorded values of normal direct radiation during the two hours centred at local noon in each available month of the three years 1945-1947. The year's mean for this group of occasions is 2.79, but instead of being practically steady from month to month as in section (a) of the Table, these independent values of turbidity have a minimum in the spring and a maximum in late summer.

(c) Angström's co-efficient of turbidity (β).

In contrast with Linke's turbidity factor which aims at giving a measure of the total extinction effects of scattering, absorption and diffuse reflexion in terms of the scattering effect as unity, Angström's co-efficient (1929) refers only to the suspended matter: the effects of pure scattering and water vapour absorption are separately estimated. The loss of radiation intensity dI_λ at a wave length λ is given by $dI_\lambda = -\beta I \frac{dm}{\lambda^\alpha}$ where the exponent α depends on the average size of the suspended particles, and, according to Angström, may normally be taken as 1.3. The loss by true scattering can be provided for by a transmission co-efficient q, so that, excluding absorption by water vapour,

$I_\lambda = I_{o\lambda}\, q_\lambda^{\,m}\, e^{-\beta m/\lambda^{1.3}}$ where $I_{o\lambda}$ and I_λ are the intensities at wave length λ before and after penetrating the air mass m. This is only valid if there is no selective absorption in the neighbourhood of λ.

The total Q_m measured at the ground is obtained from

$$Q_m = \int_0^\infty I_{o\lambda}\, q_\lambda^{\,m}\, e^{-\beta m/\lambda^{1.3}}\, d\lambda - F .$$

F in this expression is the absorption by water vapour, which is independently calculated. Angström's expression for Qm can be applied to total radiation intensity but, like Linke's turbidity factor, it is preferable to use it in conjunction with simultaneous measurements of total radiation and one of its major spectral components.

One of the obvious weaknesses in Angström's treatment arises from the necessity of assigning a fixed value to the exponent α. Almost certainly α is a complex function of the size distribution of the suspended matter in the atmosphere and therefore varies considerably with time and locality, and with the synoptic situation and previous history of the air over the station of measurement. As regards the treatment of water vapour, there is a choice between measuring separately the radiation in that region of the solar spectrum which is

free from the major bands of water vapour absorption, or of using an empirical relationship such as that due to Fowle. As this uses surface vapour pressure as a measure of the total water in the atmosphere it is therefore rather inadequate: at the best it can be used only for groups of synoptically similar days with approximately similar humidity distributions in the atmosphere.

For comparison with the values of co-efficient of turbidity (β) worked out for other localities, the data of Table 13 have been used to provide the β values for Kew given in Table 19.

It may be noted that if a relationship existed between T and β, then computed values of T might be used to estimate the amount of water vapour in the atmosphere. The corresponding values of T and β in Table 19 are, indeed, linearly related: for air mass 2 the approximate expression is $T = 1.8 + 1.7\beta$. Feussner (1932) found a similar relationship for the short wave bands of solar radiation but Kimball and Hand (1933) found that estimates of water vapour so derived did not compare successfully with more direct measurements.

...

II. Direct and diffuse short wave radiation from sun and sky on a horizontal surface

14. Introduction and instruments used.

As explained in the Introduction to Part I, equipment was set up at Kew Observatory in June 1946 for continuous recording of the total short wave radiation from sun and sky (T) which falls on a horizontal surface, and, separately, the diffuse radiation from the sky alone (D). As information on these two aspects of solar radiation is complementary to the data on direct solar radiation of normal incidence described in Part I, a summary of the first fourteen months' results may usefully be given here. It is to be noted that both the T and D data to be discussed refer only to short wave radiation within the range 0.3 μ and 3 μ : whether it comes from an apparently blue sky, when its maximum intensity is about 0.4 μ, or from dense cloud when the maximum is shifted to 0.8 μ, D is simply solar radiation which has been scattered and diffused by molecules, molecular aggregates, suspended matter or cloud particles in its progress through the atmosphere.

The two similar thermopiles used for receiving the T and D radiation are exposed horizontally on the south balustrade of the Observatory roof near the Gorczynski pyrheliograph. Each pile has 14 thermojunctions arranged to form a square of one centimetre side, set flush with the edge of a heavy cast-metal cup with which the cold junctions are in thermal contact. The thermopile surface is covered by two glass hemispherical domes one millimetre thick and 30 mm. and 50 mm. in diameter, sealed to the edge of the cup. The space above the junctions and under the inner glass dome is kept dry by a dessicator let into the bottom of the cup by an airtight screw. In the same plane as the junctions, the thermopiles are surrounded by a circular metal guard-ring to prevent radiation from below the horizontal plane through the thermopile falling on the junctions. The output of both thermopiles is led off to a thread recorder so that both T (sun and sky) and D (diffuse radiation alone) are recorded on the same chart.

To cut off the direct sun radiation from one thermopile, a ring of strip brass is mounted so that, with slight movements of the ring each day along a polar axis, direct radiation from the sun is prevented from falling on the thermopile. The breadth of the ring is such that at its distance from the thermopile (13.5 cm.) this is just effected. A correction is applied to the tabulated values of diffuse sky radiation to take account of the fraction of sky radiation which is also cut off by the ring: the correction varies from 7% in mid-summer to 3% in mid-winter. An additional constant correction is applied to take account of obstructions to a clear horizon.

Figure 1 shows the arrangement of the two thermopiles and the rest of the radiation equipment on the Observatory roof.

/15.

15. Calibration of the thermopiles.

The details of calibration of the two thermopiles supplied with the instruments have been used for reduction of the records. By eclipsing the direct solar rays from the fully exposed (T) thermopile at the same time as measurements of the intensity of direct solar radiation (I) are made with the standard Angström pyrheliometer, it has been confirmed that the calibration factor used for this thermopile is approximately consistant with the scale values adopted for the other radiation equipment. The results of more extensive tests in future months covering a greater range of solar altitudes than has been possible in this first year's recording may necessitate the adoption of a modified factor. But any such change is not likely to exceed 2% of the value hitherto used, so that the results to be discussed in subsequent paragraphs can be taken as of the same general order of accuracy as radiation measurements made elsewhere with this type of equipment, viz. within about 5%.

A brief reference should be made to the influence on the results of the two glass hemispherical domes which protect the thermopiles. Earlier models of this kind of solarimeter with only one small glass dome of thickness greater than one millimetre had been found (Gorczyński, 1927) to have a sensitivity which varied with the sun's altitude: with a single dome of 30 mm. diameter the sensitivity at altitudes below 10° was only about half of the sensitivity above 60°. The defect was attributed to the formation of caustics on the thermopile by the inner surface of the dome. By introducing a second and larger dome and also by reducing the thickness of the glass, the effects of this and other optical defects of the solarimeter are said to have been substantially lessened. According to Mörikofer (1939), the variation of the standardisation factor over the normal range of sun's altitudes is only a few percent, and is not a simple function of the altitude. This has been tested for the Kew instruments by tabulating values of the ratio $r = (T-D)/I \sin h$ where I is the direct solar intensity normal to the direction of the radiation and h is the sun's altitude. Using the maker's calibrations for the T and D thermopiles, the ratio is practically constant, though slightly greater than unity, over the range $20° < h < 60°$, then increases somewhat for values of $h < 20°$.

16. Monthly and daily totals, T and D.

Table 20 gives the monthly totals and daily means of short wave radiation from sun and sky, and the diffuse radiation from the sky alone on a horizontal surface. The table is based on the hourly tabulations of records of T and D from July 1946 to August 1947, the "year" being formed by the single representative months September 1946 to June 1947, together with the means of the two Julys and the two Augusts. Owing to minor troubles in the early stages of recording, the basic tabulations were not complete for both T and D for every day of each month. The monthly totals and daily means for T and D in the first four lines of Table 20 are derived from the same sets of complete days in each month, the totals being made up to complete months by corrections proportional to the number of missing days, while the means are derived directly from the number of days of completely recorded data, which number is the same for T and D in each month.

The next four lines of Table 20 give for comparison corresponding data for direct solar radiation of normal incidence (I). The first pair of those lines is for the same period and made up in the same way as for T and D except that there was no need to correct the monthly totals for missing days; the second pair of lines ($\bar{I}$) is extracted from Tables 2 and 4 and give the long term I values derived from the years 1933-46.

The I data in Table 20 show that the "year" represented in that table was about 5% above the 1933-46 average in direct (normal incidence) radiation, but February, March (1947) and October, November (1946) were below average, particularly February when only 842 cal./cm.^{2} were received on the Gorczynski pyrheliograph as compared with an average of 2031 for the preceding fourteen Februaries.

/The

The year's total of diffuse radiation D on a horizontal surface from the sky alone, 49,068 cal./cm.^{2}, was 56% of the total (sun and sky) radiation T, and 84% of the direct radiation from the sun (I) on a surface normal to its direction. The monthly ratio D/T varies between 0.5 and 0.75, the smaller ratios being associated with the summer months: I/T is more irregularly variable between 0.5 and 0.93, the least and greatest values occurring in the same season.

Over the year the average daily T income is 238 cal./cm.^{2} with a seasonal variation from about 50 cal./cm.^{2} in mid-winter to 470 in mid-summer: diffuse sky radiation contributes 30-35 cal./cm.^{2} on the average day in mid-winter and 250-260 cal./cm.^{2} in mid-summer.

17. Extreme values of T, D and I, and the sky conditions which produced them.

Table 21 gives the highest and lowest daily totals of T and D in the period July 1946 to August 1947 with corresponding data for I. Of greater interest than the actual values are the sky conditions when they occurred. As would have been expected from the discussion of I in the Part I of this paper, the highest I occurred on a day when the sky was not more than 1/10th clouded: the radiation from the sky alone on that day was only 16% of the total income from sun and sky together. The highest daily T, on the other hand, was recorded when the sky was about half covered with broken cumulus cloud: D was then 29% of T. On the day of highest D the sky was three quarters covered with broken low cloud (cumulus and stratocumulus) and some medium and high cloud (altocumulus, altostratus and cirrus) until near the end of the day: the total recorded sunshine was only 1.8 hours, being 11% of the possible. On that day the diffuse sky radiation was 83% of the total income from sun and sky together. On all three days of high T, D and I the visibility was very good.

The day of lowest T (which was also the day of lowest D) was uninterruptedly overcast with dense low cloud: it was continuously foggy with intermittent rain and snow.

That the sky and visibility conditions on these isolated days of extreme T, D and I values are generally representative of the conditions for high values of the three kinds of radiation was confirmed by examining the records of observations on days in each season selected for their high T, D and I totals. In comparison with the best I days which were the days of least cloud, the highest totals of sun and sky (T) radiation occurred when the sky was 3/10 - 6/10ths. covered with broken cloud of fair-weather, vertically-structured type (cumulus), with extensive reflecting surfaces which do not substantially reduce the direct solar component and contribute more by diffuse reflexion of the direct sun's radiation than they provide by internal scattering. Without exception, the highest D totals, on the other hand, occurred on days when cloud amounts were never less than 7/10ths (though not overcast): the cloud was invariably broken and mixed, cumulus, stratocumulus and altocumulus predominating. The visibility was good to very good.

Table 22 gives the mean daily totals of radiation income for the three groups of days in the period May to August 1947 selected as having the highest values of T, D and I. The second D line relates to a limited group of days selected as having the highest D/T ratios and the last line of the table gives the four-month averages for all complete days.

According to Table 22 each square centimetre of horizontal surface at Kew Observatory on the average summer day receives 431 calories from sun and sky together; 51% of this is diffuse sky radiation. But on those days when the total radiation is greatest (i.e. when there are 2/10ths. to 5/10ths. of broken cloud of cumulus type), the diffuse sky radiation falls to below 30% of the total income; and in those conditions the direct solar radiation of normal incidence has about the same intensity as the total radiation intensity on a horizontal surface from sun and sky together. When the sky is at least 7/10ths. clouded with a mixture of broken low cloud and patches of medium or high cloud, i.e. in the best conditions for high D values, the proportion of D to T increases to 70%. On certain days of this category, as shown by the second group of

values for the brightest D days in Table 22, the ratio D/T can reach 81%. When conditions are most suitable for the highest values of direct solar radiation, i.e. on days of uninterruptedly clear sky and clear atmosphere, D may be less than a quarter of T (c.f. Table 21).

18. Variation of T and D on days of highest T and D income.

Table 23 gives the hourly values of total and diffuse radiation likely to be received on a horizontal surface at Kew on the best days for these aspects of solar radiation. The manner of forming the daily variations was to select from the best radiation days those values of T (or D) when the highest readings had been attained, so building up a composite complete day. The process was similar to that used for normal direct solar intensities as described in para.6, except that 60-minute intervals were used instead of intervals of a few minutes as for I. Because of this difference the occasions of peak values of T (or D) did not require to be so accurately related to solar altitude. The hour-to-hour values of T and D on those composite days are plotted in Figure 9, together with other curves to be referred to later.

According to Table 23 the limit of daily total of T radiation is about 800 cal./cm.^{2}: this is only 8% above the highest daily total actually recorded on a particular day, viz., 730 cal./cm.^{2} on 17th June, 1947. The corresponding daily limit of D, 493 cal./cm.^{2} is 45% greater than was actually recorded on the day of highest D total, 2nd July 1947.

It is unlikely that these are true ceiling values in the sense of being the highest attainable in the most advantageous conditions at Kew, and in the sense in which the ceiling values of I were derived as explained in para.6 and given in Table 4, according to which the greatest daily total of I likely to be received at Kew is 1035 cal./cm.^{2}. Using recorded hourly values of I from only three months, May to July 1947 a composite day can be formed with a daily I total of 982 cal./cm.^{2}, so the estimated limit of 1035 is clearly not impracticable. Now I for the composite day which gave the total of 789 cal./cm.^{2}, for T was 794 cal./cm.^{2}, so confirming the first line of Table 21 that on the highest T days T is only slightly less than I. It therefore seems that a value only slightly less than 1035 cal./cm.^{2} is not an unreasonable upper limit to accept also for T. There is no similar process for allowing the limiting daily value of D (493 cal/cm.^{2}) to be modified.

It will be understood from the considerations of the preceding paragraph that the values for T and D in Table 23 relate to two different types of sky conditions.

19. Diffuse solar radiation from a cloudless sky.

The preceding paragraph has been concerned with high values of T and D radiation received on a horizontal surface at Kew. As has been explained these high values occur on days when the sky is partly covered with particular types of cloud, T with 2/10ths. to 6/10ths. cumulus cloud, and D with 7/10ths. to 9/10ths. of mixed broken cloud, preferably cumulus, stratocumulus and altocumulus. As reflexion of direct solar radiation is an important contributing factor to high T and D values on those days, it is likely that isolated vertically-structured cloud around a zone of the sky 25° - 45° above the horizon is the most effective distribution for the highest flux of diffuse radiation at the ground.

Of equal interest is the flux of diffuse radiation from a cloudless sky and also from a completely clouded sky, particularly in relation to solar altitude. Curve 4 in Figure 9 shows the mean hourly variation of D built up from a group of cloudless days and parts of days in summer. Of the complete days used, the day's total of D income ranged from 110 cal./cm.^{2} on the clearest day to 188 cal./cm.^{2} on a day of heat haze: the turbidity factors (see para.13(b) for direct solar radiation on those two days were 2.8 and 4.1. Had there been enough cloudless days in mid-summer with uniformly clear atmosphere to form a smoothed curve corresponding with 110 cal./cm.^{2} daily income curve 4 would probably have been reduced to curve 5 in Figure 9.

/Figure 10

Figure 10, which expresses D from curve 4 of Figure 9 in terms of sun's altitude (h), shows that the intensity of diffuse solar radiation on cloudless days in summer with clearness of atmosphere corresponding with turbidity 3.2 is a function of h, having a maximum at Kew of 0.23 cal./cm.^{2}/min. at h = 60°. On the clearest days this value probably does not exceed 0.20 cal./cm.^{2}. In those two sets of conditions the contributions of direct solar radiation to the total intensity on a horizontal surface would be 1.25 and about 1.28 cal./cm.^{2}/min. respectively.

20. Diffuse radiation from a cloudy sky.

For many practical purposes it is desirable to extend the analysis of the preceding paragraph to cloudy conditions, and if possible to provide a relationship by which estimates of the daily income of diffuse radiation can be derived from a knowledge of cloud amount, type and thickness. If the diffuse income (D) could be estimated in this way, it would be a simple matter to estimate the total radiation (T) from sun and sky together, because $T = D + I \sin h$, and I can be inferred in a much more direct manner than D from measures of sunshine or cloud amount. In the fourteen months over which the Kew D material extends there have, however, been few occasions when the same type of sky has persisted throughout the day, except in winter when the variation of sun's altitude is inadequate for the purpose. The analysis has therefore been based on groups of hours of apparently similar cloud amount, type and thickness: the relation between the short-interval flux of radiation on those occasions to daily totals will be developed in paragraph 21.

For reasons which will soon be obvious it has been impracticable to establish a numerical relationship between D and the state of sky: an outline of the results of the analysis are summarised in the two parts of Figure 11, Figure 11(a) being concerned primarily with the influence of changes in cloud amount on the flux of D at the ground, and Figure 11(b) with the effect of cloud density as characterised for this purpose by cloud type and thickness. The ordinates in both parts of the figure give the approximate D intensities when the sun's altitude h is $30° < h < 50°$. As has been shown in the preceding paragraph the actual intensities vary with h but more slowly than with cloud amount or density at moderate values of h.

Section A of the curve in Figure 11 shows how the diffuse radiation increases from about 0.23 cal./cm.^{2}/min. with a cloudless sky to about 0.75 cal./cm.^{2}/min. when the sky is 8/10-9/10ths. covered with cumulus, strato-cumulus or altocumulus type of cloud. As the cloud merges to form a continuous sheet so that diffuse reflexion of direct solar radiation is cut off, the sky radiation begins to decrease, at first slowly then more rapidly with increasing thickness of the cloud sheet (altostratus). If at this stage the cloud becomes thinner again, though remaining as a sheet (altostratus - cirrostratus), the radiation continues to fall to about 0.30 cal./cm.^{2}/min. Thereafter, as illustrated in Section B of the curve, further dissipation leads to disintegration of the sheet and the diffuse radiation decreases more slowly to its cloudless value of 0.20 to 0.25 cal./cm.^{2}/min. depending on the haziness of the atmosphere.

Figure 11(b) is intended to illustrate that this is not the whole story. With a completely clouded sky the intensity of diffuse radiation at ground level can assume a range of values from 0.25 cal./cm.^{2}/min. with a thin veil of cirrus to about 0.65 cal./cm.^{2}/min. as the cloud density increases through cirrus, cirrus-stratus, thin altostratus etc. to a continuous sheet of strato-cumulus. Thereafter as the strato-cumulus sheet thickens through altostratus, to stratus and nimbostratus, the intensity decreases to 0.1 cal./cm.^{2}/min., and finally, with heavy and extensive cumulonimbus and altostratus, to values of 0.05 cal./cm.^{2}/min. or less.

The foregoing analysis illustrates the impracticability of devising a formula by which diffuse radiation income, and therefore total sun and sky radiation, can be related to the state of the sky. With 8/10ths. cloud amount the D intensity can range between 0.25 cal./cm.^{2}/min. and 0.75 cal./cm.^{2}/min., and between 0.05 and 0.65 cal./cm.^{2}/min. with 10/10ths cloud. The density and structure of the cloud are at least as important as its amount, and there is no

simple way of allowing for them. Even when the structure of a cloud sheet as observed from ground level has appeared to remain the same over several hours, measurements of intensity of daylight now being made at Kew Observatory have shown that substantial changes in thickness, or density or nature of the upper surface (as regards albedo) or changes in amount and type of higher cloud layers have been in progress. Daylight intensity could of course, be taken as a measure of cloud density, and indeed there is close direct relationship between daylight intensity and T: but the maintenance of a thermopile to record radiation is no more difficult than the recording of daylight.

21. Relation between intensity and daily income of diffuse radiation.

Although the relationship between D and the state of sky is therefore too complex for a formula, the results illustrated in Figure 12 can be utilised to estimate roughly the total amount of diffuse radiation falling on a horizontal surface on a day of specified cloud amount and type. Figure 12 gives the relationship between D intensity around midday and the day's total income of D for sky conditions assumed to remain similar throughout the daylight hours. If therefore the intensity of diffuse radiation for the particular conditions of cloud type and amount in the middle of the day is deduced from Figure 11, Figure 12 gives approximately the day's total radiation in those conditions. The present objections to using this procedure is that Figure 11 takes account only of conditions of moderate sun's altitude.

22. Daily variation of T and D.

To illustrate the average relationship between T and D at various times of the year, mean daily variations of T and D were formed from all days in each month for which complete recordings were available in both T and D. The months were grouped in four sets, November to January, February to April, etc. to represent seasonal change in behaviour. Daily variations were similarly formed for solar radiation of normal incidence (I) from the same sets of complete days. The results are reproduced in Figure 13 in which the uppermost curve in each seasonal set is T, and the lowest one I.

Figure 13 shows that even fourteen months' data are sufficient to illustrate the main daily features of the three radiation elements, which are:

(1) the greater contribution of direct solar radiation to T in the a.m. and p.m. hours than in the hours immediately around noon. As this arises from a higher cloudiness and turbidity in the midday hours than when convection processes are relatively subdued, the effect is most conspicuous in summer and least in the winter months.

(2) direct radiation on a surface normal to its direction and D on a horizontal surface are of the same order of size.

(3) the direct and diffuse components of T are complementary in the sense that when one decreases the other increases to keep the total sun and sky radiation approximately constant. This is a simple effect arising from broken cloud acting as a reflector and scatterer while reducing direct radiation intensity: it is best seen in the recordings for particular days but the average result is still apparent in the I and D curves for spring and summer in Figure 13.

References

Guild, J. 1937	Proc. R. Soc. London. A.161 pp. 1-38
Dorno, C. 1919	Die Wissenschaft, 63, Braunsweig.
Linke, F. 1929	Lehrbuch der Geophysik. p.666
Fowle, F.E., 1917	Smithsonian Misc. Coll. 68. No.8 1917.
Feussner and Dubois, 1930	Ger. Beitr. z Geophysik 27 E.
Cabannes and Dufay, 1927	Journal de Physique 6e serie 8.
Fowle, F.E. 1918	Met. Zeit 31. 270.
Maten, W. 1926	Veröff. des Preuss. Met. Inst. 336. Bd. VIII Nr.4.
Linke, F. 1933	Met. Taschenbuch: Vol.II.
Angström, A. 1929	Geog. Annalen. Vols. 11, 12.
Feussner, K. 1932	Tätigkeitsber. Preuss. Met. Inst. 89.
Kimball and Hand 1933	M. Weather Rev. 61. 80.
Gorczynski, W.L. 1937	M. Weather Rev. 55, 488.
Mörikofer, W. 1939	Handbuch der biol. Arbeitsmethoden Abt. II. Teil 3.

Table 1(a). Factor F before and after reconditioning of thermopile.

	Jan.	Feb.	Mar.	Apr.	May.	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1943	13 1.10 21 1.21	3 1.15	5 1.29 9 1.24 12 1.30 16 1.23 17 1.21 18 1.29	7 1.10	13 1.17 14 1.26 18 1.08 19 1.17	28 1.06	1 1.11 2 1.13 27 1.19	17 1.23	22 1.28 23 1.19 24 1.20 27 1.23	-	10 1.14 19 1.24 25 1.31	2 1.28 22 1.27 30 1.18
1946	3 .97 12 .99	27 .97	23 .99 26 1.01	1 1.00 3 1.01 16 1.02 20 .99	26 1.02	28 1.01	3 1.00 10 1.00	-	27 .99	11 1.01 24 1.02	4 1.01	4 1.00 10 .98 20 1.00

Table 1(b). Adopted values of F.

1933 - 7: 1.00
1938: 1.03 until October.
1.04 by steps to 1.15 through November and December.
1939: 1.15 throughout.
1940: 1.16
1941: 1.17
1942: 1.18
1943: 1.20 till 31st July then 1.23
1944: 1.26
1945: 1.20 till March. After repair of the thermopile the resistance of the thermopile circuit was adjusted to restore F to its original value of 1.00.

Table 2. Monthly totals of direct radiation. (cal/cm^2)

	Jan.	Feb.	Mar.	Apr.	May.	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Means 1933-46	1167	2031	4039	5765	6913	6911	7554	7207	5422	3365	1434	1126
1933-41	1167	2015	4124	4669	7118	7035	7656	6965	5613	3427	1467	1011
1942-46	1222	2071	3855	7702	6406	6675	7494	7642	5070	3253	1515	1333
Highest (Yr.)	1635(37)	3017(39)	7169(36)	9016(42)	12203(43)	12141(40)	11234(34)	9604(43)	7433(34)	4042(35)	2324(40)	1941(36)
Lowest (Yr.)	506(41)	692(40)	2121(42)	3149(41)	5434(41)	6773(34)	6306(44)	5305(39)	2334(45)	2561(46)	918(39)	465(33)

Table 3. Annual totals of direct radiation. (cal/cm^2)

1933	34	35	36	37	38	39
62101	56554	56306	47321	51022	52069	52106

1940	41	42	43	44	45	46
60178	48044	59972	69233	56030	55108	55405

1933-40 56033
1933-41 55936
1942-46 59720

Table 4. Daily totals of direct radiation for various classes or days
Unit: cal/cm^2/min.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Mean
A	38	72	130	192	258	294	245	233	181	108	49	36	154
B	196	295	451	550	701	742	654	603	534	418	279	223	519
C	260	346	591	685	766	877	810	773	607	490	342	237	566
D	338	495	679	849	971	1035	999	895	748	571	379	291	687
E	43	74	132	199	201	310	305	272	194	98	51	32	166
F	375	525	712	882	983	1055	1018	932	766	599	414	295	712

A All days (1933-46). B days of high radiation C days of highest recorded radiation
D ceiling values at Kew Observatory.
E means from five European observatories (c.f. line A for Kew)
F computed values for clear sky in latitude 50° with turbidity factor 2.25 (c.f. line D)

Table 5. Percentage frequency distribution of daily totals (cal/cm^2) of direct radiation: all days.

	< 50	51-100	101-150	151-200	201-300	301-400	401-500	501-600	601-700	701-800	801-900	901-950
Jan.	73	12	7	5	3							
Feb.	60	12	7	8	10	3						
Mar.	40	14	11	10	9	10	4	2	< 1			
Apr.	31	11	10	8	12	11	8	5	3	1	< 1	
May	22	9	10	7	14	13	11	6	5	2	1	
June	16	10	9	9	15	12	8	9	6	3	2	< 1
July	22	11	11	8	15	11	7	7	4	3	1	< 1
Aug.	21	12	9	10	15	14	10	5	3	1	< 1	
Sept.	31	14	10	6	14	10	8	6	1			
Oct.	45	13	14	9	11	6	2	1				
Nov.	69	12	10	4	5	< 1	< 1					
Dec.	73	12	8	4	3							

Table 6. Intensity of direct radiation (I): all days (1933-46)
Unit: cal/cm^2/min.

LMT hour ending	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Jan.					.001	.020	.077	.111	.130	.124	.103	.064	.024	.001				
Feb.				.001	.021	.085	.145	.174	.195	.174	.154	.105	.035	.023	.001			
Mar.			.001	.034	.106	.166	.207	.231	.243	.231	.202	.190	.107	.022	.001			
April		.001	.029	.130	.218	.263	.298	.329	.337	.344	.334	.301	.261	.204	.116	.020	.001	
May		.019	.112	.205	.276	.331	.375	.416	.430	.395	.378	.352	.347	.295	.240	.131	.023	.001
June	.001	.054	.162	.258	.330	.382	.411	.428	.425	.401	.398	.383	.363	.330	.279	.192	.075	.001
July	.001	.040	.140	.228	.291	.310	.338	.370	.378	.340	.337	.322	.307	.277	.230	.148	.037	.001
Aug.		.003	.063	.166	.244	.314	.344	.387	.379	.375	.365	.350	.317	.275	.205	.082	.004	
Sept.			.009	.090	.196	.246	.318	.349	.344	.338	.315	.280	.240	.184	.089	.006	.001	
Oct.				.005	.064	.150	.201	.250	.259	.251	.224	.192	.139	.061	.005			
Nov.					.003	.049	.095	.135	.155	.150	.141	.107	.040	.003				
Dec.					.001	.012	.061	.111	.125	.123	.097	.057	.016	.001				

Table 7. (I)on days of high radiation: Unit: cal/cm^2/min.

LMT hour ending	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Jan.					.001	.121	.304	.421	.453	.426	.404	.243	.106	.002				
Feb.					.104	.306	.442	.417	.436	.767	.554	.451	.263	.001	.001			
Mar.			.001	.153	.439	.535	.702	.777	.832	.869	.838	.792	.645	.445	.139	.001		
Apr.			.079	.357	.599	.771	.848	.955	.977	.976	.944	.882	.768	.593	.326	.007	.001	
May		.035	.247	.504	.650	.842	1.022	1.124	1.142	1.133	1.090	1.016	1.007	.867	.650	.403	.007	.003
June		.110	.340	.592	.745	.876	1.033	1.102	1.129	1.093	1.072	1.026	.980	.862	.736	.512	.153	.004
July		.105	.367	.571	.735	.905	.903	.979	1.006	.973	.904	.891	.849	.722	.602	.388	.097	.002
Aug.		.005	.154	.431	.672	.852	.934	1.031	1.007	.990	.905	.907	.793	.636	.450	.180	.019	.001
Sept.			.023	.233	.599	.754	.902	1.020	1.037	.976	.881	.853	.753	.540	.268	.022		
Oct.				.031	.254	.629	.863	.956	.995	.921	.921	.692	.565	.247	.023			
Nov.					.028	.229	.535	.834	.870	.737	.572	.503	.266	.035				
Dec.					.010	.139	.464	.652	.734	.699	.537	.359	.077					

Table 8. (I) on selected days of highest radiation. Unit: cal/cm^2/min.

LMT Hour ending	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Jan.						.123	.517	.770	.823	.820	.666	.450	.160	.001				
Feb.					.164	.517	.802	.912	.933	.738	.686	.588	.331	.088				
Mar.				.183	.550	.807	1.037	1.177	1.210	1.217	1.117	1.033	.887	.537	.103			
Apr.			.172	.567	.753	.939	1.063	1.147	1.167	1.106	1.114	1.081	.922	.767	.492	.133		
May		.043	.190	.693	.770	.947	1.097	1.183	1.143	1.147	1.117	1.057	1.123	.990	.697	.490	.077	
June		.181	.525	.775	.892	1.006	1.172	1.253	1.272	1.272	1.244	1.172	1.131	1.006	.853	.611	.422	
July		.136	.344	.664	1.036	1.108	1.167	1.172	1.181	1.109	1.147	1.086	.909	.675	.806	.553	.194	
Aug.		.010	.257	.587	.847	.957	1.050	1.123	1.227	1.220	1.193	1.173	1.073	.907	.757	.423	.070	.003
Sept.			.031	.374	.710	.814	.957	1.107	1.084	1.04	.821	.903	.981	.800	.436	.038		
Oct.				.043	.200	.777	1.007	1.013	1.007	.903	1.047	.790	.660	.380	.027			
Nov.					.053	.263	.720	.927	.967	.847	.783	.783	.727	.340	.070			
Dec.					.017	.109	.469	.700	.792	.739	.625	.353	.058					

Table 9. (I) "ceiling" intensities. Unit: cal/cm^2/min.

LMT Hour ending	5 20	6 19	7 18	8 17	9 16	10 15	11 14	12 13
Jan.					.320	.713	.850	.925
Feb.				.353	.710	.925	1.040	1.090
Mar.			.350	.732	.997	1.133	1.207	1.233
Apr.		.340	.725	1.015	1.170	1.245	1.285	1.300
May	.163	.542	.960	1.143	1.237	1.293	1.320	1.333
June	.370	.778	1.033	1.103	1.265	1.312	1.333	1.347
July	.267	.700	.992	1.157	1.250	1.300	1.325	1.337
Aug.	.050	.442	.817	1.070	1.200	1.267	1.300	1.317
Sept.		.063	.513	.827	1.075	1.183	1.242	1.266
Oct.			.117	.503	.832	1.027	1.117	1.167
Nov.				.042	.462	.767	.905	.933
Dec.					.242	.500	.767	.830

Table 10. (I) as function of sun's altitude (h). Unit: cal/cm^2/min.

	h(°)	5	10	15	20	25	30	35	40	45	50	55	60
(a) All days	Jan.	.025	.058	.103									
	Feb.	.023	.067	.100	.132	.192							
	Mar.	.033	.095	.130	.175	.205	.247	.275					
	Apr.	.035	.100	.150	.192	.223	.252	.275	.305	.327			
	May	.052	.110	.177	.225	.262	.293	.322	.342	.353	.375	.397	
	June	.065	.132	.195	.245	.285	.310	.345	.360	.383	.397	.408	.415
	July	.055	.120	.173	.220	.253	.270	.298	.295	.317	.320	.333	
	Aug.	.053	.122	.167	.227	.265	.293	.325	.340	.365	.385		
	Sept.	.043	.112	.167	.203	.235	.260	.290	.317				
	Oct.	.035	.095	.152	.187	.223	.255						
	Nov.	.033	.07	.122	.160								
	Dec.	.020	.072	.117									
(b) Days of high total radiation	Jan.	.135	.290	.522									
	Feb.	.095	.263	.440	.625	.767							
	Mar.	.157	.347	.510	.705	.717	.802	.897					
	Apr.	.128	.300	.425	.555	.667	.770	.828	.897	.950			
	May	.100	.246	.372	.494	.612	.713	.826	.930	1.012	1.092	1.140	
	June	.150	.322	.457	.585	.695	.767	.858	.923	.970	1.037	1.077	1.105
	July	.153	.320	.467	.580	.665	.740	.792	.833	.882	.917	.950	
	Aug.	.160	.317	.453	.575	.673	.767	.843	.902	.973	1.007		
	Sept.	.140	.353	.555	.657	.767	.843	.917	.990				
	Oct.	.120	.307	.597	.757	.855	.942						
	Nov.	.172	.317	.623	.800								
	Dec.	.203	.458	.697									

/ Table 11.

Table 11. Maximum ("ceiling") intensity I as f(h) Unit: cal/cm^2/min.

h(°)	5	10	15	20	25	30	35	40	45	50	55	60	62
Jan.-Dec.	.308	.620	.867										
Feb.-Mar.	.357	.640	.827	.978	1.098	1.190	1.258	1.298					
Apr.-May	.372	.628	.812	.952	1.058	1.142	1.205	1.250	1.282	1.302	1.313	(1.333)	
June-July	.385	.648	.823	.955	1.058	1.137	1.200	1.243	1.285	1.310	1.328	1.337	(1.340)
Aug.-Sept.	.365	.635	.835	.988	1.105	1.185	1.240	1.277	1.303	1.322	1.340		
Oct.-Nov.	(.400)	.657	.895	1.032	1.110	1.152	1.183						

Table 12. Direct radiation falling on various surfaces on clearest days.
Unit: cal/cm^2.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Normal to radiation	330	495	679	849	971	1035	999	895	748	571	379	291
Vertical towards sun	327	467	602	695	739	753	748	701	643	526	364	285
Vertical facing south	291	376	405	356	279	235	230	231	287	397	316	251
Vertical facing north				8	43	70	52	16	1			
Vertical facing west	64	117	186	257	298	319	286	246	217	145	77	53
Horizontal	81	152	283	449	575	642	569	465	356	209	99	59

Table 13. (I)as function of air mass (m). Unit: cal/cm^2/min.

High radiation days.

m	6.0	5.0	4.0	3.0	2.0	1.5	1.25
Jan.	.300	.383	.547				
Feb.	.300	.375	.500	.700			
Mar.	.353	.400	.570	.580	.900		
Apr.	.253	.330	.410	.542	.770	.920	
May	.225	.300	.380	.453	.713	.960	1.130
June	.360	.447	.530	.657	.853	.997	1.090
July	.327	.367	.465	.587	.753	.865	.970
Aug.	.310	.370	.457	.597	.793	.970	1.050
Sept.	.342	.412	.547	.673	.902	1.047	
Oct.	.367	.467	.608	.782	.997		
Nov.	.395	.508	.683	.870			
Dec.	.508	.595	.713				

Occasions of "ceiling" intensity.

Dec.-Jan.	.580	.697	.833				
Feb.-Mar.	.615	.698	.803	.963	1.183		
Apr.-May	.600	.697	.783	.933	1.137	1.260	1.307
June-July	.620	.697	.803	.948	1.137	1.265	1.327
Aug.-Sept.	.607	.697	.810	.970	1.183	1.288	1.333
Oct.-Nov.	.630	.727	.862	1.013	1.157		

Table 14. Transmission coefficient for molecular scattering.

Air mass (m)	1	2	3	4	6	8
Corresponding sun's altitude, h(°).	90	30	19.3	14.3	9.3	6.8
Coefficient q	.907	.915	.921	.926	.935	.942
Transmitted I (cal/cm^2/min.)	1.76	1.62	1.52	1.43	1.30	1.20

Table 15. Reduction in I by water vapour absorption Unit: cal/cm^2/min.

	Jan.	Apr.	July	Oct.	Year
m = 2	.154	.166	.211	.132	.178
m = 4	.210	.233	.323	.264	.255

Table 16. Depletion of direct radiation by various agencies on clearest days.
Unit: cal/cm²/min.

	Jan. Dec.	Feb. Mar.	Apr. May	June July	Aug. Sept.	Oct. Nov.	Mean
(a) Air mass 2							
Total depletion		.789	.774	.740	.722	.812	.767
Molecular scattering		.322	.311	.306	.311	.321	.314
Ozone		.020	.017	.015	.013	.014	.016
Water vapour		.159	.173	.205	.206	.175	.184
Suspended matter		.288	.273	.214	.192	.302	.253
(b) Air mass 4							
Total depletion	1.172	1.174	1.128	1.074	1.095	1.107	1.125
Molecular scattering	.531	.524	.507	.496	.505	.521	.514
Ozone	.017	.020	.017	.015	.013	.014	.016
Water vapour	.217	.218	.246	.310	.311	.249	.258
Suspended matter	.407	.412	.358	.253	.266	.323	.337

Table 17. Depletion of direct radiation on selected days of high radiation.
Unit: cal/cm²/min.

	(a) Air mass 2.				(b) Air mass 4.			
	Mar.	May	June	Sept.	Jan.	Apr.	July	Oct.
Total depletion	1.161	1.180	1.125	1.026	1.458	1.517	1.413	1.344
Scattering	.320	.309	.306	.313	.531	.510	.496	.518
Ozone	.020	.017	.015	.012	.017	.018	.015	.013
Water Vapour	.163	.179	.211	.199	.210	.233	.323	.264
Suspended matter	.658	.675	.595	.502	.700	.756	.579	.549

Table 18. Transmission coefficients (q')

(a) "Ceiling" radiation intensities

		Jan. Dec.	Feb. Mar.	Apr. May	June July	Aug. Sept.	Oct. Nov.
Kew	m = 2		.775	.771	.778	.788	.767
	m = 4	.803	.798	.800	.809	.807	.813
Potsdam	m = 4	.830	.815	.797	.783	.787	.817

(b) Days of high total radiation.

		Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Kew	m = 2			.639	.632	.615	.674	.633	.648	.682	.715		
	m = 4	.722	.706	.711	.679	.664	.729	.705	.701	.724	.747	.766	.772

Table 19. Turbidity factor (T) and coefficient of turbidity (β)

(a) "Ceiling" radiation intensities.

		Dec. Jan.	Feb. Mar.	Apr. May	June July	Aug. Sept.	Oct. Nov.
Air mass 2	T		2.94	2.90	2.80	2.86	2.97
	β		.062	.073	.064	.052	.068
Air mass 4	T	2.87	2.94	2.92	2.77	2.79	2.70
	β	.060	.065	.064	.048	.046	.049

(b) Days of high total radiation

		Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
m = 2	T			5.00	5.13	5.44	4.41	5.11	4.85	4.28	3.75		
	β			.190	.204	.225	.151	.136	.175	.142	.116		
m = 4	T	4.24	4.49	4.45	5.05	5.36	4.13	4.56	4.64	4.22	3.81	3.48	3.37
	β	.132	.145	.139	.134	.179	.114	.121	.122	.111	.099	.089	.083

(c) Occasions of highest recorded values.

T	2.80	2.89	2.73	2.29	2.67	2.92	2.90	3.07	2.81	2.82	2.81	2.81

Table 20. Monthly totals (t) and daily means (m) of T and D radiation on a horizontal surface, and of direct (normally incident) radiation I.
Unit: cal/cm^2.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
T(t)	2128	1679	4641	9897	11888	14130	13727	12095	9105	4255	2028	1531	87104
T(m)	66	60	150	330	383	471	443	390	304	137	67	49	238
D(t)	1389	1251	3070	5089	6327	7082	8013	6017	5638	2810	1364	1018	49068
D(m)	45	45	99	170	204	236	258	194	188	91	45	33	134
I(t)	1860	842	2838	7796	8297	10070	8111	8903	4515	2561	1240	1416	58449
I(m)	64	30	91	260	268	323	257	281	163	81	44	46	159
$\overline{I}$(t)	1137	2031	4029	5765	8013	8811	7594	7207	5422	3365	1484	1126	56033
$\overline{I}$(m)	38	72	130	192	259	294	245	233	181	108	49	36	154
D/T(%)	65	75	66	52	52	50	58	50	62	66	67	67	56

Table 21. Highest and lowest daily totals of T, D and I.

	T				D				I			
	cal/cm^2	Date	Corresponding D	Corresponding I	Cal/cm^2	Date	Corresponding T	Corresponding I	cal/cm^2	Date	Corresponding T	Corresponding D
Highest	730	17/6/47	211	764	339	2/7/47	411	55	880	28/5/47	696	110
Lowest	7	6/2/47	3	0	3	6/2/47	7	0	0	6/2/47	7	3

Table 22. Daily totals T, D and I on selected groups of days.
Unit: cal/cm^2.

	No. of days.	T	D	I	D/T (%)	I/T (%)
Days of highest T	13	652	192	667	29	102
" " " D	16	430	301	170	70	41
" " " D	5	531	309	72	61	19
" " " I	13	628	169	693	27	110
All complete days	117	431	220	294	51	68

Table 23. Highest hourly values T and D radiation
Unit: cal/cm^2/hr.

Hour ending (L.A.T.)	4	5	6	7	8	9	10	11	12	
T	1	6	16	31	48	61	73	81	87	
D	1	6	15	24	31	39	44	48	50	
Hour ending	13	14	15	16	17	18	19	20	21	Total
T	87	80	68	56	42	30	17	4	1	789
D	49	47	40	34	26	20	12	6	1	493

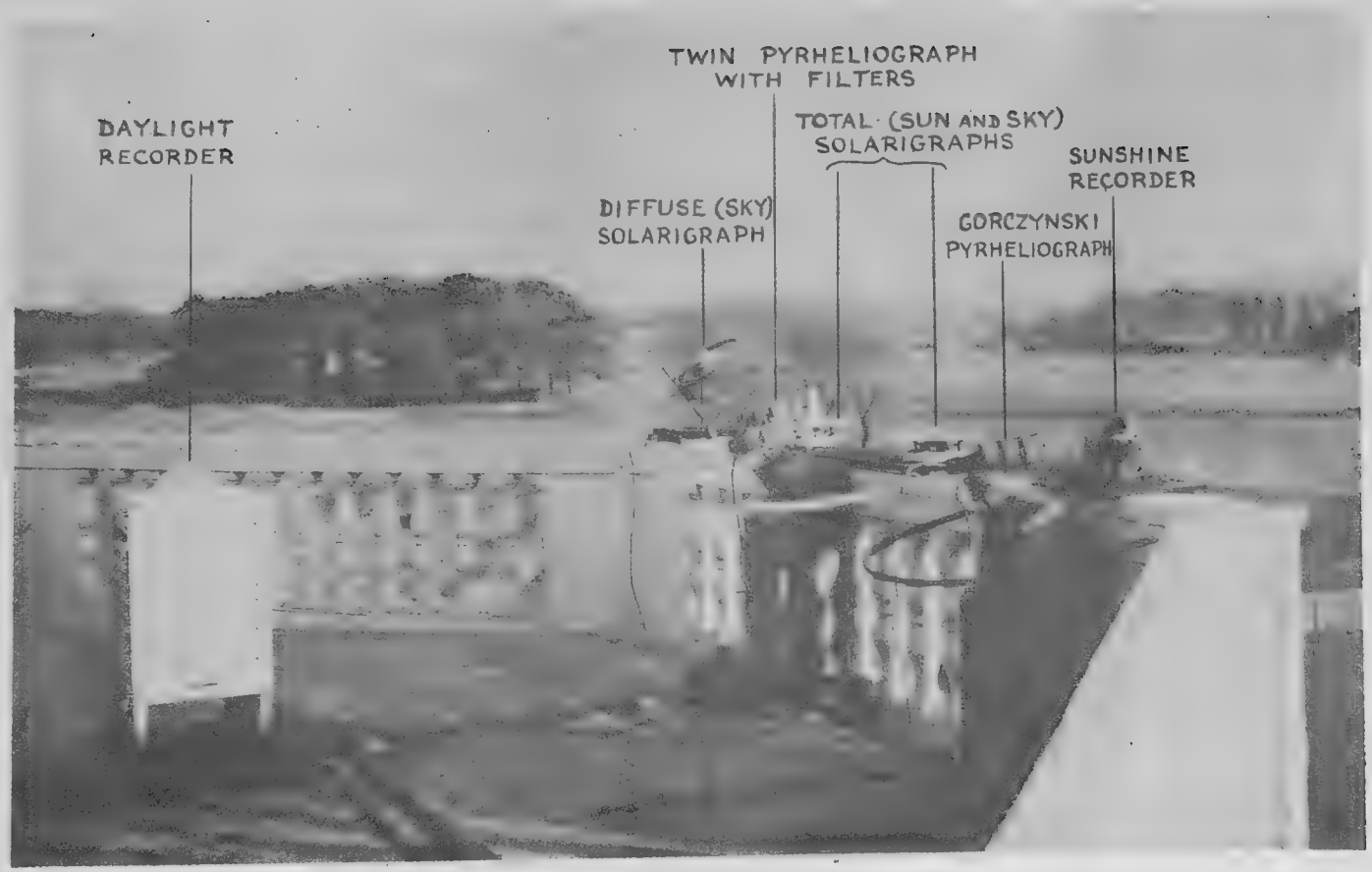


Fig 1. Solar Radiation equipment at Kew Observatory.

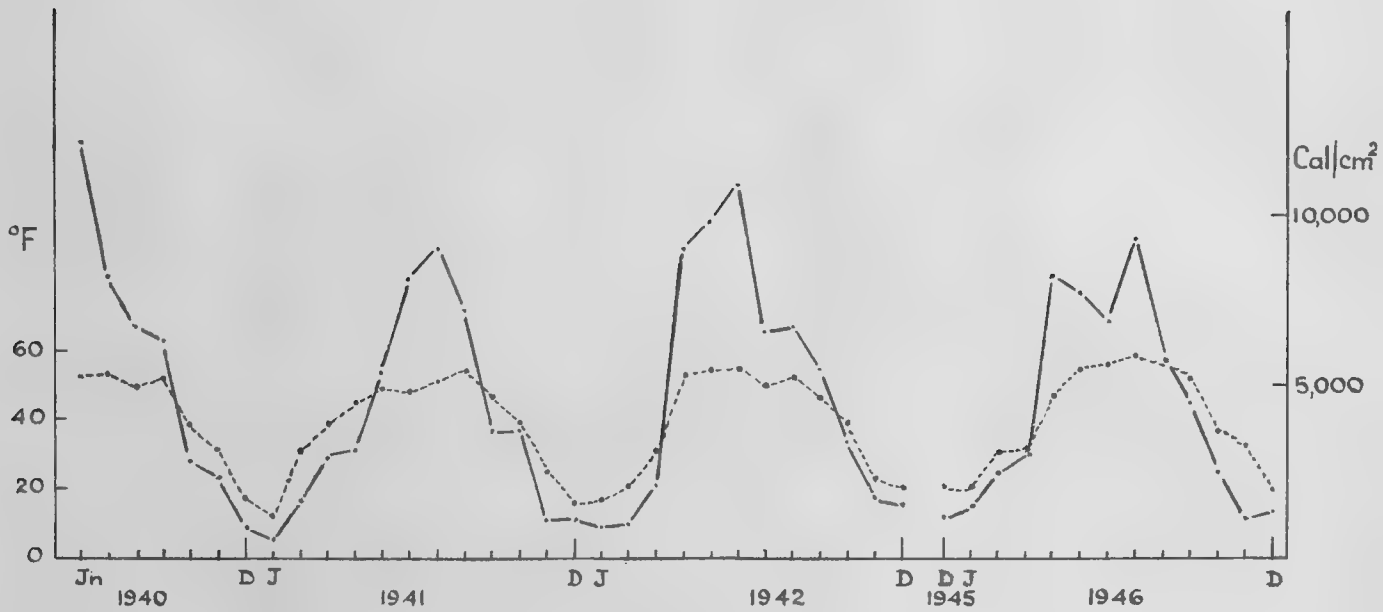


Fig. 2. Direct radiation and (solar max.-screen) temperature.

•·····• Temperature.

•———• Radiation

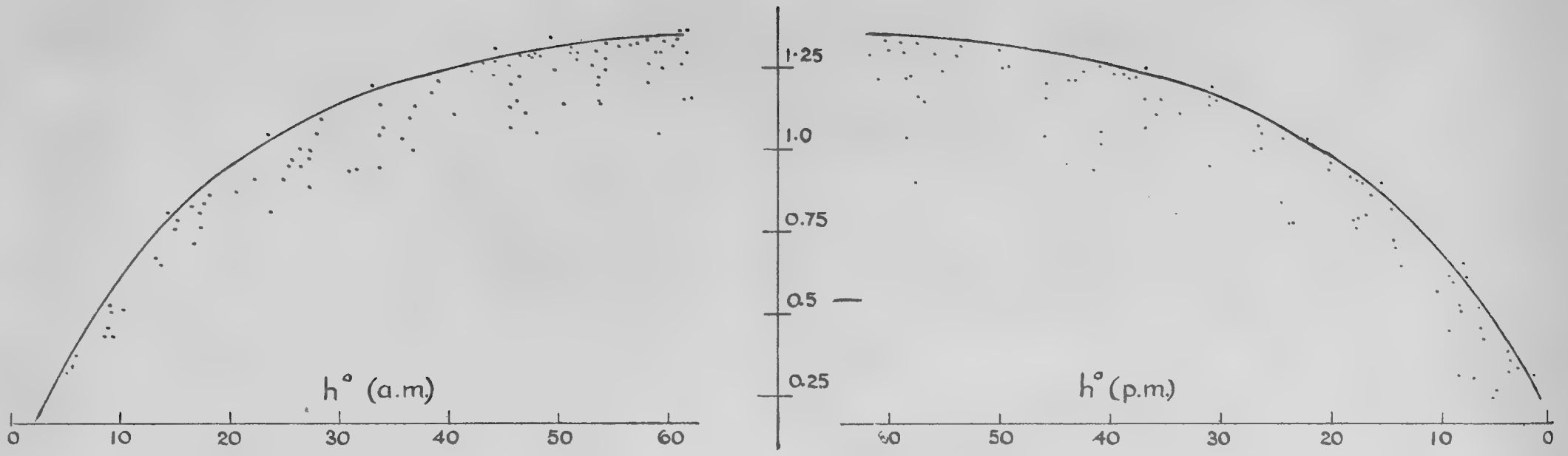


Fig. 3. I (cal./cm.²/min.) on clearest occasions as f(h). (June and July together).

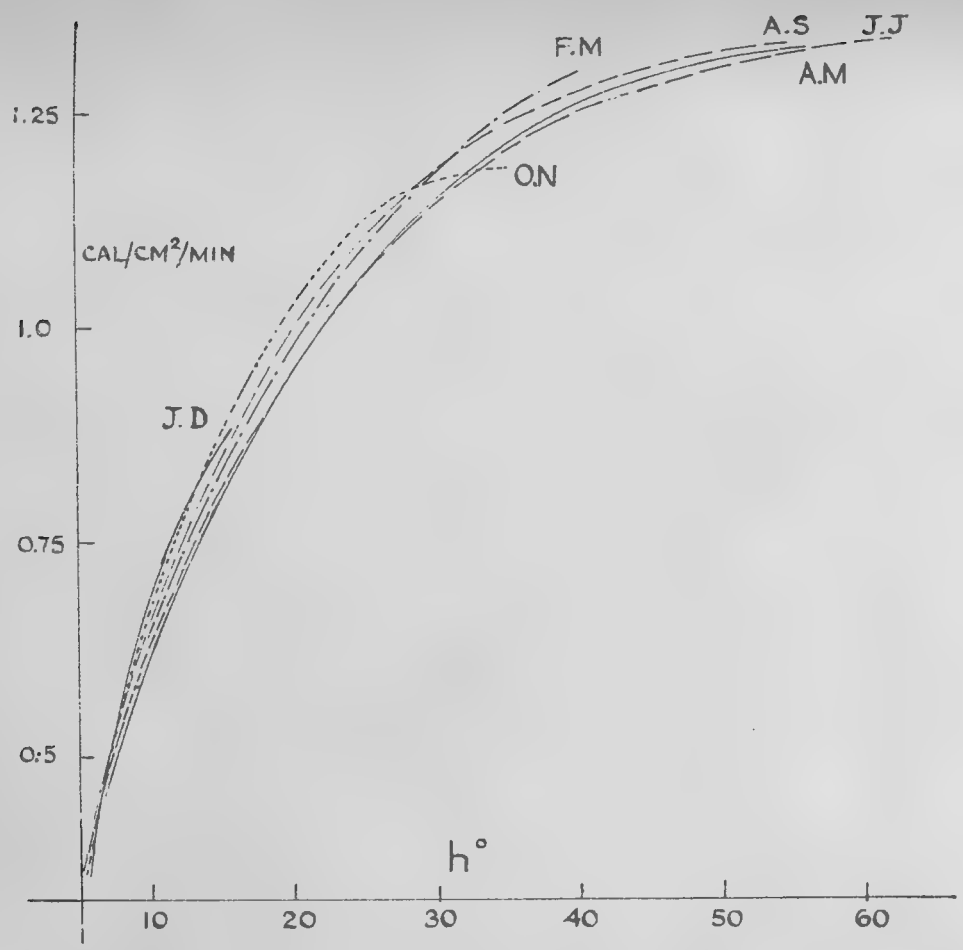


Fig. 4. I (cal./cm.²/min.) as f(h) on clearest occasions in pairs of months.

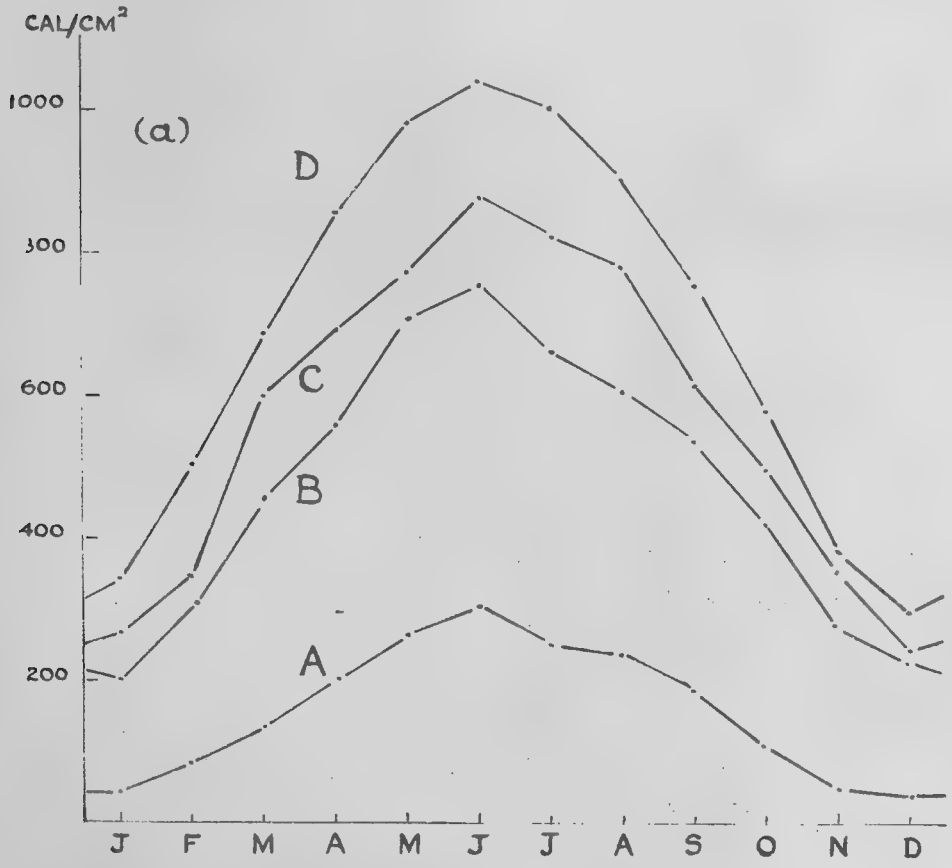


Fig. 5. Daily totals of direct radiation (cal./cm.²).

A all days
B days of high radiation
C complete days of highest radiation
D ceiling radiation values

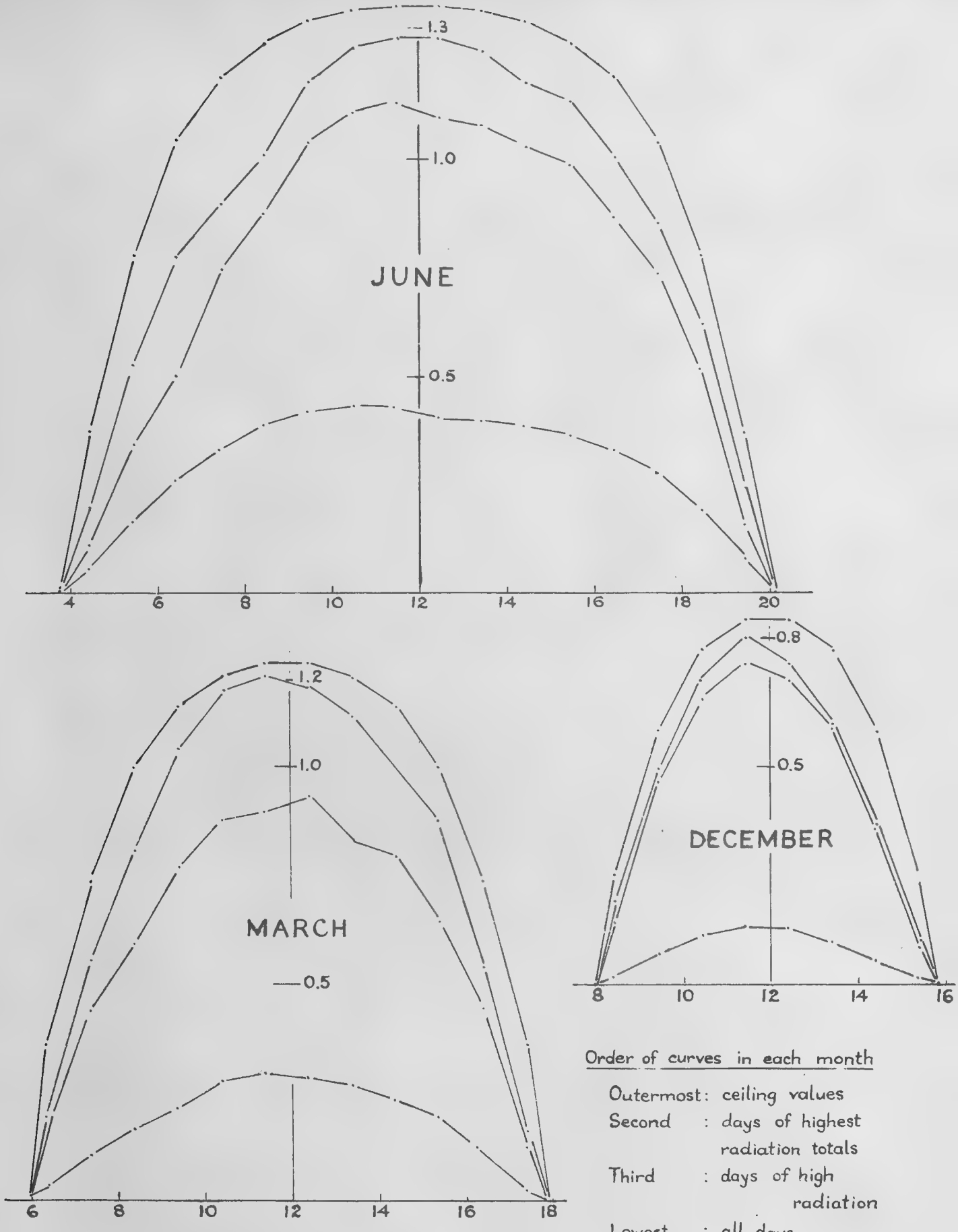


Fig. 6. I (cal./cm.²/min.) as function of local time.

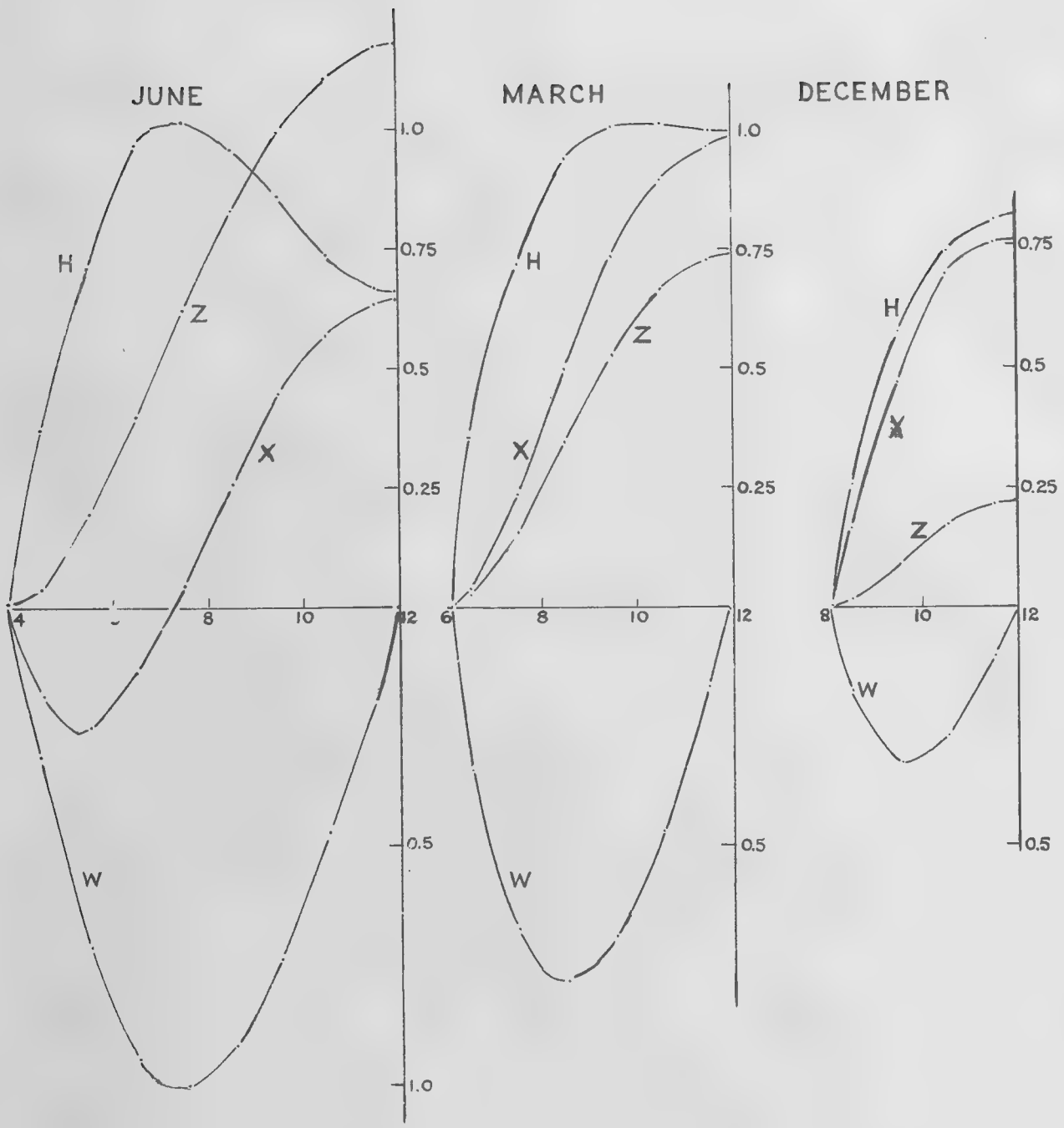


Fig. 7. Component intensities of direct radiation (cal./cm.²/min.)

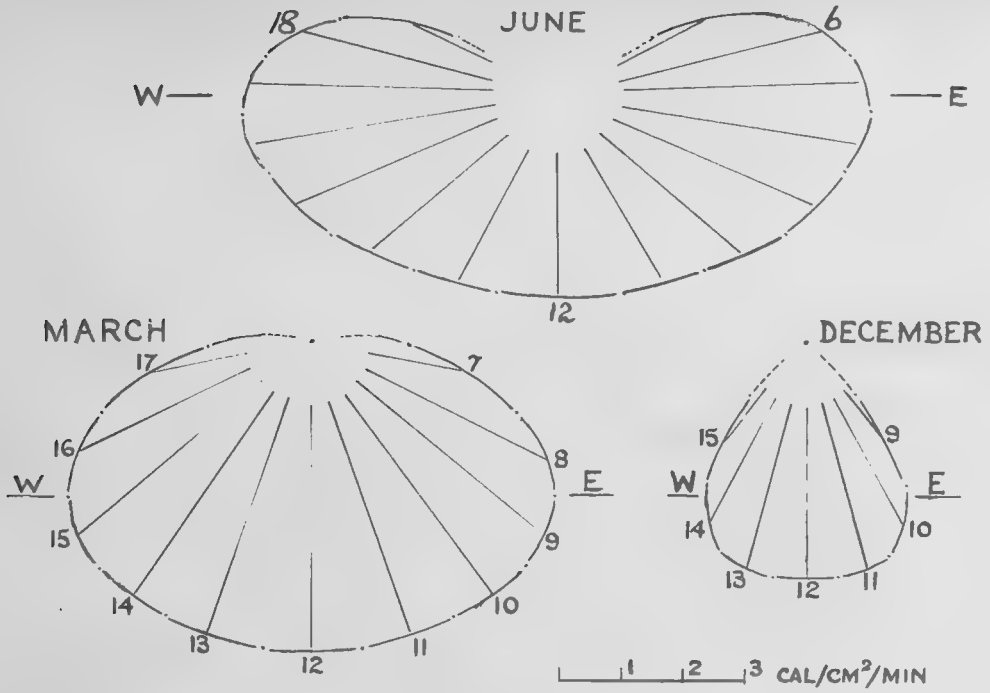


Fig. 8. Horizontal component intensities

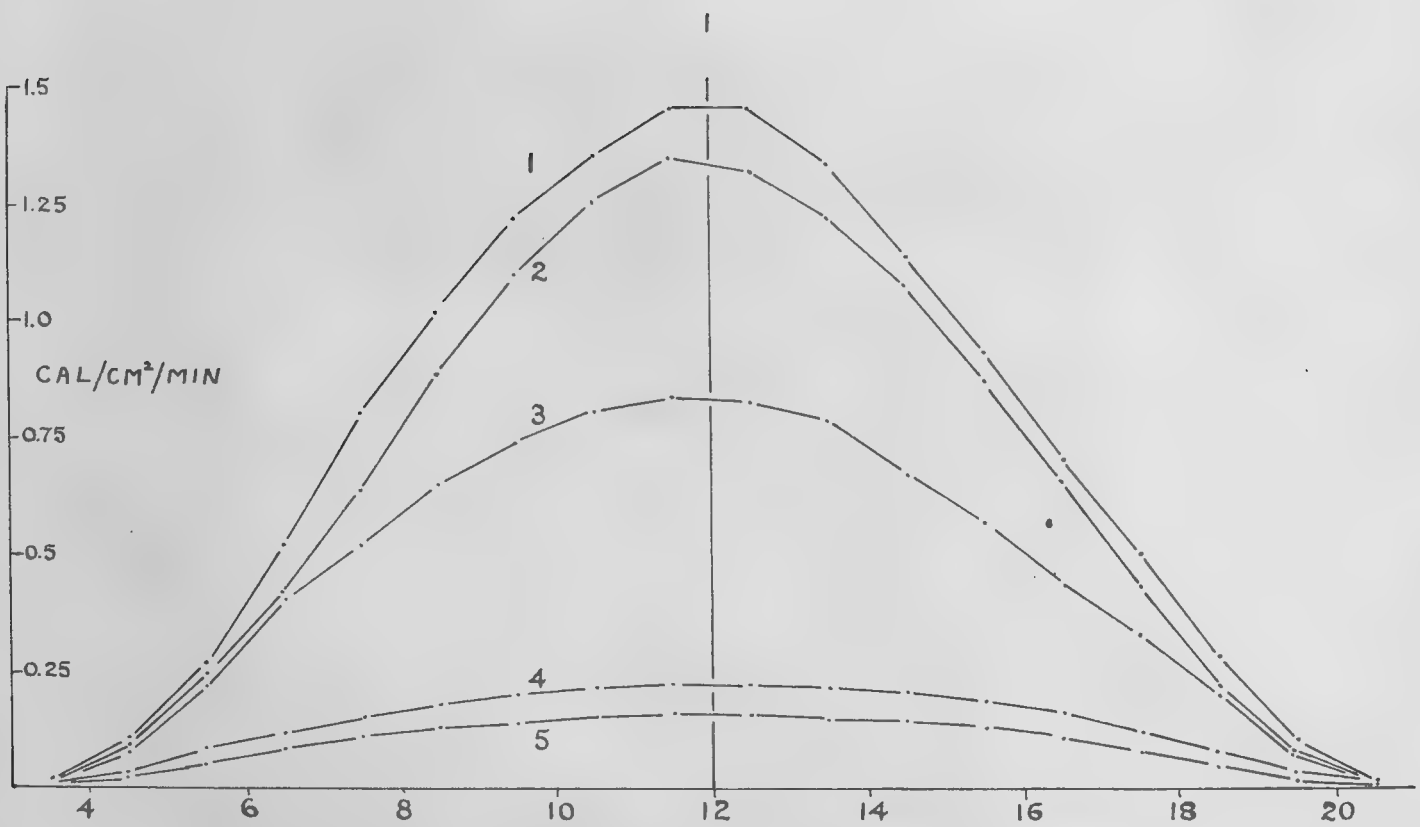


Fig. 9. Daily variation of total and diffuse radiation.

1. Ceiling T.
2 & 4. T and D on cloudless days.
3. Ceiling D.
5. D on cloudless and clear days.

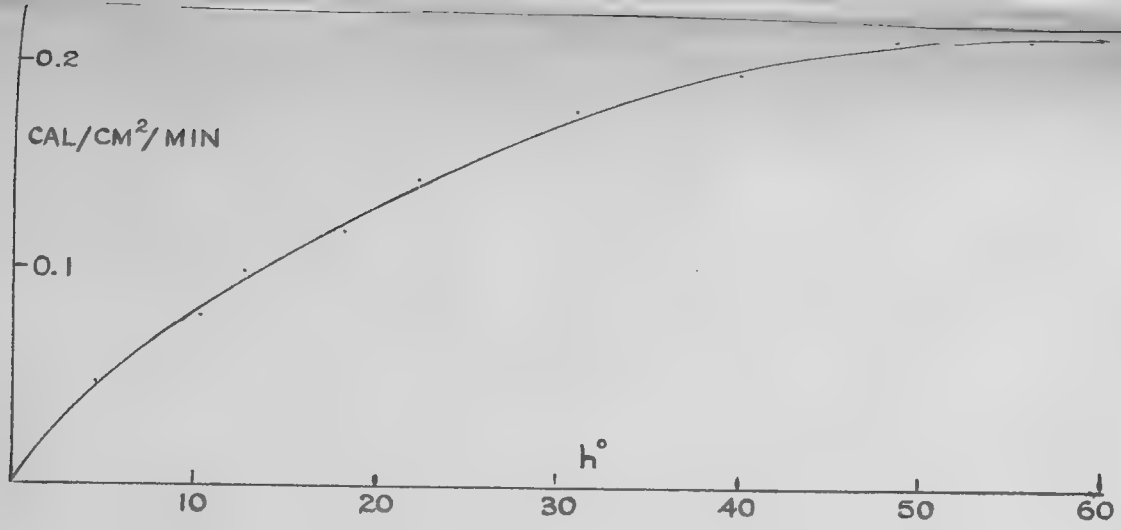


Fig. 10. D (cal./cm.²/min.) on cloudless days as $f(h)$.

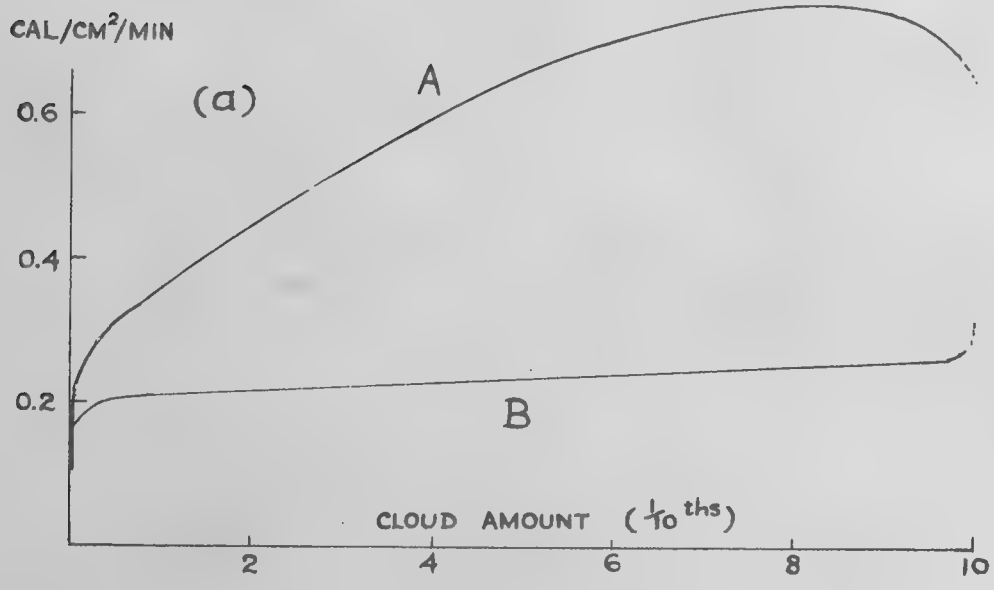


Fig. 11 (a). Variation of D with cloud amount.

A. Low and medium cloud, mainly Cu, Sc, Ac, thick As.
B. High cloud, Cs, Ci, thin As.

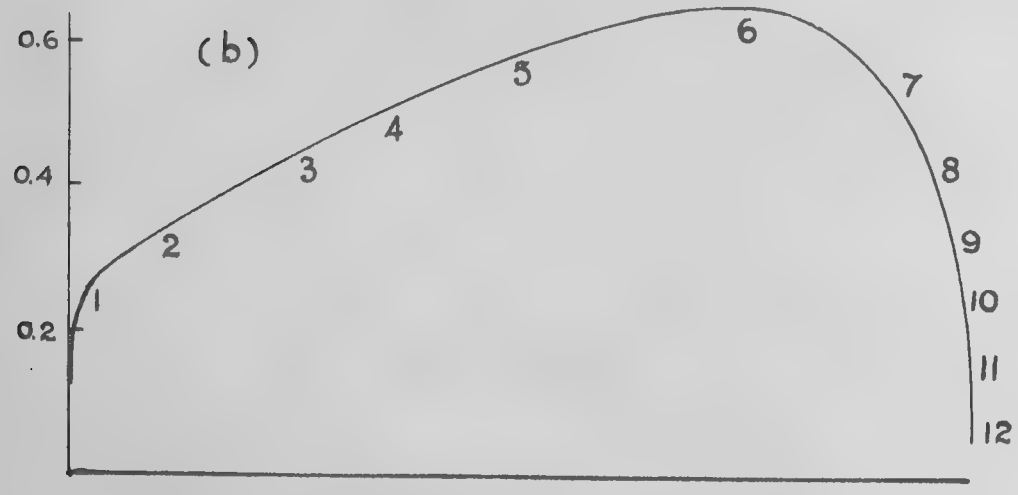


Fig. 11 (b). Variation of D with cloud type and density with 10/10 ths. clouded skies.

1. Ci
2. Ci, Cs
3. Cs, As
4. As
5. Ac, Sc
6. Sc, Cu
7. Sc (heavy) As
8. As (thick)
9. As, St
10. St
11. Ns
12. Cb, As.

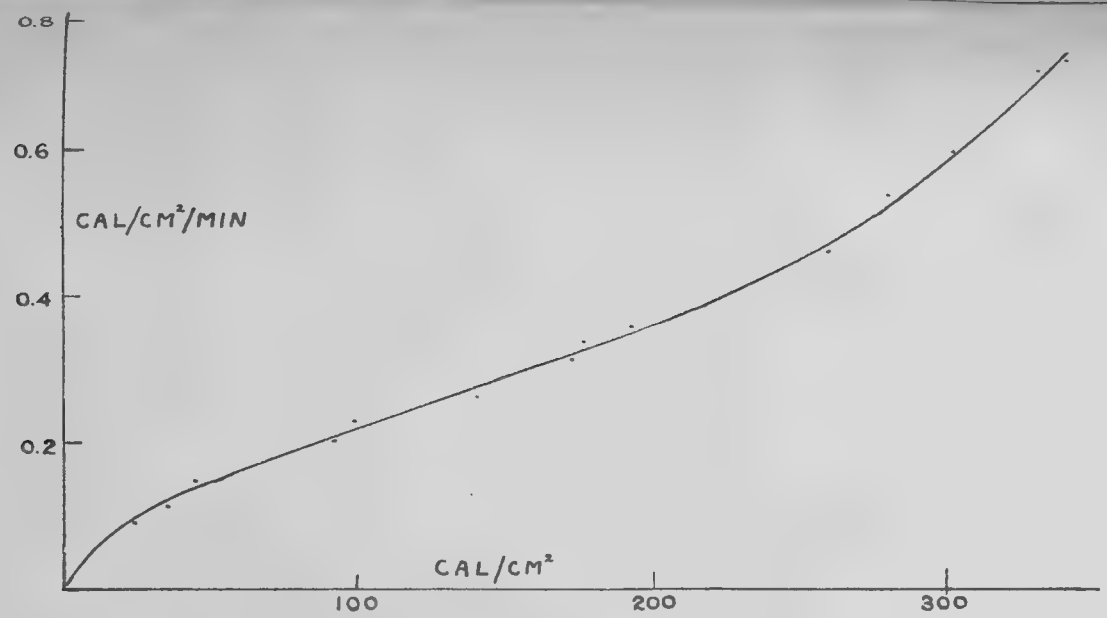


Fig. 12. Relation between midday intensities of D (cal./cm.²/min.) and daily totals of D (cal./cm.²).

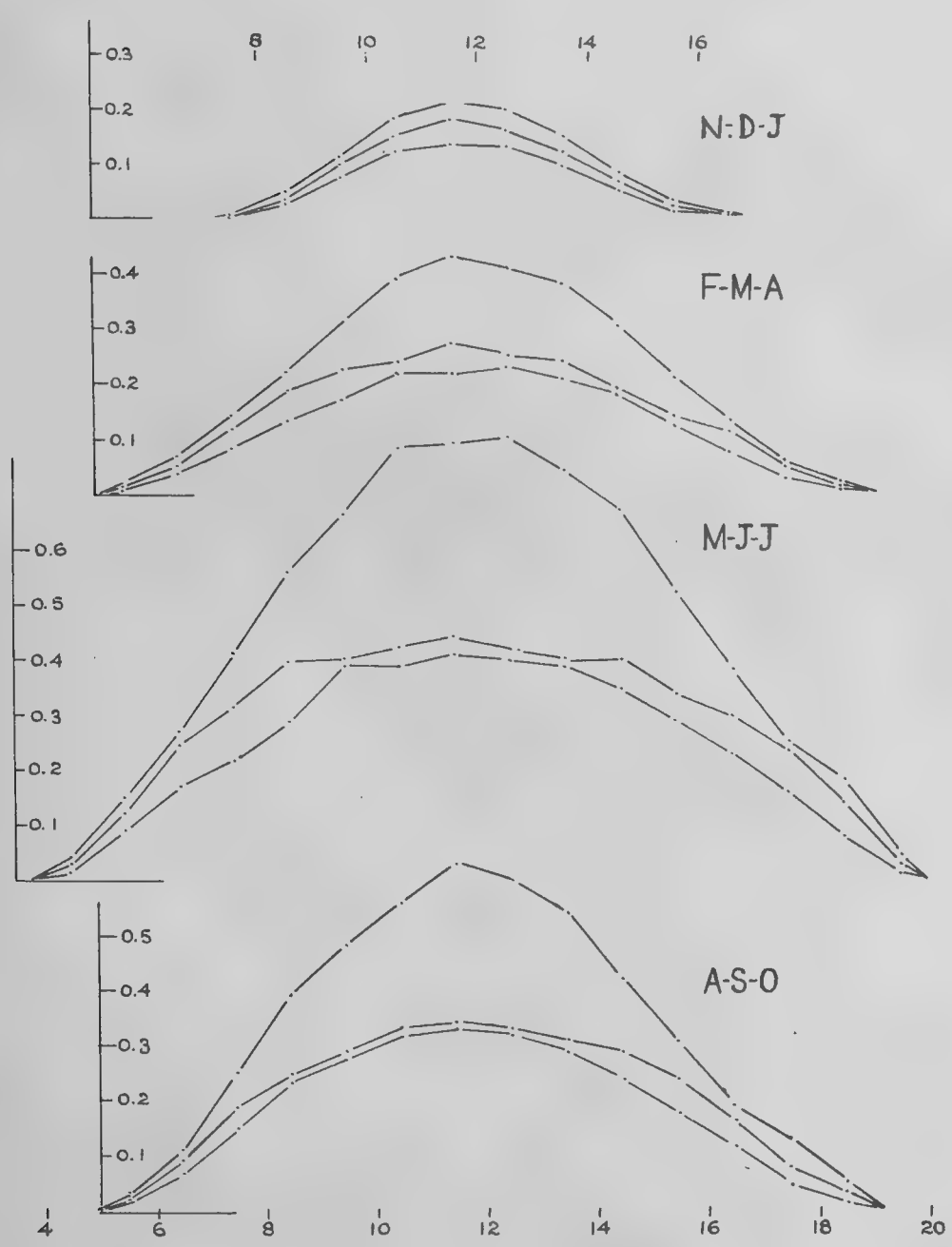


Fig. 13. Daily variations of T, D and I (cal./cm.²/min.). In each set T is highest and D the lowest curve.

MRP's 376, 382 383, 387, 392

This report has not yet been published and should be treated CONFIDENTIAL. Its contents must not be quoted.
C.L.F. 22.1.48

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE.

M.R.P. 376
15th December, 1947.

Acc. 48.5488

THE MAGNITUDE OF THE CASUAL VARIATIONS OF TEMPERATURE IN THE FREE AIR.

By

R. Frith.

Introduction.

This note is intended to give an indication of the degree of variability of the free air temperature.

On a number of occasions (ten) we have been able to incorporate with other work, level flights of at least 10 minutes duration, taking frequent readings of temperature and humidity.

The situations were not selected synoptically, and the only restriction imposed was that there should be no cloud at the level of the flight. Generally there was no cloud, above or below, within 6,000 feet.

No attempt has been made to analyse the variations found.

2. Instruments.

Humidity readings were made with the Dobson-Brewer Frost Point Hygrometer.[1] Temperature readings were made with the flat plate electrical thermometer.[2]

With the electrical flat plate thermometer readings are generally accurate to within 0.2°(F) (Apart from a biased error due to some uncertainty about the speed correction coefficient. This may be as large as 1°F., but it is practically constant in any one run and does not affect the temperature variations found).

3. Four Typical Cases.

Figures 1 to 4 present graphically the results of four typical runs. Every reading made is shown, and the time scale is indicated in the figures.

Humidity variations do not concern us in the present note, but readings of frost point have been included in the diagrams (on a greatly reduced scale) as a matter of general interest.

4. Discussion.

There are irregular variations on each side of the mean of the order of one degree; and the "period" of the fluctuations appears to be of the order of one or two minutes - i.e. three to six miles.

In M.R.P. 338 it was remarked that there are fluctuations both of surface wind and of winds at 4,000 feet with "eddy-size" of two to ten miles. The two effects are probably due to the same cause.

5. Practical Importance of the Work.

The nature of the variations which we have shewn to exist should be borne in mind when consideration is being given to the questions of

(a) What accuracy;

(b) What lag,

are desirable in thermometers for use in aircraft.

Thermometers intended for use in "spot" observations, i.e. on met. recce. aircraft, ought, for example, to have sufficient lag to smooth out the irregularities.

/On

On the face of it, it would appear to be impracticable to have sufficient lag to smooth out completely the irregularities. There would, therefore, appear to be little justification in aiming at an accuracy much greater than 0.5°F.

6. Speed Correction Coefficient.

We have previously remarked that there is some uncertainty as to the true value of the speed correction coefficient to be used with the flat plate thermometer.

It is clear that, in order to smooth out the erratic variations of temperature which are found in level flight during speed runs, a large number of runs must be made in order to obtain an accurate value for the correction coefficient.

Speed runs are generally made just below a sheet of cloud, but even here the temperature is far from steady.

In Fig. 5 we reproduce the result of readings taken during a five minutes run, at a constant speed, just below 10/10 Sc. at a height of 2,000 feet.

7. Conclusions.

We have shewn that variations of temperature of the order of 1°F. about a mean value are quite normal in the free air; and that the size of the "eddies" of temperature is of the order of 3 to 6 miles - the same as found from a study of wind speed variations.

The importance of these variations in

(i) determining what lag and what accuracy are desirable in aircraft thermometers;

(ii) determining speed correction coefficients

are discussed.

1. M.R.P. 126.
2. M.R.P. 112

M.R.P. 376

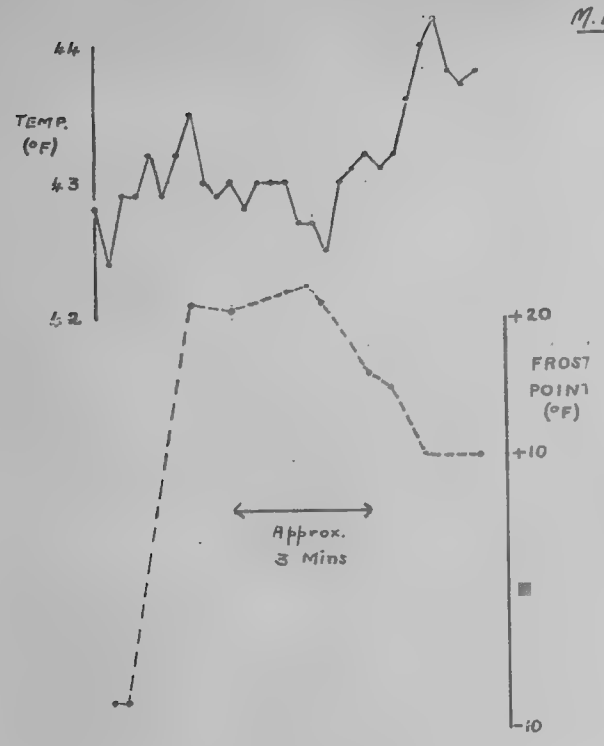


FIG 1
JUNE 16 1947
5,600 FT.
BETWEEN TWO LAYERS OF CLOUD.

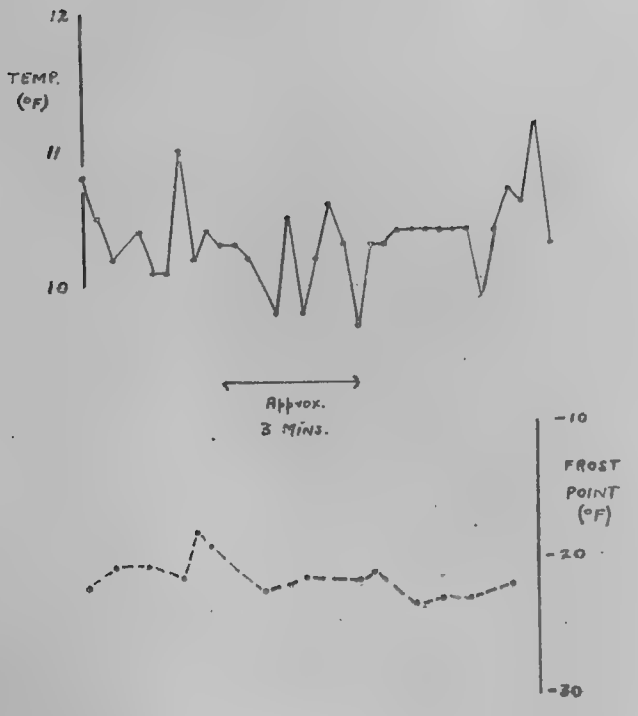


FIG 2
SEPT 2 1947
16,000 FT.

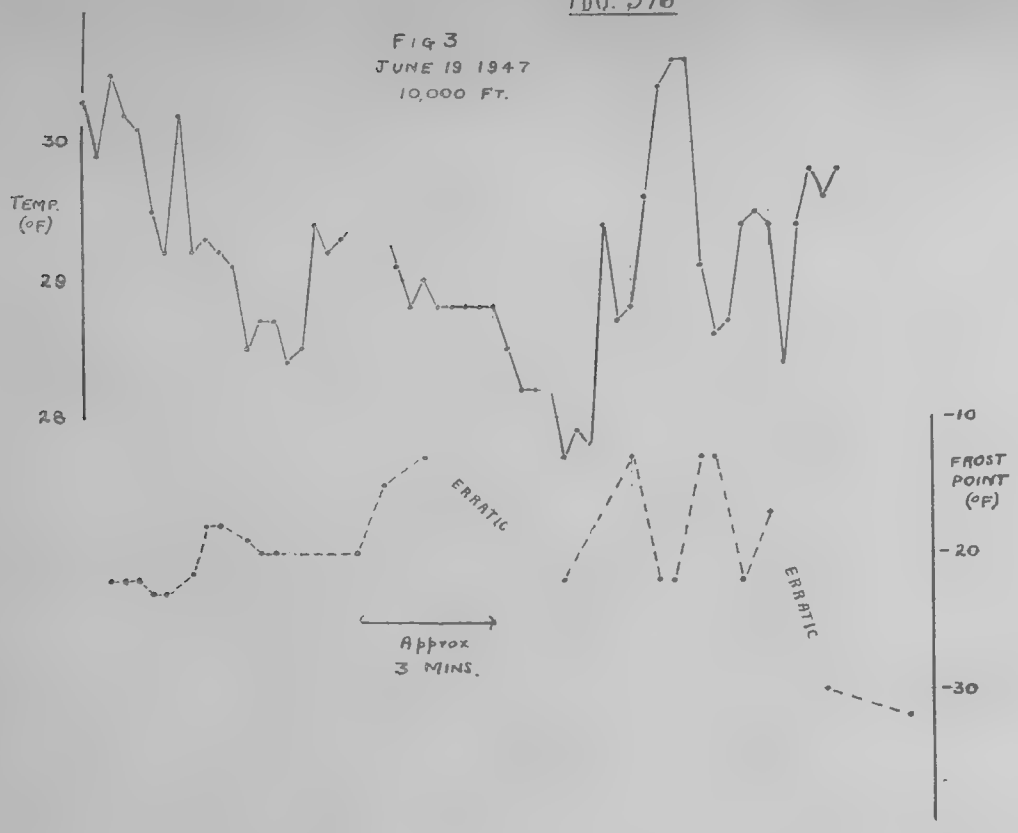


FIG 3
JUNE 19 1947
10,000 FT.

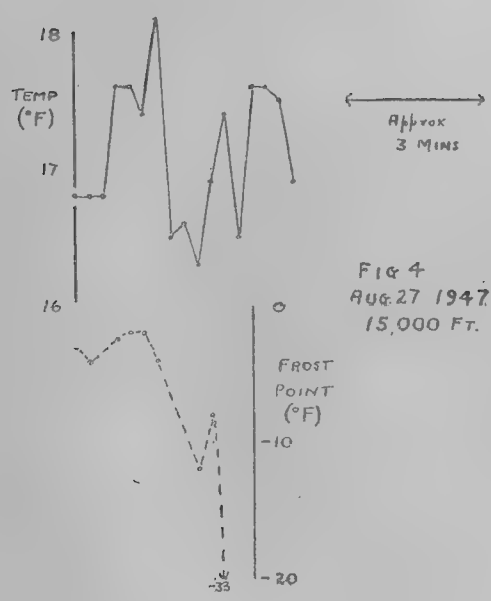
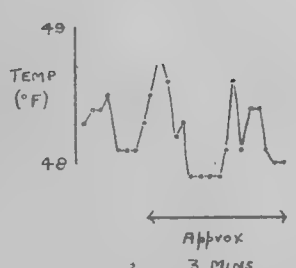


FIG 4
AUG. 27 1947
15,000 FT.

FIG 5
Nov. 11 1947
2,000 FT.
JUST BELOW 10/10
SHEET Sc.
BUMPY.



AIR WEATHER SERVICE
REFERENCE FILES

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

in the lowest layers of the atmosphere.)

conditions in summer. The data required are set out, and justified by reference

for N is supplied by means of a radiation chart. The data then allow the
of the convective transfer C in terms of its surface value C(0)
The usual equation for convective transfer is

l interest only, and are incidental to

ever, are only approximately true, the
" case is investigated. This is possible
the surface (dew deposition), hence the
is known in terms of the previously assumed

light adjustments of the assumed humidities,

. (para. 6)

es with height is then examined. It is shown

This report has not yet been published and should be treated as CONFIDENTIAL. Its contents must not be quoted.

M.R.P.377

16th December, 1947

Gr. Brit. Air Ministry

ATI 43043

Meteorological Research Committee

(A contribution to the study of convection in the lowest layers of the atmosphere.)

by

G.D. Robinson

1. Summary.

The observations necessary for a computation of heat transfer near the ground are detailed (para. 1.). Since no such complete set of observations has been published two hypothetical cases are considered, representing afternoon and night conditions in summer. The data assumed are set out, and justified by reference to measurements at Porton (Best) and Kew (para. 2). From the data the radiative transfer R is computed by means of a radiation chart. The data then allow the computation of non-radiative transfer C in terms of its surface value C(0) (para. 3). The usual equation for convective transfer is

$$C = -\rho c_p K\left(\frac{\partial T}{\partial z} + \Gamma\right)$$

A minimum value for C(0) is estimated from the molecular value of K and a likely minimum value of $\partial T/\partial z$ at the surface, based on Bests measurements of temperature at 1". This minimum C(0) is compared with estimates from energy measurements. (para. 4). The eddy conductivity K is then computed on four assumptions.

(a) C(0) having its "minimum" value

(b) C(0) having three times its "minimum" value

(c) C(0) being zero

(d) K assumed constant with height (and hence independent of C(0))

(c) and (d) are regarded as of historical interest only, and are incidental to the main argument, (para. 5).

On the assumption that eddy conductivity and eddy diffusivity are equal, the transport of waste vapour in the "night" case is investigated. This is possible since saturation has been assumed at the surface (dew deposition), hence the gradient of humidity at the surface is known in terms of the previously assumed temperature gradient. The computations are made for minimum C(0) and for three times this value. It is found that slight adjustments of the assumed humidities, in the permissible direction, allow a balanced flow of water vapour with a rate of dew deposition of the correct magnitude. (para.6)

Summarising the position, it has been shown that the data initially assumed are consistent with a simple relation between eddy conduction and eddy diffusion, and also with magnitude of the respective coefficients which yield reasonable values for the fluxes, (para. 7).

The variation of the coefficients with height is then examined. It is shown to be represented up to 120 cm by a power law which has the same index for both afternoon and night cases. (para. 8).

Some results of Best on the variation with height of the mean wind and its eddy components are quoted. By means of power laws which are approximately valid over the range considered it is shown that the results are not consistent with equality of eddy conductivity and eddy viscosity. (A well known result, but as far as is known not previously demonstrated in an investigation which considers radiative transfer). Making certain assumptions, for which however there is no

Acc. 48. 5329

evidence, regarding the inveriance with height of the correlation between certain elements of the eddy motion, the power laws are shown to be self consistent in a scheme which involves non conservation of momentum, but which is consistent with Taylors (two dimensional) theory of eddy transfer of vorticity (para. 9).

2. Factors governing the transfer of heat near the ground.

In investigation of the transfer of heat in the atmosphere close to the ground it has been usual to neglet the effects of radiation, on the assumption that they are small in comparision with those due to convective transfer (see e.g. Cowling and White, 1941, and Sutton, unpublished). Methods are now available which permit the computation of this radiative transfer if the radiative temperature of the earth's surface and the temperature and humidity structure of the layer of air concerned are known.

3. Data available and assumed.

No series of simultaneous observations of these values appears to have been made. Best (1935) has made detailed measurements of temperature in the lowest 120 cm., and adjacent instruments allowed him to extend the record to 1700 cm. The computations which follow use Best's air temperatures with hypothetical humidities and surface temperatures which are believed to be a reasonable representation of cloudless summer conditions in southern England over a short grass surface with very light winds. The times chosen are 0300 and 1400 hrs. corresponding roughly to minimum and maximum values of surface temperature. As temperature gradient is also near its extreme values at these times the maximum radiative transfer of heat within the layer is to be expected. The data assumed are shown in Tables I and II.

TABLE I

0300 G.M.T. Clear nights in June. Temperature and humidity data and computed radiative flux

Z cms.	T °A	$\frac{\partial T}{\partial z}$ deg. cm $^{-1}$	$\frac{\partial^2 T}{\partial z^2}$ deg. cm $^{-2}$	$\frac{\partial T}{\partial t}$ deg. sec $^{-1}$	Vapour press mb.	Integrated Water path g. cm $^{-2}$	R(z) cal. cm^{-2} sec^{-1}
0	280.0	-	-	-	10	0	R(0)
20	283.4	$+2.3\times10^{-2}$	$-(5\times10^{-3})$	-5×10^{-5}	12.6	1.8×10^{-4}	R(0) + 6×10^{-6}
40	283.7	$+1.0\times10^{-2}$	-5×10^{-4}	-5×10^{-5}	12.9	3.7×10^{-4}	R(0) + 20×10^{-6}
80	283.9	$+4.0\times10^{-3}$	-1.2×10^{-4}	-5×10^{-5}	13.0	7.7×10^{-4}	R(0) + 36×10^{-6}
120	284.0	$+2.5\times10^{-3}$	-2.4×10^{-5}	-5×10^{-5}	13.1	11.8×10^{-4}	R(0) + 54×10^{-6}
710	285.0	$+1.1\times10^{-3}$	-2×10^{-6}	-5×10^{-5}	14.0	7.1×10^{-3}	R(0) + 125×10^{-6}
1710	285.5	$+4\times10^{-4}$	-6×10^{-7}	-5×10^{-5}	14.5	18.1×10^{-3}	R(0) + 260×10^{-6}

TABLE II

1400 G.M.T. Clear days in June. Temperature and humidity data and computed radiative flux

Z cms.	T °A	$\frac{\partial T}{\partial z}$ deg. cm $^{-1}$	$\frac{\partial^2 T}{\partial z^2}$ deg. cm^{-2}	$\frac{\partial T}{\partial t}$ deg. sec^{-1}	Vapour press mb.	Integrated Water path g. cm^{-2}	R(z) cal. cm^{-2} sec^{-1}
0	313	-		-	10	0	R(0)
20	298.3	-3.5×10^{-2}	$(+1.5\times10^{-2})$	0	10	1.2×10^{-4}	R(0)-3×10^{-5}
40	297.8	-1.5×10^{-2}	$+8\times10^{-4}$	0	10	2.4×10^{-4}	R(0)-8×10^{-5}
80	297.5	-7×10^{-3}	$+2\times10^{-4}$	0	10	4.8×10^{-4}	R(0)-18×10^{-5}
120	297.3	-4×10^{-3}	$+7\times10^{-5}$	$+1\times10^{-4}$	10	7.2×10^{-4}	R(0)-24×10^{-5}
710	296.4	-1×10^{-3}	$+5\times10^{-6}$	$+1\times10^{-4}$	10	4×10^{-3}	R(0)-57×10^{-5}
1710	296.1	(-5×10^{-4})	$+8\times10^{-7}$	$+1\times10^{-4}$	10	1×10^{-2}	R(0)-71×10^{-5}

The air temperatures are obtained by interpolation from Best's mean values for the appropriate hour on clear days in June; the following considerations form the basis of the surface temperatures and humidities. The difference between the surface radiative temperature and the screen temperature at 0300 hrs. is estimated from a number of results obtained at Kew on clear nights, for example:-

28 May, 1947	2230 G.M.T.	Screen 288.5°A	Surface 285°A.
27 June, 1947	2230 G.M.T.	Screen 291°A	Surface 288°A.
14 Sept. 1946	0200 G.M.T.	Screen 283°A	Surface 278.5°A

/The

The humidities of Table I represent saturation at the corresponding temperatures. There are obvious objections to this and the question will be reconsidered later (para. 6). For the present it is sufficient to note firstly that it is not unusual for screen humidities to remain at values of 95% or more for long periods, with a falling temperature and deposition of dew, but with no fog formation, and secondly that the radiative transfer is not very sensitive to humidity changes of the order of 10%. Some relevant observations selected from the Kew records may be quoted here.

15 April, 1947

Time G.M.T.	2200	2320	2359
Surface temperature °A	-	276.5	-
Screen temperature °A (Stevenson 4ft)	283.5	282.5	281.5
Relative Humidity % " "	96	97	95

Dew forming. Ground fog formed at screen after midnight.

14 Sept., 1946.

Time G.M.T.	0001	0200
Surface temperature °A	279.1	278.6
Screen temperature °A (North Wall 10 ft)	283.5	282.9
Relative humidity % " " "	84	88
Water content (g/kg) " " "	8.0	8.0

Dew forming. Water content corresponds to saturation at 281°A.

The surface temperature associated with the 1400 hr. values was in the first instance estimated from the results of Penman (1943). A recent restricted series of readings at Kew using radiometers has shown that it may be unusual for the surface temperature so measured to exceed screen temperature by as much as 15°C, but this has been observed on one occasion -25 June, 1947 1300 G.M.T., surface 310°A, Screen 295°A - and 10°C difference has been measured twice - 26 June 1947 1030 G.M.T. and 30 September 1947 1400 G.M.T. Very dry ground conditions are probably necessary for such large differences. The assumption of constant humidity (corresponding to 35% R.H. at screen temperature) in Table II is obviously unsound, but, as with the 0300 values it has been found that a reasonable divergence from these humidities does not introduce a serious change into the computed radiative transfer. On the basis of these arguments it is submitted that, with the above provisos regarding humidity, the data set out in Tables I and II are a reasonable representation of conditions likely to occur in nature. They are not claimed to be average conditions. The derived values of $\partial T/\partial z$ and $\partial^2 T/\partial z^2$ were obtained by graphical and difference methods.

4. Computation of radiative transfer and its relation to convective transfer

The computation of the radiative flux at height Z cm., R(Z), which is shown in the last column of Tables I and II was made using an empirical radiation chart (Robinson 1947), but the results would not differ appreciably if the chart developed by Elsasser (1942) were used. For layers of air of the thickness of those under consideration these charts have empirical confirmation in observations of isothermal emissivity, particularly by Elssasser (1942) and Brooke (1941). These observations extend down to paths of 10^{-4} cm. water, i.e. 20 cm. of air with a water vapour pressure of 10mb., and this is therefire the lower limit of layer thickness for which the radiation charts may safely be used. This restriction was observed in the present computations. We are now in a position to examine the relative importance of radiative transfer and transfer by the other available processes - convection and molecular conduction, the word convection being used in its fullest sense of transfer by mass movement of air, whether "free" or "forced". Denoting the flux of heat due to these two processes together at height Z by C(Z), and assuming zero horizontal gradients, we may write the equation of conservation

$$-\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} (\text{total flux}) = \frac{\partial R(z)}{\partial z} + \frac{\partial C(z)}{\partial z} \quad \ldots\ldots\ldots 1$$

This is more conveniently used in its approximate form

$$c_p (z_1 - z_2) \left\{ \rho \frac{\partial T}{\partial t} \right\} = (R(z_2) - R(z_1)) + (C(z_2) - C(z_1)) \quad \ldots\ldots\ldots 2$$

the quantity in the square brackets being the mean value for the layer. This should obviously be applied to the smallest possible intervals $Z_2 - Z_1$ but we have seen that the radiation chart sets a lower limit of about 20 cm. Values of $\partial T/\partial t$ may be obtained from Best's observations; they are small at the times in which we are interested, and great accuracy is not required. The figures are shown in Tables I and II, and the computed values of C(Z) in Tables III and IV, R(0) and C(0) being respectively the radiative and convective plus conductive fluxes at the surface.

5. The non-radiative transfer of heat at the surface

The usual expression for the convective flux of heat is

$$C(z) = \left(-\rho c_p K \left(\frac{\partial T}{\partial z} + \Gamma \right) \right)_z \quad \ldots\ldots\ldots 3$$

where Γ is the adiabatic lapse rate and K the eddy conductivity. The expression may be derived (e.g. Brunt 1941 p.225) by considering the heat transferred by an eddy which starts its motion at distance l from the reference level Z and crosses it with a vertical component of velocity w. K is identified with the mean product $\overline{wl}$. Two notable assumptions are made in this derivation. The first, which has been criticised by Priestley and Swinbank (1947) is that a vertically moving eddy is a fair sample, with respect to heat content, of its level of origin; the second is that $\partial T/\partial z$ is constant over distances comparable with l. To compute K from 3 and the values of C in Tables III and IV it is necessary to know C(0), and no reliable measurements of this quantity have yet been made. We may however set a lower limit to C(0). K cannot fall below the value appropriate to molecular conduction, 0.18 cm^2 sec^{-1}. (By inference from the figures quoted by Roberts 1945 p.225, see also Best, loc. cit.), and we may estimate a lower limit of $|(\partial T/\partial z)_0|$ from Best's values of temperature at 2.5 cm. The temperature gradients assumed as being the lowest likely values at the surface were 3.0 deg. $cm.^{-1}$ for the 0300 hr. case, and -15.0 deg. cm^{-1} at 1400 hrs; in each case approximately three times the mean value of the gradient between the surface and 2.5 cm. The corresponding minimum values of C(0) are -1.6×10^{-4} and $+8.0 \times 10^{-4}$ cals. cm^{-2} $sec.^{-1}$. K was computed for these values of C(0) and also for values three times as great, and the results are shown in Tables 3 and 4. We may note here that Sutton (loc. cit.), using an entirely different approach, has estimated that C(0) lies between 2.0×10^{-4} and 20×10^{-4} cal. $cm.^{-2}$ $sec.^{-1}$ for the hours around noon on a clear June day in southern England. The direct measurement of C(0) is possible in theory by taking the heat balance at the surface, but this is an experiment in which it would be difficult to achieve the necessary precision.

6. Computation of the eddy conductivity

Attempts have been made to derive values of K from temperature profile measurements without assumptions regarding C(0). From 3 we have

$$\frac{\partial C}{\partial z} = -c_p \frac{\partial}{\partial z} \left(K_\rho \left(\frac{\partial T}{\partial z} + \Gamma \right) \right) \quad \ldots\ldots\ldots 4$$

If $\partial R/\partial z$ (and, more justifiably, $\frac{\partial \rho}{\partial z}$) is neglected, this equation reduces, using (1) to

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left(K \left(\frac{\partial T}{\partial z} + \Gamma \right) \right) \quad \ldots\ldots\ldots 5$$

TABLE III

0300 hrs. Convective flux and eddy conductivity on various assumptions.

Z cm.	$(z_1-z_2)\rho c_p \frac{\partial T}{\partial t}$ cal cm^{-2} sec^{-1}	C(z) cal cm^{-2} sec^{-1}	$-\frac{\partial C}{\partial z} \Big/ \rho c_p \frac{\partial^2 T}{\partial z^2}$ cm^2 sec^{-1}	$\frac{-C(z)}{\rho c_p(\frac{\partial T}{\partial z}+\Gamma)}$ $cm^2 sec^{-1}$ C(0) =0	C(0)=1.6 x10^{-4}	C(0)=4.8 x10^{-4}	$\frac{1}{\frac{\partial T}{\partial z}+\Gamma}$ cm deg^{-1}
0		C(0)		-	-	-	
	$+3\times10^{-7}$		-0.2				
20		C(0)-5.7x10^{-6}		0.8	24	70	43.5
	$+3\times10^{-7}$		-4.5				
40		C(0)-19x10^{-6}		6	60	167	100
	$+6\times10^{-7}$		-11				
80		C(0)-35x10^{-6}		30	158	420	245
	$+6\times10^{-7}$		-60				
120		C(0)-52x10^{-6}		70	272	680	385
	8.9×10^{-6}		-175				
710		C(0)-114x10^{-6}		320	760	1650	835
	1.6×10^{-5}		-670				
1710		C(0)-233x10^{-6}		1550	2950	4700	2000

TABLE IV

1400 hrs. Convective flux and eddy conductivity on various assumptions.

Z cm.	$(z_1-z_2)\rho c_p \frac{\partial T}{\partial t}$ cal cm^{-2} sec^{-1}	C(z) cal cm^{-2} sec^{-1}	$-\frac{\partial C}{\partial z} \Big/ \rho c_p \partial^2 T/\partial z^2$ cm^2 sec^{-1}	$-\frac{C(z)}{\rho c_p(\frac{\partial T}{\partial z}+\Gamma)}$ cm^2 sec^{-1} C(0) =0	C(0)=8 x10^{-4}	C(0)=24 x10^{-4}	$\frac{1}{\frac{\partial T}{\partial z}+\Gamma}$ cm deg^{-1}
0		C(0)					
	0		-0.4				
20		C(0)+3x10^{-5}		3.5	79	230	285
	0		-12				
40		C(0)+8x10^{-5}		20	196	550	670
	0		-50				
80		C(0)+18x10^{-5}		103	475	1250	1430
	-2×10^{-7}		-84				
120		C(0)+24x10^{-5}		240	890	2250	2500
	-1×10^{-5}		-450				
710		C(0)+57x10^{-5}		2480	5070	11000	11100
	-3×10^{-5}		-(550)				
1710		C(0)+71x10^{-5}		(6700)	12600	26000	25000

which has been solved for certain boundary conditions together with $K \propto z^p$. Best (loc. cit.) and Johnson and Heywood (1938), however, attempted to approximate to K by assuming independence with height over limited ranges and reducing 5 to

/

$$\frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial z^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$

Cowling and White (1941) have emphasised the objections to this procedure and conclude that the Ks so obtained may be in error by an order of magnitude. Nevertheless, since values of K obtained in this way are still quoted by workers in this field, it seemed of interest to compute K from

$$\frac{\partial G}{\partial z} = -\rho c_p K \frac{\partial^2 T}{\partial z^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 7$$

which is the equivalent of 6 allowing for radiative transfer, and the results are shown in tables III and IV, Col.4. It is seen that meaningless negative values are obtained, and inspection of columns 6 and 7 shows that the neglected term in (5), $\partial T/\partial z . \partial K/\partial z$ is indeed of greater magnitude than, and opposite sign to, the retained term $K\partial^2 T/\partial z^2$. In column 5 will be found values of K obtained by using (3) with the assumption $C(0) = 0$; these are approximately equal to the values derived by Best (loc.cit.) using (6). Since $C(0)$ cannot fall below the value appropriate to molecular conduction the difference between columns 5 and 6 is therefore a measure of the minimum errors in K's determined from (6). These are of the same order as the errors estimated by other methods by Cowling and White. Returning to equations 6 and 7 we may recall that in deriving (3) it was assumed that $\partial^2 T/\partial z^2 = 0$. The inconsistency of using 6 to derive K values is thus clear, but the use of 3 itself, near the surface, is also open to serious criticism on these grounds, a fact which must be borne in mind in any application of the numerical values of K from tables III and IV.

7. The transfer of water vapour. Eddy diffusivity.

An equation similar to 3 may be derived for any property which is conserved during convection, so long as there is no correlation between excess or defect of this property and the values of w and $\bar{1}$ as defined in para. 4. In such a derivation a coefficient is identified with $\overline{wl}$ so that the simple theory leads to the same numerical values for the coefficients of transfer of all properties which obey the above conditions. We now consider in particular the water vapour content e, which we will express in grams per gram of air. Writing $E(Z)$ for the vertical flux of water vapour at height Z and D for the eddy diffusivity of water vapour in air, the equation of transfer is

$$E(z) = \left(-\rho D \frac{\partial e}{\partial z}\right)_z \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 8$$

and the equation of conservation

$$-\rho \frac{\partial e}{\partial t} = \frac{\partial E}{\partial z} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 9$$

the approximate expression

$$(z_1 - z_2)\left(\rho \frac{\partial e}{\partial t}\right) = E(z_2) - E(z_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 10$$

being again more convenient in use. Equating D to the values given for K in Table III we may apply (8) and (10) to investigate the flux of water vapour in the 0300 hr. example. Values of $\partial e/\partial t$ and $\partial e/\partial z$ are required and, since saturation has been assumed throughout, they may be obtained from the $\partial T/\partial t$ and $\partial T/\partial z$ values of Table I by multiplying by the value of $\partial e/\partial T$ appropriate to saturation at the temperature of the level concerned (obtained from Meteorological Office Hygrometric Tables (M.O.265) 1940 p.p.42 and 45) Computation of E(Z) was carried out for the values of D (= K) appropriate to $C(0) = -1.6 \times 10^{-4}$ cal. cm.$^{-2}$ sec^{-1} and $C(0) = -4.8 \times 10^{-4}$ cal. cm^{-2} sec. In each case D(0) was assumed equal to the molecular diffusivity, for which the numerical value 0.25 was taken (Brunt, 1941, p.421. There is considerable doubt regarding this figure). The results obtained are shown in Table V and the terms of equation 10 in Table VI.

TABLE V

0300 hrs. Flux of water vapour

Z cm.	$\frac{\partial e}{\partial z}$ cm^{-1}	$\frac{\partial e}{\partial t}$ sec^{-1}	E(Z) g. cm^{-2} $sec.^{-1}$	
			for C(0)=-1.6 x 10^{-4}	for C(0)=-4.8 x 10^{-4}
0		(-2.0×10^{-8})	-3.96×10^{-7}	-11.88×10^{-7}
20	1.23×10^{-5}	-2.6×10^{-8}	-3.57×10^{-7}	-8.62×10^{-7}
40	5.45×10^{-6}	-2.6×10^{-8}	-3.92×10^{-7}	-9.10×10^{-7}
80	2.21×10^{-6}	-2.6×10^{-8}	-4.20×10^{-7}	-9.30×10^{-7}
120	1.39×10^{-6}	-2.6×10^{-8}	-4.50×10^{-7}	-9.45×10^{-7}
710	6.45×10^{-7}	-2.8×10^{-8}	-5.87×10^{-7}	-10.65×10^{-7}
1710	2.39×10^{-7}	-2.9×10^{-8}	-8.5×10^{-7}	-11.25×10^{-7}

TABLE VI

0300 hrs. Balance of water vapour transfer.

Z cm.	$(z_1-z_2)(\rho\frac{\partial e}{\partial t})$ g. cm^{-2} sec^{-1}	E(Z2) - E(Z1) g. cm^{-2} sec^{-1}	
		for C(0)= -1.6 x 10^{-4}	for C(0)=-4.8 x 10^{-4}
0-20	$+4.8\times10^{-10}$	$+4\times10^{-8}$	$+3.2\times10^{-7}$
20-40	$+6.2\times10^{-10}$	-3.5×10^{-8}	-5×10^{-8}
40-80	$+1.24\times10^{-9}$	-3.8×10^{-8}	-2×10^{-8}
80-120	$+1.24\times10^{-9}$	-3×10^{-8}	-1.5×10^{-8}
120-710	$+2\times10^{-8}$	-1.4×10^{-7}	-1.2×10^{-7}
710-1710	$+3.5\times10^{-8}$	-2.6×10^{-7}	-2.8×10^{-7}

It is seen that the assumed conditions do not lead to a balance, but that Table VI shows an accumulation of water above the 20 cm. level of the order of 5×10^{-10} g. $cm.^{-3}$ sec^{-1}, that is about 2g. per $metre^3$ per hour, sufficient to form a thick fog in a few minutes. Below 20 cm. there is removal of water vapour. These conditions are contrary to experience. Table VI shows that the term in $\partial e/\partial t$ is small compared with δE and it is neglected in the following consideration, since it is clear that a very slight adjustment of E(Z) will accommodate it. We therefore seek to make E(Z) constant. This may be done wither by adjustment of D(Z) or of $\partial e/\partial z$. Whilst multiplication of D by a constant, of approximately the magnitude of the ratio of molecular diffusivity and conductivity, makes E(0) = E(20), further constancy of the flux cannot be attained in this way. To obtain constant E by adjusting D it is necessary to give to D a form of variation with height different from that of K. A slight adjustment of $\partial e/\partial z$ is however sufficient to provide constant flux above 20 cm. without

further change in D values, and this adjustment is in the permissible direction involving a decrease of relative humidity with height. Table VII shows the values of $\partial e/\partial z$ which give constant E, and also the concentration e consistent with these gradients, obtained by a graphical integration of the $\partial e/\partial z$ $-z$ curve. The corresponding relative humidities are also shown. It

TABLE VII

0300 hrs. Values of humidity and humidity gradient which give no accumulation of water. D (molecular) assumed equal to 1.4 K (molecular) in each case *

Z	$C(0) = -1.6\times10^{-4}$ D=1.1 K			$C(0) = -4.8\times10^{-4}$ D=1.4 K		
cm.	$\frac{\partial e}{\partial z}$ cm^{-1}	e	R.H.%	$\frac{\partial e}{\partial z}$ cm^{-1}	e	R.H.%
20	1.23×10^{-5}	9.55×10^{-3}	100	1.23×10^{-5}	9.55×10^{-3}	100
40	4.96×10^{-6}	9.74×10^{-3}	99	5.15×10^{-6}		99
80	1.88×10^{-6}	9.84×10^{-3}	99	2.05×10^{-6}		99
120	1.10×10^{-6}	9.87×10^{-3}	99	1.27×10^{-6}		99
710	3.94×10^{-7}	10.29×10^{-3}	98	5.21×10^{-7}	10.42×10^{-3}	99
1710	1.00×10^{-7}	10.47×10^{-3}	96	1.82×10^{-7}	10.74×10^{-3}	98.5

will be seen that the changes of humidity are too small to make an appreciable change in the radiative flux computations which are implicit in the assumed D values. Finally it is seen (Table V) that the rate of transport of water to the surface is 4×10^{-7} and 12×10^{-7} g. cm^{-2} sec^{-1} in the two cases, i.e. 0.015 and 0.045 mm. per hour. These are reasonable rates for the deposition of dew, in particular it is considered that 0.05 mm. per hour is a generous upper limit with a screen temperature as low as 10°C. If this is admitted it follows that the upper limit of C(0) is 5×10^{-4} cal. $cm.^{-2}$ sec^{-1}, i.e. about 2 cals. per cm^2 per hour.

8. Recapitulation and summary.

If then the temperatures and humidities of Table I, slightly amended by Table VII are admitted to represent possible conditions, a contention supported by the evidence given in para. 2, it is seen that the equations of conservation and transport of water vapour and heat can be satisfied by the following self consistent scheme.

(a) Conduction and diffusion at the surface are entirely by molecular processes.

(b) The gradient of temperature in which this molecular conduction occurs is of the order of 3 to 10 times the mean gradient between the surface and 2.5 cm., and the gradient of humidity is that corresponding to saturation in this temperature gradient.

(c) The coefficients of eddy conduction and diffusion bear at all heights approximately the same ratio as that of the molecular coefficients.

/The

* In this table saturation is assumed up to 20 cm. There is no need for this restriction, but the ratio D/K necessary to secure a balance of E in the saturated layer will depend on its thickness. It is tempting to choose this thickness to make the ratio D/K equal throughout to the corresponding molecular ratio, a result which appears by chance for a thickness of 20 cm. with $C(0) = -4.8\times10^{-4}$ cals. cm^{-2} sec^{-1}.

The arguments which have led to these conclusions can be applied to any case in which temperatures and humidities are known, but one essential feature, the relation of temperature and humidity gradients at the surface cannot be estimated unless there is saturation i.e. deposition of dew or evaporation from a free water surface. In particular the method cannot be extended to the 1400 hr. case, but there is nothing to suggest that the above conclusions do not hold.

9. Variation of eddy conductivity and diffusivity with height:.

It is usual to attempt to express the variation of a property with height near the ground by means of a power law, and quite apart from any physical significance of such laws they form a useful means of seeking functional relations between the various properties. Examination of the present figures shows that the fluxes and conductivities may be fitted to laws of this type in the region below 120 cm. In Fig. 1. $\log \frac{K(Z)}{K(20)}$ is plotted against $\log \frac{Z}{20}$ for both assumed values of C(0). The surprising feature emerges that the law of variation with height is the same for both 0300 hrs. and 1400 hrs., the index 1.35 corresponding to minimum C(0) and 1.27 corresponding to three times this value of C(0). This agreement may be considered indirect support for the view that the assumed magnitude of C(0) at 1400 hrs. (which could not be checked by considering the water vapour transport) is approximately correct, and that the conclusions of para. 7 are valid for both 0300 and 1400 hrs. If equation (3) is written

$$K(z) = -\frac{1}{\rho c_p} \frac{C(z)}{\left(\frac{\partial T}{\partial z} + \Gamma\right)_z} = -\frac{1}{\rho c_p} \left\{\frac{C(0) + \delta C}{\frac{\partial T}{\partial z} + \Gamma}\right\}_z$$

it is seen that the variation of K(Z) with Z approaches that of $\left(\partial T/\partial z + \Gamma\right)^{-1}$ with Z as the magnitude of C(0) increases relative to δC. Plotting $\log\left(\partial T/\partial z + \Gamma\right)^{-1}$ against log Z we find that the relation is represented below 120 cm. by a power law with index 1.20 at both 0300 and 1400 hrs.* This gives a lower limit to the index of Z in the law connecting K and Z, and is approximately the index which would be found if the effects of radiative transfer were neglected. Combining the K(Z) and $\left(\partial T/\partial z + \Gamma\right)^{-1}$ relations it is seen that C(Z) varies as a power of Z, the index being 0.15 for minimum C(0) and 0.07 if C(0) has three times this value. This may be confirmed by a direct plot of log C(Z) against Z. It must be remembered that these relations are valid only in the range 20-120cm. The figures given for K at night cannot be fitted to a power law between 120 and 1710 cm.; the daytime Ks fit reasonably well with an index of Z about 0.9 to 1.0, according to C(0), but the data are too few and not sufficiently precise for definite conclusions to be drawn. Below 20 cm. there are no data, owing to the limitation on layer thickness imposed by the radiation chart. Extrapolation of the laws quoted shows that the molecular value of K is reached at a height of 0.1 to 0.2 cm.; Fig. 2 illustrates this. This is of the expected order of magnitude (see e.g. Brunt 1941 p.258), so it may not be unreasonable to apply to laws below 20 cm.

10. The relation of eddy conductivity to eddy viscosity

It is well known (e.g. Best loc. cit. p.39) that a variation of K with height of the form $K \propto z^p$ is inconsistent with the observed wind velocity profile when $p > 1$ if the assumption is made that eddy viscosity and eddy conductivity are equal (or in constant proportion). We may examine this inconsistency by means of the power laws developed in para. 8 coupled with observations made on various components of wind velocity over a short grass surface. We have shown that eddy conductivity is represented by the same power law in conditions of strong lapse and inversion, it is therefore reasonable to expect the same relation to hold in conditions approximating to no lapse of temperature. In such conditions Best (loc. cit.) has made a very complete investigation of wind structure; Table XXXIV of his paper includes measurements of mean wind speed U and mean horizontal and vertical eddy velocities u, v, and w, for heights from 25 cm. to 200 cm. This table is for a particular wind velocity (370 cm. sec^{-1} at 200 cm.); there is not sufficient evidence to show the effect of reducing this value. Best's method of

determining the mean values of the eddy velocities was necessarily arbitrary, and we are not justified in distinguishing between his means and the standard deviations of the respective wind components. Fig. 3 is a plot of U, σ_w, and $\sigma_w\sigma_u$ against height, the σ's being the standard deviations of the wind components in the vertical and in the direction of the mean wind. It is seen that a power law gives a reasonable representation of the variation of these elements with height. Now the elementary heat transport theory identifies eddy conductivity with the mean product $\overline{wl}$, w being the vertical velocity of a given eddy at the reference level, and l its initial vertical displacement. The mean product may be written $r_{lw}\sigma_w\sigma_l$ where r_{lw} is the correlation coefficient and the σs the root mean square values of w and l. We have therefore, from Figs 1 and 3, the following power laws.

$$U \propto z^{0.17} \qquad \ldots\ldots 11$$

$$\sigma_w \propto z^{0.32} \qquad \ldots\ldots 12$$

$$\sigma_u\sigma_w \propto z^{0.45} \qquad \ldots\ldots 13$$

$$\overline{wl} = r_{wl}\sigma_w\sigma_l \propto z^{1.27} \text{ to } z^{1.35} \qquad \ldots\ldots 14$$

The rate at which momentum crosses unit area at a level Z is given by $-\rho\overline{wu}$, w and u being observed at the level. This may also be expressed on the simple transport theory as $\rho\overline{wl}\,\partial U/\partial z$ the assumption being made that momentum is conserved by the moving eddy. Hence

$$-\rho\overline{wu} = -\rho r_{uw}\sigma_w\sigma_u = -\rho\overline{wl}\,\frac{\partial U}{\partial z}$$

Relations 11, 13 and 14 are consistent with this if r_{uw} is independent of height; the expression on the left being then proportional to $z^{0.45}$ and that on the right to $z^{0.44}$ to $z^{0.52}$. It is seen however that the condition that $-\rho\overline{wu}$ is not constant with height implies destruction of momentum in the mixing process, since $\partial U/\partial t = 0$. Scrase (1930) has made simultaneous measurements of u and w and found a high negative correlation. He also found that the eddy shearing stress $-\rho\overline{wu}$ varied by a factor of about 4 between 1.5m. and 19m., but this variation can be seen to be due mainly to a variation of the correlation. It has been seen that the power laws 11 to 14 are applicable only below 1.5m., and Scrase's results are mentioned here only as a reminder that variation of $\overline{uw}$ in the lower atmosphere has been observed. The relation which we have proposed

$$\overline{uw} = \overline{wl}\,\frac{\partial U}{\partial z} \propto z^{0.47}$$

is explicable on the simple mixing length theory of eddy transport only if the moving eddy conserves the momentum of its level of origin during its vertical movement, but during the mixing at the final level, a proportion of the momentum is used in processes which do not contribute to the mean momentum.

Taylor (1915-1932) has constructed a theory of the variation of velocity with height on the basis that vorticity is conserved in the transfer process. For two dimensional flow he derives the expression $\rho\overline{wl}\,\partial^2U/\partial z^2$ for the rate of loss of momentum per unit volume at height Z. Using 11 and 14 it is seen that this quantity obeys a power law with index -0.56 to -0.48 which is in agreement with the index of $\partial(\rho\sigma_u\sigma_w)/\partial z$ i.e. -0.55. Taylor's theory does not demand conservation of momentum in the eddy process, so we see that it is consistent with the present results if the correlation coefficient r_{uw} is independent of height.

/A further

A further useful relation may be obtained from the power laws, 12 and 14 giving

$$r_{wl}\,\sigma_l \propto z^{0.95} \quad \text{to} \quad z^{1.03}$$

If the correlation coefficient is independent of height, σ_l is proportional to Z, and σ_l must bear a simple relation to the mixing length, so the expression reduces to $l \propto z$. Best (loc. cit. p.55) produces evidence that σ_w / U probably varies with height in the same way in both lapse and inversion, but in lapse the index in relation 11 is decreased and in inversion it is increased. The index in relation 12 is therefore lower in lapse conditions and higher in inversions, whilst the index in 14 remains constant. If the correlation coefficient r_{wl} is unaffected this means that the variation of mixing length is less rapid than the height in inversions, and more rapid than the height in lapses, in agreement with accepted views.

There is an obvious lack of precision about these arguments, and it must be emphasised that it is not suggested that the power laws have any basis in physical processes, but they are good approximations to the behaviour of the quantities involved and, with the possible exception of that for U , yield values within the range of the errors of observation of the quantities. The restriction to two dimensional treatment and the discontinuous nature of the assumed eddy process are defects shared with many currently accepted theories and so can hardly be made a basis for criticism directed solely at the present argument. Finally it should be noted that all the quantities involved are observable, in particular the quantity $\rho r_{uw} \sigma_u \sigma_w$ could be investigated by an extension of Scrase's cine camera methods, and in conjunction with simultaneous profile measurements its variation would provide a searching test of the argument.

References

Best. A.C. 1935 Geophys. Mem. 7. No 65.

Brooks F.A. 1941. Pap. Phy. Ocean Met. Cambridge, Mass. 8. No. 2.

Brunt D. 1941 "Physical and Dynamical Meteorology" 2nd ed. Cambridge. C.U.P.

Cowling T.G. and White A. 1941 Q.J.R. Met. Soc. 67 p.276.

Elsasser W.M. 1942. Harvard. Met. Studies. No.6.

Johnson N.K. and Heywood. G.S.P., 1938 Geophys Mem. 9. No.77.

Penman. H.L. 1943 Q.J.R. Met. Soc. 69. p.1.

Priestley C.H.B. and Swinbank, W.C., 1947 Proc. R. Soc. A. 189. p.543.

Roberts J.K. 1940 "Heat and Thermodynamics" 3rd ed. London, Blackie.

Robinson G.D. 1947 Q.J.R. Met. Soc 73 p. 127

Scrase. F.J. 1930. Geophys Mem. 6 No. 52

Sutton. O.G. Unpublished. (1943 "Note on the convection of heat in the lower atmosphere") (M.R.P.146)

Taylor G.I. 1915 Phil. Trans. R.S. A.215 p.1

" " 1932 Proc. R.S. A.135 p.685.

FIG. 1

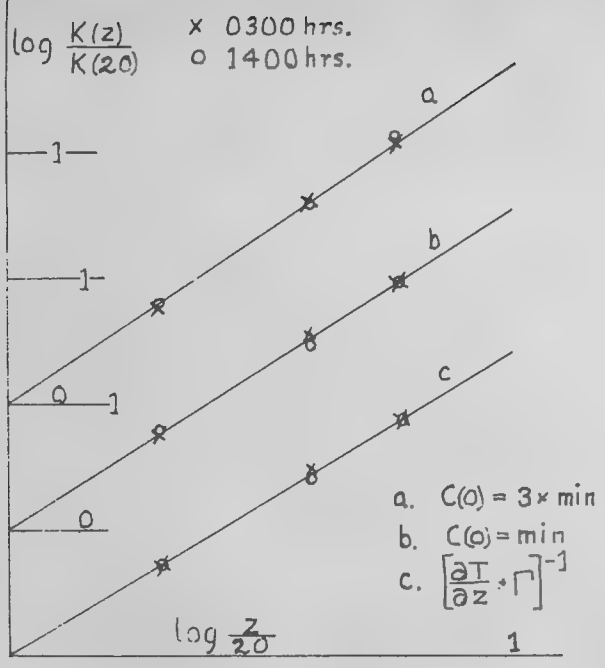


FIG. 3

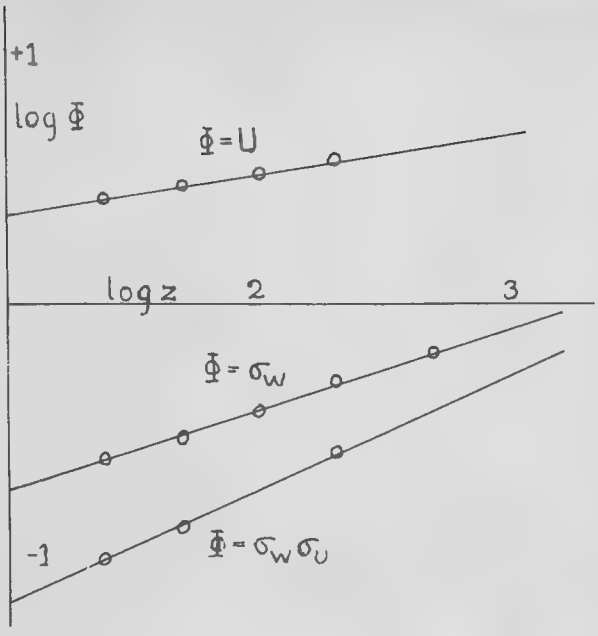
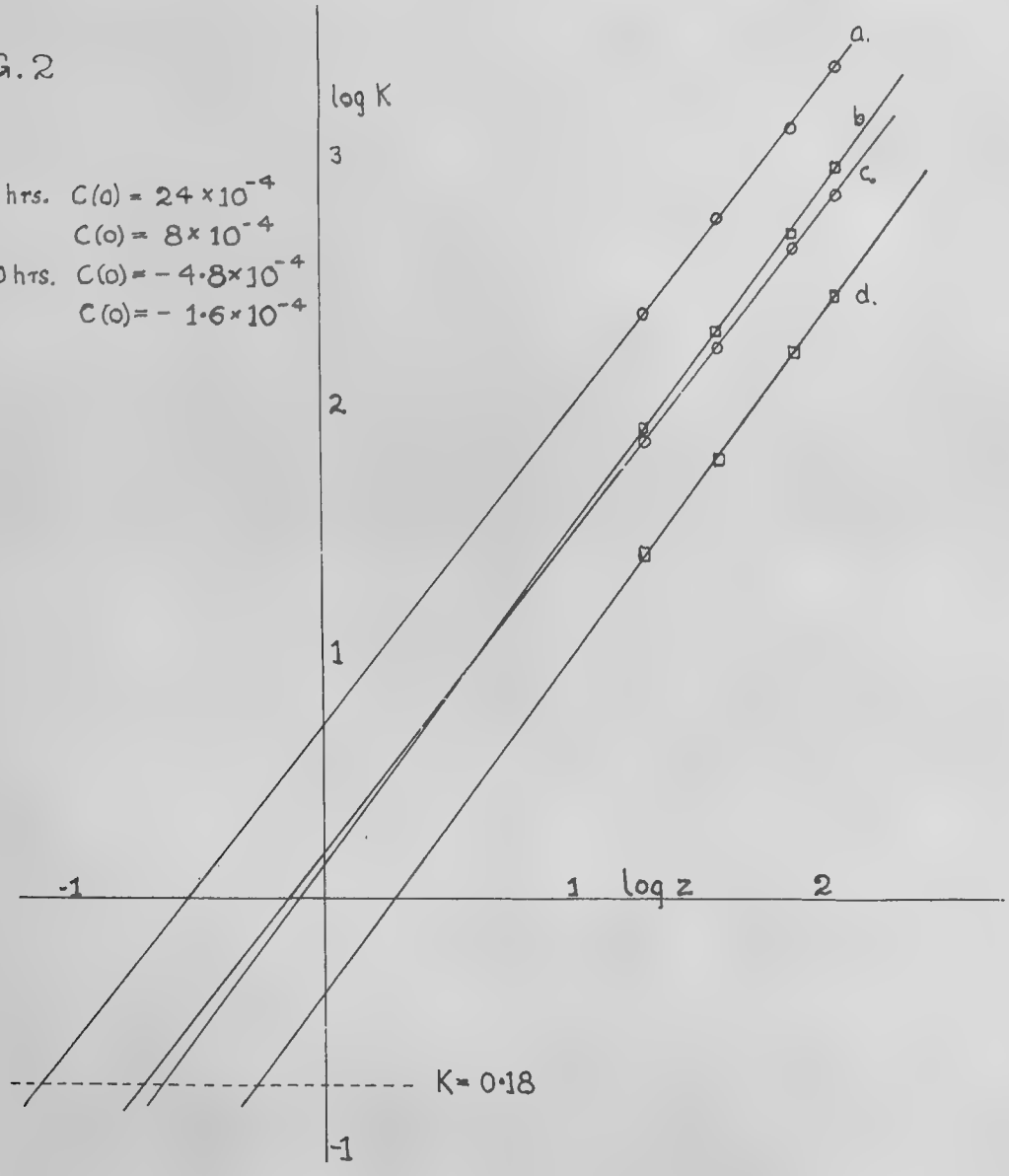


FIG. 2

a. 1400 hrs. $C(0) = 24 \times 10^{-4}$
b. " $C(0) = 8 \times 10^{-4}$
c. 0300 hrs. $C(0) = -4{\cdot}8 \times 10^{-4}$
d " $C(0) = -1{\cdot}6 \times 10^{-4}$



1/4391

This report is not to be published or quoted.

M.R.P. No. 383. S.C.II/2

28th January, 1948.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

A Note on the Insufficiency of the Gradient Wind Approximation in Disturbed Conditions

by

J.K. Bannon

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

Introduction and Summary

In temperate latitudes the gradient wind is usually assumed to be a good approximation to the actual wind, that is, it is assumed that the wind in the free air is close to that required to balance the pressure gradient and the cyclostrophic forces. There is little doubt that this approximation (or the geostrophic approximation) is sufficiently accurate for many of the forecaster's purposes, e.g. the estimation of the mean wind to be expected when flying over a long route. Meteorologists, however, realise that in specific cases the gradient wind may not be a good approximation to the true wind.

The purpose of this note is to investigate the conditions under which the gradient wind is a good approximation to the true wind and to emphasise that although there are many occasions when the gradient wind approximately satisfies the equations of motion, there are other occasions when the gradient wind in temperate latitudes cannot be regarded even as a reasonable approximation to the true wind. Occasions when the gradient wind is of little use to the forecaster may occur when there are appreciable up or down currents over a wide area and when there is a large vertical wind shear. In such circumstances the equations of motion may not be satisfied even approximately by the gradient wind.

Investigation of eight occasions of development over the Hebrides showed that in at least half the cases, the up-currents, (as deduced from rainfall records), and wind shear were so large that the gradient wind could not even be a first approximation to the actual wind.

The horizontal divergence of the wind field is a useful indicator of dynamic development and a comparison of the horizontal divergence of the gradient wind field and of the actual wind field, (as deduced from rainfall), in the upper troposphere on the eight occasions mentioned above, showed that the divergence of the actual wind field was appreciably larger than the divergence of the gradient wind field. The calculations may be in some doubt, however, as, it may be argued, the net-work of upper-air observations which is used to construct pressure - contour charts is not sufficiently close to enable accurate contours, (from which the gradient wind is measured), to be drawn. In two of the cases investigated, however, the rainfall was over a wide area and of several hours duration so that there is little doubt that the extent of the horizontal divergence deduced from rainfall was much greater than could be covered by a sinuosity in the contours too small to be detected.

Thus there are strong grounds for thinking that not only are the deviations of the actual wind from the balanced or gradient wind important in the development of active depressions, but also these deviations may be appreciable over limited areas.

Much of the rather laborious mathematics, including a discussion of the relative magnitudes of the terms in the general equations of motion, has been relegated to an appendix.

/ 1.

1. General Equations of Motion in the Atmosphere, neglecting Friction but allowing for Small Vertical Motion

The general equations of motion in the free air, neglecting friction and assuming that the vertical component of wind is small compared with the horizontal, are, (see Appendix 1) :-

$$\frac{du}{dt} - lv = -\frac{1}{\rho}\frac{\partial p}{\partial x} \quad \ldots\ldots\ldots (1)$$

$$\frac{dv}{dt} + lu = -\frac{1}{\rho}\frac{\partial p}{\partial y} \quad \ldots\ldots\ldots (2)$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \quad \ldots\ldots\ldots (3)$$

where :-

l = $2\omega \sin \phi$
ω = earth's angular velocity
ϕ = latitude

u,v,w are the component velocities in two horizontal directions, (at right angles), and the vertical respectively.

ρ = density of the air
g = acceleration of gravity
d/dt denotes differentiation following the motion
p = atmospheric pressure.

If the axes are taken along the tangent and normal to the isobar through the point of reference, these become, (see Appendix 1 (e)) :-

$$\frac{du_s}{dt} - u_n\frac{d\psi}{dt} - lu_n = 0 \quad \ldots\ldots\ldots (1A)$$

$$\frac{du_n}{dt} + u_s\frac{d\psi}{dt} + lu_s = lJ \quad \ldots\ldots\ldots (2A)$$

where :-

J = geostrophic wind speed.
u_s and u_n are horizontal components of wind along tangent and normal to the isobar respectively.
ψ = angle the isobar makes with a fixed direction.
$\partial/\partial s$ and $\partial/\partial n$ are differentiation along the tangent and normal to the isobar respectively.

2. Wind along the Isobars

If the wind blows along the isobars then, from Eqn.(1A) :-

$$\frac{du_s}{dt} = 0$$

i.e. the speed of each particle of air remains the same.

3. Wind approximately along the isobars

Let us now investigate an airflow which is approximately along the isobars ; i.e. when u_n/u_s is small.

From Eqns. (1A) and (2A) :-

/ $\frac{du_s}{dt}$

$$\frac{du_s}{dt} = u_n \left(1 + \frac{d\psi}{dt} \right) \quad \ldots\ldots\ldots\ldots (4)$$

and

$$u_s \left(1 + \frac{d\psi}{dt} \right) = 1J - \frac{du_n}{dt}$$

$$= 1J \text{ , approximately, } \quad \ldots\ldots\ldots (5)$$

since u_n/u_s is small.

Since the motion is very nearly along the isobars, the isobars will be the streamlines, approximately, and hence $d\psi/dt$ will be the same, to first order, as $d\theta/dt$, the rate of change of the wind direction, following the fluid.

But $$V\sigma = \frac{d\theta}{dt} \quad \ldots\ldots\ldots (6)$$

approximately, where V = horizontal wind speed
and σ = horizontal curvature of the trajectory.

(Equation (6) is strictly true if $w = 0$; when w/V is small, as is assumed in all this work, then the equation is true to first order).

But $$V \doteqdot u_s$$

hence

$$\frac{d\psi}{dt} \doteqdot u_s\sigma$$

Thus Eq. (5) may be written , to first order :-

$$u_s (1 + u_s\sigma) = 1J \quad \ldots\ldots\ldots (7)$$

The gradient wind is defined as blowing along the isobars with speed G such that :-

$$G (1 + \sigma G) = 1J \quad \ldots\ldots\ldots (8)$$

and also $$G (\sigma - \sigma_s) = \frac{\partial\psi}{\partial t} \quad \ldots\ldots\ldots (9)$$

where σ_s = curvature of the isobars, (streamlines in this case). (Priestley, 1).

Thus u_s as defined by Eq. (7) is the gradient wind G , approximately, Blaton's equation, Eq. (9) above, being implicit in Eq.(6).

Hence, if the wind blows approximately along the isobars at every point, it will have the gradient wind speed, to first order.

Returning to Eq.(4), if the assumption that $u_n/u_s \ll 1$ is to be valid, we must have :-

$$\frac{1}{1 + d\psi/dt} \frac{1}{u_s} \frac{du_s}{dt} \ll 1$$

The condition of reality of the roots of Eq.(7) gives a lower limit of 1/2 for $1 + u_s\sigma$ or $1 + d\psi/dt$. Values of $1 + d\psi/dt$ greater than 21 are seldom, if ever, attained in our latitude.

It is thus necessary to examine carefully the expression

$$\frac{1}{1} \frac{1}{u_s} \frac{du_s}{dt}$$

/ to see

to see if, in fact, the assumption that the wind blows approximately along the isobar at every point is justified.

Expanding the differentials following the motion gives :-

$$\frac{1}{u_s}\frac{du_s}{dt} = \frac{1}{u_s}\left(\frac{\partial u_s}{\partial t} + u_s\frac{\partial u_s}{\partial s} + u_n\frac{\partial u_s}{\partial n} + w\frac{\partial u_s}{\partial z}\right)$$

$$\doteq \frac{1}{u_s}\left(\frac{\partial u_s}{\partial t} + u_s\frac{\partial u_s}{\partial s}\right) + \frac{w}{u_s}\frac{\partial u_s}{\partial z} \qquad \ldots\ldots\ldots (10)$$

since it is assumed that $u_n/u_s \ll 1$, and obviously $\partial u_s/\partial n$ will not be of higher order than $\partial u_s/\partial s$.

The first term on the right hand side of Eq.(10) is, with our assumption that $u_n/u_s \ll 1$, approximately the proportionate rate of change of the wind following the horizontal wind, and this, since we have shewn the wind to be approximately the gradient wind under these conditions, is to first order the proportionate rate of change of the gradient wind following the motion under gradient wind, which must be zero, of course, from Section 2 above. i.e. the first term on the right of Eq. (10) is automatically small if u_n/u_s is small.

Since w is small the second term on the right of Eq.(10), $\frac{w}{u_s}\frac{\partial u_s}{\partial z}$, is usually small also. When, however, there is considerable development taking place, though w/u_s is small the whole term may not be. For example, w may be of the order 0.3 mph (13 cm/sec), (Bannon, 2), and $\partial u_s/\partial z$ may be as much as 30 mph/mile, so that in latitude 55°, where $l = 0.41$ hours^{-1},

$$\frac{1}{l}\frac{w}{u_s}\frac{\partial u_s}{\partial z} \quad \text{may be of order} \quad \frac{22}{u_s}$$

which is <u>not</u> very much less than 1 unless u_s is very great.

The assumption that $u_n/u_s \ll 1$ may not be justified, therefore, when considerable development is taking place.

4. Applicability of the Gradient Wind

One method of proving or disproving the justification of the assumption $u_n/u_s \ll 1$, and therefore of approximate gradient wind, would be to study consecutive isobaric or constant pressure contour charts, compare the gradient wind measured at a certain point on one chart with the gradient wind as measured on following charts at points which would have been reached from the first point by gradient motion, and thus check if the rate of change of gradient wind, as measured, is in fact small compared with the velocity multiplied by l. This is checking the first term on the right of Eq.(10) ; if it is not small then the assumption $u_n/u_s \ll 1$, which enables the deduction that it should be small to be made, is not justified. This presents considerable difficulty, however, for the gradient wind is difficult to assess with accuracy owing to inaccuracies in drawing isobars or contours and in estimating the curvature of the trajectory.

The second term on the right of Eq.(10) is more easily assessed in specific cases, however, and this has been done for eight occasions. For the purposes of demonstrating that $u_n/u_s \ll 1$ is not always justifiable, it is sufficient to consider the expression

$$\frac{1}{l}\frac{w}{V}\frac{\partial V}{\partial z}$$

/where

where V is the horizontal velocity, since if u_n/u_s is very small, $u_s \doteqdot V$.

In Table I below, values of $\frac{1}{I}\frac{w}{V}\frac{\partial V}{\partial z}$ are given for eight cases investigated over the Hebrides. The values of w are averages over most of the troposphere and were estimated from the rate of rainfall. (Bannon,2). V and $\partial V/\partial z$ were directly observed at Stornoway, or were estimated from observations at Lerwick, Aldergrove or Liverpool and mean values over depths of from 4,000 to 8,000 ft were used, usually between 10,000 and 18,000 ft, depending on the observations available. The values of $\frac{1}{I}\frac{w}{V}\frac{\partial V}{\partial z}$ will thus be averages over a considerable depth and will therefore probably be exceeded over several thousand feet. They show that in half the cases, at least, the assumption that u_n/u_s is small is not justified ; the gradient wind, in fact, cannot even be a first approximation to the true wind.

Table I

Date (1946)	w (mph)	$\partial V/\partial z$ (mph/mile)	$\frac{1}{V}\frac{\partial V}{\partial z}$ (miles^{-1})	$\frac{1}{I}\frac{w}{V}\frac{\partial V}{\partial z}$
22nd January	.3	Very small	Very small	Very small
22nd February	.3	7	.3	.2
17th March	.4	20	.8	.8
5th June	.3	12	.2	.15
24th June	.25	24	.8	.5
4th July	.4	12	.4	.4
17th November	.2	Small	Small	Small
22nd December	.1	19	.5	.1

5. Comparison of the Horizontal Divergence of the Gradient Wind and the Actual Wind Fields

Horizontal convergence (or divergence) of the wind field is an indication of dynamic development ; it is associated with up- (or down) currents and areas of convergence (or divergence) are necessarily areas of changing circulation. (See, for example, Brunt, Physical and Dynamical Meteorology, 2nd Ed, p.177).

A comparison of the horizontal divergence of the gradient wind field and of the horizontal divergence of the actual wind field, (as deduced from rainfall), will thus be an indication of the ability of the gradient wind to satisfy the requirements of dynamic development.

(a) Divergence of the Gradient Wind

The gradient wind, G, is given by the equation :-

$$\sigma G^2 + lG - lJ = 0 \qquad \ldots\ldots\ldots\ldots \quad (11)$$

(Equation (8) above).

The gradient wind field is thus the vector field $G\underline{t}$ where $\underline{t}$ is unit vector along the tangent to the isobars, or we may write it $\underline{G}$.

Then it can be shewn (Appendix, 2), that the horizontal divergence of this wind field, $\nabla_2 \cdot \underline{G}$, is given by :-

/ $\nabla_2 \cdot \underline{G}$

$$\nabla_2 . \underline{G} = - \frac{G^2 \frac{\partial}{\partial s}(\sigma J)}{J(1 + 2\sigma G)} - \frac{l J_y \cot \phi}{a(1 + 2\sigma G)} \quad \ldots\ldots\ldots\ldots (12)$$

when horizontal variations in density are neglected ##, where :-

J_y = component of the geostrophic wind northwards
a = radius of the earth.

Equation (12) is in a form suitable for computation from surface pressure, or pressure contour charts. J may be measured by a geostrophic scale and σ may be estimated from the formula :-

$$\sigma = \sigma_i (1 - \frac{C}{G} \cos \alpha) - \frac{C}{G} \sin \alpha \frac{\partial \psi}{\partial n} \quad \ldots\ldots\ldots (13)$$

where C = speed of movement of the system.
σ_i = curvature of the isobar or contour
α = angle between $\underline{t}$ and the direction of movement of the system.

(See Appendix, 3 for derivation of this equation or Hogben,(3)).

An idea of the magnitude of $\nabla_2 . \underline{G}$ in the upper troposphere may be obtained thus :-

σ is usually 1/1,000 miles or less in absolute magnitude, and the change from $\sigma = 0$ to this value occurs in 500 miles or more, extreme cases, 300 miles. (In cases of larger σ, e.g. in an old closed low, σ changes but little and G is also not large).

In latitude 55° , $l = .41$ hours^{-1}. The first term in Eq.(12) will thus seldom be greater than

$$\frac{J^2}{.41} \times \frac{1}{1000} \times \frac{1}{500} \text{ hours}^{-1}, \text{ when } J \text{ is in mph.}$$
$$= 5 J^2 \times 10^{-6}$$
$$= 5 \times 10^{-6} \times (\frac{J}{60})^2 \text{ sec}^{-1}$$

The second term in Eq.(12) will be of order $\frac{J_y}{21} \times 10^{-6}$ sec^{-1}

Thus with a strong wind and high curvature, $\nabla_2 . \underline{G}$ may be of order 10^{-5} , (from the first term of Eq.(12)), but is usually less.

(b) Horizontal Divergence of the Actual Wind in the Upper Troposphere

The mean divergence of the actual wind in the upper troposphere may be deduced from estimates of vertical currents based on rainfall rates. (Bannon,2).

It seems reasonable to suppose that vertical velocities in the atmosphere at the tropopause or at some level but little below it, are very small. Vertical movement of the tropopause probably does take place ; (Palmen, as quoted by Haurwitz, Dynamic Meteorology, p.327). It is, however, difficult to visualise a vertical motion of the tropopause of the same order as the rain

$\partial\rho/\partial y$ = about 2×10^{-14} gm/cc/cm for a horizontal variation in temperature of about 10°C per 100 miles. A variation of 10 mb in 100 miles would make $\partial\rho/\partial y$ = approximately 5×10^{-15} gm/cc/cm. The part of $\nabla_2 . \underline{J}$ due to variations in ρ will thus be of order $\frac{1}{l\rho^2} \frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x}$.

i.e. less than $J_y \times 10^{-11}$, taking above figures as extremes, as compared with $J_y \times 10^{-9}$ of the latitudinal term.

/producing

producing up-currents of the troposphere, since such a velocity would cause much larger perturbations than are observed. Available evidence seems to indicate that, not only is the stratospheric air practically distinct from tropospheric, but that the movement of the boundary between them, as observed above one point on the earth's surface, can be explained in almost all cases by the horizontal advection of lower or higher tropopause, and thus the vertical motion of the air at that height must necessarily be very small. (M.R.P. 213 refers. Schedler and J.Bjerknes have pointed out that perturbations of the tropopause can be explained by horizontal advection ; the references for these are given in Brunt's textbook, p.365. This was confirmed by the writer from a study of the variation of tropopause height above active depressions over the British Isles during 1945-46).

The conservation of mass leads to the equation :-

Rate of flow of air into a vertical column of unit cross section through the bottom

= the rate of flow outwards through the sides of this column, when the outflow from the top is known to be zero.

Applying this to a column from level h to the tropopause, at T , leads to the approximate equation :-

$$\overline{\nabla_2 \cdot \underline{V}} = \frac{\rho_h}{\overline{\rho}} w_h \frac{1}{T - h} \qquad \ldots\ldots\ldots\ldots\ldots \quad (14)$$

where :-

$\underline{V}$ = the wind vector

$\overline{\nabla_2 \cdot \underline{V}}$ = the mean horizontal divergence of the wind between h and T.

w_h = up-current at height h.

$\overline{\rho}$ = mean density from h to T.

(See also Appendix,4)

If, for example, h = 10,000 ft, T = 30,000 ft, then $\rho_h/\overline{\rho} = 7/4$ approximately, at temperatures usual to these heights in this country. Then :-

$$\overline{\nabla_2 \cdot \underline{V}} = w_h \times \frac{7}{4} \times \frac{1}{6{,}200 \times 10^2} \ \text{sec}^{-1},$$

if w_h is in cm/sec

$$= \frac{w_h}{36} \times 10^{-4} \ \text{sec}^{-1}$$

The mean up-current through a rising air mass is often between 5 and 15 cm/sec (2), so that w_h will often have at least these values and sometimes more.

Thus $\overline{\nabla_2 \cdot \underline{V}}$, between 10,000 and 30,000 ft in such conditions will lie between 1.4×10^{-5} and 5×10^{-5} sec^{-1} and will sometimes be more. This is a mean value of $\nabla_2 \cdot \underline{V}$ and thus $\nabla_2 \cdot \underline{V}$ will usually be greater than this over a considerable range.

(Fleagle (4) quotes values of $\nabla_2 \cdot \underline{V}$ of the order of 10^{-5} sec^{-1}, and Durst (5) deduced values up to 5×10^{-5} sec^{-1} , in one instance, at a height of 5 km, from the wind field. Houghton and Austin, (6), state that values greater than 10^{-5} sec^{-1} are common .)

/ (c)

(c) Comparison of $\nabla_2 . \underline{G}$ and $\nabla_2 . \underline{V}$ - Examples

From (a) above it is seen that in the upper troposphere the divergence of the gradient wind may sometimes be of order 10^{-5} sec^{-1} but is usually less. From (b) above, the divergence of the actual wind is deduced as often being of order 10^{-5} and sometimes of order 10^{-4} sec^{-1}.

The eight occasions considered in Table I have been investigated and the results are shewn in Table II. In each case the divergence of the gradient wind at 500 mb, (18,000 ft approximately), has been deduced roughly from the contours as published in the Daily Weather Report. The figures stated may be over-estimates.

The mean horizontal divergence of the actual winds was calculated from the mean up-current deduced from the rate of rainfall, assuming that all the upper divergence occurs between 10,000 ft and the tropopause. The figures stated may thus be under-estimates.

Table II

Horizontal Divergence of Actual and Gradient Winds

Date and time (1946)	Divergence of gradient wind at 500 mb (sec^{-1})	Mean divergence of actual wind, 10,000 - 30,000 ft. (sec^{-1})
22nd January Evening	3×10^{-6}	3×10^{-5}
22nd February Afternoon and evening	7×10^{-6}	3×10^{-5}
17th March Morning	5×10^{-6}	5×10^{-5}
5th June Early morning	2×10^{-5}	4×10^{-5}
24th June Early morning	6×10^{-6}	3×10^{-5}
4th July Mid-day and p.m.	2×10^{-5}	7×10^{-5}
17th November Early morning	2×10^{-6}	3×10^{-5}
22nd December Early morning	Less than 10^{-6}	2×10^{-5}

It may be argued that the contours which can be drawn over the British Isles from the available upper-air soundings are not sufficiently accurate to allow of calculations of divergence of the gradient wind measured from them. For example, if the contours had much sharper curves in the neighbourhood of fronts, (almost kinks), than are usually drawn, the divergence of the gradient wind appropriate to such contours would be appreciably greater over a small area than for the smoother contours. And it is difficult to refute this argument in all cases.

It can be said, however, that, from a study of pressure contours over that area of the British Isles with the closest net-work of upper air observing stations, (i.e. over England), sharp curves are rare and smooth contours appear to be the rule in the middle and upper troposphere.

In two of the cases quoted in Table I it is possible to be fairly certain that the gradient wind field over a small area could not explain the development deduced from the rainfall. In the 22nd February case the rainfall was very wide spread and of several hours duration and much of it could not be

/associated

associated with the one warm front in the vicinity ; the depression which was approaching was moving quickly so that there was no question of the horizontal divergence in the upper troposphere being confined to a narrow band which could be associated with very sharply curved contours. Again in the case of 4th July, the rainfall was wide spread over the whole of Scotland and much besides and continued for many hours ; the disturbance had appreciable movement also and the development was certainly not confined to a narrow belt. In this case too the upper air observations gave quite a good net-work from which to draw the contours.

Thus, though the evidence is not entirely conclusive, there are strong grounds for thinking that the gradient wind field is quite incapable of explaining observed development in disturbed conditions.

6. Conclusion

Section 3 has shewn that the gradient wind may be a good approximation to the true wind in undisturbed conditions, but Section 4 demonstrates that it cannot always satisfy the conditions when development is taking place. Section 5 also, though not so conclusively, indicates that the gradient wind field cannot always explain observed phenomena.

It would seem, therefore, that for a true understanding of the processes of dynamical development, attention should be diverted from the wind field of the conventional pressure-contour maps, often assumed to be sufficient approximation to the true wind, to the essential un-balance between wind and pressure gradient, when large changes are taking place. (Sutcliffe,(7)).

References

1. C.H.B.Priestley — Q.J.Roy.Met.Soc., Vol 73, p.65, 1947
2. J.K.Bannon — M.R.P. 353, 1947
3. G.L.Hogben — Q.J.Roy.Met.Soc.,Vol 72, No 314, 1946
4. R.G.Fleagle — Journal of Met., Vol 3, No 4, 1946
5. C.S.Durst — Q.J.Roy.Met.Soc.,Vol 66, No 285, p.171, 1940
6. Houghton and Austin — Journal of Met., Vol 3, No 3, 1946
7. R.C.Sutcliffe — Q.J.Roy.Met.Soc.,Vol 64, No 276, 1938

It seems safe to assume, therefore, that above 1,000 metres, and not in unstable conditions, K is less than 2×10^5 cgs.

If v = 2×10^3 cm/sec, (45 mph), is a typical velocity, the left side of Eq.(15A) is of order, in latitude 55°,

$$\frac{.41}{60^2} \times 2 \times 10^3 = 2 \times 10^{-1} \text{ cgs units.}$$

An extreme case of increase of wind with height would be

10^3 cm/sec at 1 km
2×10^3 " " 2 km
4×10^3 " " 3 km

giving

$$\frac{\partial^2 u}{\partial z^2} \text{ approximately } = \frac{10^3}{10^{10}} = 10^{-7} \text{ cgs units.}$$

$K \frac{\partial^2 u}{\partial z^2}$ will thus, in extreme cases have an upper limit of 2×10^{-2} which is smaller than the other main terms of the equation.

As a first approximation it is therefore permissible to neglect / the

Early morning

It may be argued that the contours which can be drawn over the British Isles from the available upper-air soundings are not sufficiently accurate to allow of calculations of divergence of the gradient wind measured from them. For example, if the contours had much sharper curves in the neighbour hood of fronts, (almost kinks), than are usually drawn, the divergence of the gradient wind appropriate to such contours would be appreciably greater over a small area than for the smoother contours. And it is difficult to refute this argument in all cases.

It can be said, however, that, from a study of pressure contours over that area of the British Isles with the closest net-work of upper air observing stations, (i.e. over England), sharp curves are rare and smooth contours appear to be the rule in the middle and upper troposphere.

In two of the cases quoted in Table I it is possible to be fairly certain that the gradient wind field over a small area could not explain the development deduced from the rainfall. In the 22nd February case the rainfall was very wide spread and of several hours duration and much of it could not be

/associated

Appendix

1. General Equations of Motion in the Free Air

The equations of motion of the free air may be written, using the usual symbols :-

$$\frac{du}{dt} + 2\omega (w \cos \phi - v \sin \phi) = -\frac{1}{\rho}\frac{\partial p}{\partial x} + K\frac{\partial^2 u}{\partial z^2} \quad \ldots\ldots (15A)$$

$$\frac{dv}{dt} + 2\omega u \sin \phi = -\frac{1}{\rho}\frac{\partial p}{\partial y} + K\frac{\partial^2 v}{\partial z^2} \quad \ldots\ldots (15B)$$

$$\frac{dw}{dt} - 2\omega u \cos \phi = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \quad \ldots\ldots (15C)$$

where the axes of x , y , z are taken eastwards, northwards and vertically upwards respectively. (Brunt, Physical and Dynamical Meteorology, 2nd Ed., p.276)

The importance of the various terms in these equations is now considered.

(a) The K terms

The frictional K terms may not be exact as the true nature of eddies is not known ; they do, however, afford a means of estimating the importance of the turbulent nature of the airflow.

Experiment shows that K increases with height up to the limit of the surface friction layer and then decreases again. (Mildner, Schwandtke, Rossby and others, quoted by Haurwitz, Dynamic Meteorology, p.210). The depth of this friction layer will vary with the type of air mass, of course, but is probably below 1,000 metres except in unstable conditions. (Taylor, as quoted by Brunt, Physical and Dynamical Met., p.255).

K will also vary with the air mass, being larger the less the stability. Mildner, (loq. cit.), found on one occasion a maximum of 4×10^5 cgs at 240 metres, falling off to 6×10^4 cgs in the first 500 metres ; other investigators have found values up to about 2×10^5 cgs in the first 1,800 metres, (see Shaw, Manual of Meteorology, Vol IV, p.140), but usually less than 10^5 cgs.

It seems safe to assume, therefore, that above 1,000 metres, and not in unstable conditions, K is less than 2×10^5 cgs.

If $v = 2 \times 10^3$ cm/sec, (45 mph), is a typical velocity, the left side of Eq.(15A) is of order, in latitude 55°,

$$\frac{.41}{60^2} \times 2 \times 10^3 = 2 \times 10^{-1} \text{ cgs units.}$$

An extreme case of increase of wind with height would be

10^3 cm/sec at 1 km
2×10^3 " " 2 km
4×10^3 " " 3 km

giving

$$\frac{\partial^2 u}{\partial z^2} \text{ approximately } = \frac{10^3}{10^{10}} = 10^{-7} \text{ cgs units.}$$

$K\frac{\partial^2 u}{\partial z^2}$ will thus, in extreme cases have an upper limit of 2×10^{-2} which is smaller than the other main terms of the equation.

As a first approximation it is therefore permissible to neglect

/ the

the K terms in the equations of motion, above the surface friction layer, i.e. above about a kilometre, when there is no great measure of instability.

(b) Terms in w

w may be large in violent convection, of the order of 50 metres/sec. (M.R.P. 192, and Brunt, loq.cit.)

For large scale upward motion, however, as is found in the active depressions of temperate and high latitudes, w = 30 cm/sec would be a large value and in most cases w will be less than 20 cm/sec. (M.R.P. 353). For downward motion, w is probably less than 2cm/sec, (1.7 km/day). (Brunt, loq. cit.)

Thus except in the (relatively) small scale convection, w in general is small compared with the horizontal velocity. (Further, it is probable that large values of w are usually associated with large values of the horizontal velocity).

The term $2\omega w \cos\phi$ in Eq.(15A) may thus be neglected, as a first approximation.

The terms $w\,\partial u/\partial z$, $w\,\partial v/\partial z$ in Eq. (15A) and (15B) respectively, are not , however, negligible in all cases.
e.g. if $\partial u/\partial z = 2 \times 10^{3} \times 10^{-5} = 2 \times 10^{-2}$ as assumed in para.(a) above, the terms $w\,\partial u/\partial z$ and $2\omega w \cos\phi$ will be in the ratio 124/1 for $\phi = 55°$. (See also Durst and Sutcliffe, Q.J.Roy.Met.Soc., No 273,January 1938 and No 281, 1939.)

(c) The terms in Equation (15C)

$2\omega u \cos\phi$ is negligible compared with g. (Brunt, loq.cit.). dw/dt will also be very small compared with g except in strong convection, since w is of order 30 cm/sec or less.

(d) Equations of motion - first approximation

The general equations (15) for the motion of the free air may therefore be written as under, as a first approximation, above the friction layer, when the air is not markedly unstable :-

$$\frac{du}{dt} - 2\omega v \sin\phi = -\frac{1}{\rho}\frac{\partial p}{\partial x} \quad \ldots\ldots\ldots\ldots (16A)$$

$$\frac{dv}{dt} + 2\omega u \sin\phi = -\frac{1}{\rho}\frac{\partial p}{\partial y} \quad \ldots\ldots\ldots\ldots (16B)$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \quad \ldots\ldots\ldots\ldots (16C)$$

It should be noted that these equations do not omit the effect of (small) vertical motion as the w terms are retained in du/dt and dv/dt. They will, also, retain the same form though the axes are rotated about the vertical so that x and y are no longer E and N respectively.

(e) Equations of motion (first approximation) referred to axes along and perpendicular to the isobars

Equations (16) above, when referred to fixed axes momentarily

/coinciding

coinciding with the tangent and normal to the isobars and the vertical, respectively, become :-

$$\frac{du}{dt} - lv = 0$$

$$\frac{dv}{dt} + lu = lJ$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

where $l = 2\omega \sin\phi$, and J = geostrophic wind $= -\frac{1}{l\rho}\frac{\partial p}{\partial y}$.

If we refer the motion of a particle of air to axes which are always along the tangent and normal to the isobars and the vertical respectively, as the particle moves, then these axes will have a rotation $d\psi/dt$ about the vertical, where ψ is the angle between the tangent to the isobar and a fixed direction, and the equations of motion will then become :-

$$\frac{du_s}{dt} - u_n\frac{d\psi}{dt} - lu_n = 0$$

$$\frac{du_n}{dt} + u_s\frac{d\psi}{dt} + lu_s = lJ$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

where u_s, u_n are the velocities along tangent and normal to the isobar, respectively.

2. Divergence of the Gradient Wind

If $\underline{G}$ = gradient wind $= (G_x, G_y)$

$= (G\cos\psi, G\sin\psi)$

$= G\underline{t}$

where $\underline{G}$ makes ψ with the x axis, then

$$\sigma G^2 + lG - lJ = 0 \qquad \ldots\ldots\ldots\ldots (17)$$

(Equation (8) in the text).
Then

$$\nabla_2 \cdot \underline{G} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y}$$

$$= \frac{\partial G}{\partial x}\cos\psi + \frac{\partial G}{\partial y}\sin\psi - G\sin\psi\frac{\partial\psi}{\partial x} + G\cos\psi\frac{\partial\psi}{\partial y}$$

Putting $\psi = 0$, $\frac{\partial}{\partial x} = \frac{\partial}{\partial s}$, $\frac{\partial}{\partial y} = \frac{\partial}{\partial n}$

where s and n are along the tangent and normal to the isobars, respectively, we get :-

$$\nabla_2 \cdot \underline{G} = \frac{\partial G}{\partial s} + G\frac{\partial\psi}{\partial n} \qquad \ldots\ldots\ldots\ldots (18)$$

Similarly $$\nabla_2 \cdot \underline{J} = \frac{\partial J}{\partial s} + J\frac{\partial\psi}{\partial n} \qquad \ldots\ldots\ldots\ldots (19)$$

/since

since $\underline{J}$ and $\underline{G}$ are in the same direction.

If the axes of x and y are taken eastwards and northwards respectively, then :-

$$\underline{J} = (J_x, J_y)$$

where

$$J_x = -\frac{1}{l\rho}\frac{\partial p}{\partial y}, \quad J_y = \frac{1}{l\rho}\frac{\partial p}{\partial x}$$

Neglecting horizontal variations in ρ, (see p. 6 of text, footnote),

$$\nabla_2 \cdot \underline{J} = -\frac{1}{l^2\rho}\frac{\partial p}{\partial x}\frac{\partial l}{\partial y}$$

But

$$\frac{\partial l}{\partial y} = 2\omega\cos\phi/a = \frac{1}{a}\cot\phi$$

where a = radius of the earth.
Therefore

$$\nabla_2 \cdot \underline{J} = -\frac{J_y}{a}\cot\phi \qquad \ldots\ldots\ldots (20)$$

Therefore from (19)

$$\frac{\partial J}{\partial s} + J\frac{\partial \psi}{\partial n} = -\frac{J_y}{a}\cot\phi \qquad \ldots\ldots\ldots (21)$$

Differentiating Eq.(17)

$$\frac{\partial G}{\partial s}(2\sigma G + 1) - l\frac{\partial J}{\partial s} + (G - J)\frac{\partial l}{\partial s} + G^2\frac{\partial \sigma}{\partial s} = 0$$

i.e. $$\frac{\partial G}{\partial s}(2\sigma G + 1) - l\frac{\partial J}{\partial s} + (G - J)\frac{1}{a}\cot\phi\sin\psi + G^2\frac{\partial \sigma}{\partial s} = 0$$

and therefore

$$\frac{\partial G}{\partial s} = \frac{1}{2\sigma G + 1}\left\{ l\frac{\partial J}{\partial s} - G^2\frac{\partial \sigma}{\partial s} - \frac{1}{a}(G - J)\cot\phi\sin\psi \right\}$$

and hence from (18) and (21),

$$\nabla_2 \cdot \underline{G} = \frac{1}{2\sigma G + 1}\left[\left\{ l\frac{\partial J}{\partial s} - G^2\frac{\partial \sigma}{\partial s} - \frac{1}{a}(G - J)\cot\phi\sin\psi \right\} - \frac{G}{J}\left\{ \frac{J_y}{a}\cot\phi + \frac{\partial J}{\partial s} \right\} \right]$$

$$= \frac{1}{J(2\sigma G + 1)}\left[\left\{ \frac{\partial J}{\partial s}(lJ - lG - 2\sigma G^2) - JG^2\frac{\partial \sigma}{\partial s} \right\} - \frac{\cot\phi}{a}J_y\left\{ l(G - J) + G(2\sigma G + 1) \right\} \right]$$

$$= -\frac{1}{J(2\sigma G + 1)}\left\{ G^2\frac{\partial J}{\partial s} + G^2 J\frac{\partial \sigma}{\partial s} + lJ\frac{J_y\cot\phi}{a} \right\} \text{ from Eq.(17)}$$

$$= -\frac{G\frac{\partial}{\partial s}(\sigma J)}{J(2\sigma G + 1)} - \frac{1}{a}\frac{J_y\cot\phi}{2\sigma G + 1} \qquad \ldots\ldots\ldots (22)$$

It should be noted that from Eq(17), $2\sigma G + 1$ is always positive for real G, and has its minimum of 1/2 when $1 + 4\sigma J = 0$, i.e. when G has its maximum value of 2J.

/3.

3. Relation between curvature of the trajectory and curvature of the streamline

If $\underline{V}$ is the velocity of the air, supposed horizontal,
σ the curvature of the trajectory
σ_s . . .. . . streamline

then

$$V\sigma = \frac{d\psi}{dt}$$, i.e. the rate of change of ψ following the particle.

$$= \frac{\partial\psi}{\partial t} + V\frac{\partial\psi}{\partial s}$$

and therefore

$$\frac{\partial\psi}{\partial t} = V(\sigma - \sigma_s) \qquad \ldots\ldots\ldots\ldots (23)$$

(Blaton's formula).

If the system is moving with velocity $\underline{C}$, then

$$\frac{\partial\psi}{\partial t} + \underline{C}.\nabla\psi = 0$$

and therefore

$$\frac{\partial\psi}{\partial t} = -C\cos\alpha\,.\,\sigma_s - C\sin\alpha\frac{\partial\psi}{\partial n}$$

where α is the angle between $\underline{C}$ and $\underline{V}$.

Hence from Eq (23) ,

$$V(\sigma - \sigma_s) + C\sigma_s\cos\alpha + C\sin\alpha\frac{\partial\psi}{\partial n} = 0$$

i.e. $$\sigma = \sigma_s\left(1 - \frac{C}{V}\cos\alpha\right) - \frac{C}{V}\sin\alpha\frac{\partial\psi}{\partial n} \qquad \ldots\ldots\ldots (24)$$

(See G.L.Hogben. Q.J.Roy.Met.Soc., Vol 72, No 314, 1946, for a full discussion of the estimation of the curvature of air trajectories.)

4. The estimation of the horizontal divergence in the upper troposphere

The well known tendency equation of J.Bjerknes, (Met. Zeitschr. 1937), can be written

$$\left(\frac{\partial p}{\partial t}\right)_h = -\int_h^\infty g\,\nabla_2.(\rho\underline{V})\,dz + g(\rho w)_h \qquad \ldots\ldots (25)$$

where h is some specified height.

As seen in section 5 (b) of the main paper, $w \doteq 0$ at or near the tropopause and hence we may write

$$\left(\frac{\partial p}{\partial t}\right)_T = -\int_T^\infty g\,\nabla_2.(\rho\underline{V})\,dz$$

where T is the height of the tropopause or some level just below it where the vertical velocity is zero.

Equation (25) may be written

$$\left(\frac{\partial p}{\partial t}\right)_h - \left(\frac{\partial p}{\partial t}\right)_T = -\int_h^T g\,\nabla_2.(\rho\underline{V})\,dz + g(\rho w)_h \qquad \ldots\ldots (26)$$

/When

When w_h , $(\partial p/\partial t)_h$, $(\partial p/\partial t)_T$ are known, Eq (26) may be used to assess the horizontal divergence of the air in the troposphere above the height h.

The two terms on the left hand side of Eq (26) are of the same order and usually tend to cancel each other. (See, for example, the reproduction of Penner's results in Haurwitz's Dynamic Meteorology, p.323). They are easily computed from radio-sonde observations, when the surface pressure tendencies are known.

In cases of large development, $\partial p/\partial t$ will be of the order 4 mb in 3 hours, i.e. $(4 \times 10^3)/(3 \times 60^2)$ = about 1/3 dyne/cm^2/sec, so that the left hand side of Eq (26) would be of order 2/3 at most, taking the two tendencies to be of opposite sign

The second term on the right hand side of Eq (26) , in the middle troposphere, is of order w dyne/cm^2/sec , if w is measured in cm/sec.

In disturbed conditions w is often 5 to 15 cm/sec, or even more, (MRP 353). In fact the tendency terms on the left of Eq (26) are usually much smaller than the others when development is large, and hence in these conditions we may write

$$\int_h^T g \nabla_2 \cdot (\rho \underline{V}) \, dz = g (\rho w)_h \qquad \ldots\ldots\ldots (27)$$

(In all cases considered in Table II of the text, the tendency terms were very much smaller than the values assumed above and were thus negligible.)

The variation of ρ in the horizontal is very small and may be neglected. (See footnote of p.6 of text.) Thus Eq (27) may be written

$$\rho_h w_h = (T - h) \overline{\nabla_2 \cdot \underline{V}} \, \bar{\rho}$$

where the bar denotes an average over the interval h to T.

Hence

$$\overline{\nabla_2 \cdot \underline{V}} = \frac{\rho_h}{\bar{\rho}} w_h \frac{1}{T - h} \qquad \ldots\ldots\ldots (28)$$

This report has not yet been published and is to be treated CONFIDENTIALLY. Its contents must not be quoted.

M.R.P. 387,
S.C. I/1.
26th February, 1948.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

AD 611965

The Errors of GL III observations

by D.N. Harrison

Contents.

Summary

1. Introduction
2. Instrumental details
3. Data
4. Mean differences
5. Mean deviation (scatter)
6. Mean change of difference in 1 minute
7. Mean deviation of mean of ascent (drift)
8. Azimuth and elevation errors from theodolite trials
9. Conclusions.

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

Summary

Simultaneous observations with two GL IIIs have been made on 14 ascents. The results show observational errors little greater than the discrimination of the set, but considerable systematic errors, especially in range. Comparison is made with the collected data of theodolite tests of GL IIIs.

2. Mean deviation (scatter)

3. Mean change of difference in 1 minute, without regard to sign

and For all ranges together.

4. Mean deviation of the mean of the ascent from the mean of all (drift).

The results are summarized in Table I. Means (1) are given for every ascent, but means (2), (3) and (4) only for the whole series.

4. Mean differences

In range, these are less than 10 yards throughout, the mean for all ranges being 7 yards. This appears to be very good agreement, but it will be seen later that it is fortuitous.

In azimuth the mean difference of 0.9° is rather large, but an error of this amount would not be very serious for wind measurements.

/The

* The effect of these errors on wind determinations is discussed in MRP 340 by J.K. Hannon.

The Errors of GL III observations

1. Introduction

Since the GL III equipment was adopted by the Meteorological Office for wind measurements, a number of comparisons against pilot-balloon theodolites have been made on balloon-borne targets, with the object of assessing the errors of the GL III observations.* It is a simple matter to place a theodolite close to the GL and by synchronized readings to obtain a direct measure of the GL errors in azimuth and elevation, since the theodolite is much the more accurate. Range comparisons, however, cannot be made, since it is impossible to locate the balloon with sufficient accuracy by theodolite observations. It is believed that kine-theodolites have been used for this purpose, but the technique is difficult and requires specially trained staff, and this method has not been attempted by the Meteorological Office. Apart from the use of fixed reference objects on the ground, the only other method by which range errors can be assessed is direct comparison of one radar set with another.

Such a comparison was made at Larkhill in September 1947. A GL III was borrowed from the School of Artillery, and simultaneous observations were made with this and with the set belonging to the radio-sonde station on sixteen ascents on five consecutive days. The data, presented here, represent a considerable effort, and much credit is due to those who organized and those who took part in the experiment, which was done under the direction of Mr. R.J. Murgatroyd, Officer in charge of the station.

2. Instrumental details

The M.O. set (No. 1) was the one used for routine radio-sonde ascents; it possessed a range doubler and type CV 120 magnetron. The borrowed set (No.2) had a type CV 232 magnetron working at medium power, and the range was limited to 36,000 yards. Both sets were in good order. The levels were checked at the beginning of each day, and readings on a fixed reference object were made before and after each ascent. Three operators were employed on each set. Readings were taken at $\frac{1}{2}$ minute or 1 minute intervals, and as far as possible the "breaks" on the range tubes were kept of the same height in the two sets. Synchronization errors were negligible. Standard 3 foot corner reflectors of metallized paper were used.

3. Data

660 simultaneous observations were obtained; the differences between the readings of range, azimuth and elevation, corrected for the distance of 16 yards between the sets, form the material for this analysis. The following mean values have been found for each ascent, for ranges 0-10, 10-20, 20-30 and 30-36 thousand yards:-

1. Algebraic (systematic) mean difference

2. Mean deviation (scatter)

3. Mean change of difference in 1 minute, without regard to sign

and For all ranges together.

4. Mean deviation of the mean of the ascent from the mean of all (drift).

The results are summarized in Table I. Means (1) are given for every ascent, but means (2), (3) and (4) only for the whole series.

4. Mean differences

In range, these are less than 10 yards throughout, the mean for all ranges being 7 yards. This appears to be very good agreement, but it will be seen later that it is fortuitous.

In azimuth the mean difference of 0.9° is rather large, but an error of this amount would not be very serious for wind measurements.

/The

* The effect of these errors on wind determinations is discussed in MRP 340 by J.K. Bannon.

The difference of about 0.2° in elevation is serious, since it implies zero errors of at least ±0.1°. The existence of systematic errors of this magnitude has been shown repeatedly by theodolite comparisons (see Table II). The resulting error in height is proportional to the horizontal range; for 0.1° it is 100 ft. at 19,000 yards.

There is no significant systematic change with increasing range in azimuth or elevation; in range, the mean differences are practically constant, but the values for individual ascents show considerable systematic changes (see section 7).

5. Mean deviation

This is taken from the mean of the individual range group, and thus the effect of any systematic variation with range is eliminated. The over-all mean deviation is 18 yards for range, 0.12° for azimuth and 0.10° for elevation. If the mean deviation is proportional to the standard deviation and is the same for two sets, the m.d. of a single reading is $1/\sqrt{2}$ of these values.

The m.d. of A and E is largest in the first range group. This is due to the first two or three readings, which always tend to show more scatter, presumably because the angles are changing rapidly. If these are excluded the m.d. increases with range, but only slowly; this is probably due to the fact that with the transmitting valves used, good signal strength is obtained at the range limit of 36,000 yards.

6. Change of difference in 1 minute

This is the important quantity for a wind determination by slide-rule without smoothing. If the errors of observation are completely random, these values should be $\sqrt{2}$ times the m.d., but any tendency for the observations to run in groups, high and low, will reduce the ratio. The ratio, as shown in the table, is actually very close to $\sqrt{2}$, although at other trials with theodolites a ratio near to 1.0 has been found (see Table II).

7. Mean deviation of mean of ascent (drift).

Treating the mean difference for an ascent as a single observation, we may obtain the m.d. The ratio of this to the m.d. previously found will depend on the number of observations per ascent (roughly 40) and on the magnitude of long-period changes. If the latter were zero, it would be $1/\sqrt{40}$, or 0.16. For A and E the ratio is about 0.5, which shows that there are probably real, though small, changes of zero from one ascent to another. For R the ratio is 1.2, indicating a drift larger than the error of observation.

In A the variation from one ascent to another is practically random. In E there is a tendency to a continuous drift over several consecutive ascents, the total change being 0.3°. In R the changes are large; the following are the mean values for the separate days:-

1st day	(ascents 1-4)	- 1 yd.
2nd day	(" 5-8)	+ 24 "
3rd day	(" 9-12)	- 37 "
4th day	(" 13,14)	- 7 "
5th day	(" 15,16)	+ 11 "

Thus the mean difference in range measurement between two sets, both carefully maintained and thought to be in good condition, may change by more than 60 yards from one day to the next. The possibility of even larger errors, of the same sign in the two sets, is not excluded.

The mean differences for the separate range groups are also shown in Table 1. In A and E there is no systematic variation with increasing range, but the mean difference in R shows a large systematic variation of two distinct types, (a) decreasing and (b) increasing with range. Ascents Nos. 1 - 4 (the first day)

/and

and No. 6 are of type a, the rest of type b. (The number of observations in the last range group is in some cases only 2 or 3). The magnitude of the change during one ascent varies between 10 yards and 60 yards or more.

The mean differences between the readings on the reference object before and after each ascent are also given in Table 1. There is some correspondence, but not very much, between these and the values for the ascents.

8. Azimuth and elevation errors from theodolite trials.

In Table II are given the results of all comparisons between GL III and theodolite observations made at M.O. stations. In these trials either one theodolite close to the GL was used and the differences were corrected for parallax, or two theodolites symmetrically placed, the mean of their readings being taken. The difference (GL - theodolite) is taken as the GL error. The data are grouped as follows:-

	Place	Date	No. of ascents	No. of obs.	Magnetron
1	Larkhill*	1944	14	356	CV 41
2	Larkhill	June 1947	4	236	
3	Larkhill	June 1947	1	41	CV 120
4	Lerwick	May-Sept.1947	7	202	or
5	Downham, Fazakerley, Leuchars	Aug-Oct. 1947	8	165	CV 232

The set used in Nos. 2 and 3 was the same as that referred to as set No. 1 in the earlier part of this report. In all these trials 3 foot corner reflectors of metallized paper were used. At most of the trials No. 1, at Nos. 2, 4 and probably No. 5, two GL operators were employed, one controlling range and time-keeping, the other azimuth and elevation; this is normal practice at M.O. stations. It has often been remarked that when the signal is weak the second operator cannot get correct settings of A and E simultaneously on the exact minute, but tends to get one set beforehand and then to concentrate on the other. The data of the four ascents in group 2 were examined with this in view, and in some cases a small correlation was found between the error and rate of change of A or E, the sign indicating an early setting. At No. 3, therefore, three operators were employed, and although on this occasion the signal was weak the mean deviations are smaller than on any of the other four ascents. It is considered that this is a real effect of the third operator.

Comparing Tables I and II, it is seen that the mean deviations of A and E in Table I are about the same as the best of the results of theodolite comparisons. Since Table I represents the difference between two GLs, the separate observations must have been appreciably better. No doubt this is largely due to the employment of three operators on each set.

It may be noted here that errors of setting in azimuth, depending as they do on the changes of received signal strength with angle measured in a plane through the axis of the beam, should be proportional to sec. E. This probably accounts for the fact that azimuth errors are found to be larger than those of elevation.

9. Conclusions

The conclusions may be summarized as follows:-

1. Under the best conditions the mean error of setting and reading the GL III is less than 18 yards in range, and less than 0.1° in azimuth and elevation. It is consistently less for elevation than for azimuth. There is only slight deterioration with increasing range up to 36,000 yards, when CV 120 or CV 232 magnetrons are used.

* See Addent No. 1 to SDTM No. 64

/2.

2. Other things being equal, the scatter is less with three operators than with two.

3. The observational error is generally practically random, but sometimes shows a systematic trend over several minutes (probably when only two operators are employed).

4. At any one ascent, systematic errors in range of at least ± 20 yards, and probably more, may occur. These may increase or decrease systematically with range, and may vary erratically between one ascent and the next.

5. For azimuth and elevation the systematic errors are a few tenths of 1°.

6. The systematic errors in R and E may give rise to important differences in height measurements between two stations, or at one station at different times.

7. It would be valuable if this experiment could be repeated with two sets, both having range doublers.

TABLE I

Mean difference, set No. 2 - set No. 1

Ascent No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Mean diff.	Mean dev.	Mean change in 1 min.	Drift
Range 1000's yards										Range (yds)										
0-10	+ 34	+ 8	+ 24	+ 12	+ 20	+ 11	+ 26	+ 19	- 29	- 48	- 44	- 52	+ 5	- 31	+ 2	- 18	- 9	17	25	-
10-20	+ 11	- 24	+ 18	- 3	+ 38	+ 8	+ 37	+ 22	- 26	- 44	- 39	- 37	+ 12	- 23	- 7	+ 13	- 5	19	28	-
20-30	- 5	- 22	- 13	- 54	+ 36	+ 5	+ 34	+ 27	- 15	- 41	- 25	- 44	-	- 15	+ 27	+ 23	- 8	19	29	-
30-36	-	- 33	- 45	- 80	+ 40	- 38	+ 53	+ 49	0	- 52	- 26	- 59	-	- 13	+ 54	+ 53	- 5	21	34	-
0-36	+ 14	- 15	+ 8	- 13	+ 32	+ 4	+ 34	+ 25	- 20	- 45	- 35	- 46	+ 8	- 21	+ 13	+ 8	- 7	18	28	22
R.O.	- 10	- 10	0	- 20	0	0	0	+ 10	+ 10	0	- 10	- 20	0	- 20	+ 10	- 15	-	-	-	-
										Azimuth (°)										
0-10	-1.09	- .97	- .86	- .83	- .83	- .89	- .92	- .96	- .86	-1.01	- .95	-1.02	- .90	- .80	- .85	- .75	- .91	.15	.19	-
10-20	- .96	- .93	- .79	- .91	- .83	- .85	- .92	- .89	- .89	- .86	- .90	- .95	- .75	- .76	- .93	- .83	- .88	.09	.13	-
20-30	- .80	-1.06	- .87	- .80	- .69	- .65	- .92	- .93	- .94	- .86	- .98	-1.03	-	- .84	- .97	- .93	- .89	.14	.16	-
30-36	-	-1.20	- .75	- .77	- .88	- .64	- .82	- .80	- .92	- .65	-1.04	-1.05	-	- .73	-1.30	- .76	- .87	.14	.19	-
0-36	- .96	-1.01	- .83	- .84	- .80	- .79	- .91	- .91	- .90	- .89	- .96	-1.00	- .83	- .79	- .97	- .81	- .89	.12	.16	.07
R.O.	- .70	- .70	- .60	- .65	- .70	- .70	- .80	- .85	- .55	- .60	- .70	- .55	- .60	- .70	- .70	- .70	-	-	-	-
										Elevation (°)										
0-10	- .14	- .20	- .21	- .25	- .42	- .25	- .33	- .33	- .18	- .12	- .18	- .14	- .15	- .10	- .22	- .23	- .22	.12	.17	-
10-20	- .29	- .26	- .20	- .23	- .27	- .33	- .27	- .33	- .28	- .21	- .10	- .27	- .07	- .07	- .22	- .19	- .22	- .08	.11	-
20-30	- .30	- .26	- .12	- .22	- .25	- .22	- .12	- .43	- .26	- .30	- .15	- .11	-	- .07	- .05	- .23	- .21	.10	.14	-
30-36	-	- .23	- .05	- .20	- .22	- .24	- .30	- .60	- .28	- .29	- .22	- .27	-	- .07	+ .08	- .28	- .23	.09	.13	-
0-36	- .24	- .23	- .17	- .23	- .30	- .26	- .27	- .38	- .25	- .21	- .16	- .18	- .11	- .07	- .13	- .23	- .22	.10	.14	.05
R.O.	- .75	- .70	- .70	- .60	- .70	- .70	- .60	- .65	- .50	- .50	- .50	- .50	- .40	- .45	- .80	- .55	-	-	-	-

Set No. 1: CV 120 magnetron and range doubler (not used).

Set No. 2: CV 232 magnetron, range limit 36,000 yards.

Three operators on each set. R.O., reference object.

TABLE II - GL errors from theodolite observations

Date	No. of obs.	Range 1000's yds.	Azimuth Mean error °	Azimuth Mean dev. °	Azimuth Change in 1 min °	Azimuth Drift °	Elevation Mean error °	Elevation Mean dev. °	Elevation Change in 1 min °	Elevation Drift °	Remarks
LARKHILL											
1944	356	0-36	- .05	.16	.16	.07	+ .03	.11	.11	.04	CV 41 magnetron, 14 ascents.
June 1947	236	0-25	+ .05	.18	.20	-	- .28	.12	.15	-	4 ascents.
June 1947	41	0-32	- .20	.11	.16	-	+ .10	.10	.11	-	1 ascent. 3 GL operators.
LERWICK											
8. 5.47	29	2-34	+ .19	.13	.12		+ .53	.11	.15		
10. 5.47	44	2-40	+ .14	.13	.17		+ .11	.13	.16		
12. 5.47	13	4-18	+ .46	.11	.13		+ .19	.10	.12		
2. 7.47	18	1-11	+ .21	.15	.21		+ .08	.09	.15		
7. 7.47	21	1-17	+ .51	.08	.13		+ .20	.07	.07		
21. 7.47	45	2-27	+ .10	.10	.11		+ .11	.07	.11		
12. 9.47	32	0-64	+ .05	.11	.09		+ .01	.07	.08		
Mean (7 ascents, equal weight to each)				.12	.14			.09	.12		
DOWNHAM MARKET											
15. 8.47	53	2-33	- .04	.17	.21		- .45	.10	.14		
16. 8.47	32	3-21	+ .02	.12	.16		- .45	.08	.10		
18. 8.47	31	1-19	- .08	.31	.30		- .42	.11	.11		
FAZAKERLEY											
Aug-Sept.47	25	0- 3	+1.00	.20	.29		± .09	.08	.11		4 ascents, treated as 1.
LEUCHARS											
13.10.47	24	1-50	+ .11	.17	.20		- .20	.05	.07		
Mean (5 ascents, equal weight to each)				.24	.23			.08	.11		

CV 232 working at high power or CV 120 magnetron was used and 2 GL operators employed, except where otherwise stated.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

Amendment to M.R.P.392 - The Angular Deviation of the Wind from the Isobars, at Liverpool.

The following corrections are necessary, to M.R.P.392; the need for these arises from an uncorrected slip in the printed version of S.D.T.L.75 Pt. VII.

Page 7, at the foot, to read:-

2. a systematic error, of average value:-

700 mb	500 mb	
+ 3 ft.	+17 ft.	Maximum
+20 ft.	- 7 ft.	Minimum (darkness)

Page 13, 3rd paragraph, lines 9 and 10 to read:-

"Observations of the height of the 700 mb surface have systematic errors of up to 3ft. Thus the" - etc.

Page 13, 3rd paragraph, lines 11 and 12 to read:-

"to have a systematic error, compared with the other observations over the British Isles, of 0 to - 9 ft; this would" - etc.

0494736

C.202872/D.S./8/48.

36 15

The deviations measured at 3,000 ft were probably not typical, for it seems likely that, even at 3,000 ft the mountains have an appreciable effect on the wind direction. Analysis of the deviations showed no correlation with the stability of the air.

An attempt was made to correlate the deviations at 700 and 500 mb with the general weather situation and development with some apparent success. It was found that, at both levels, there was a negative deviation on the boundary of an anticyclone, i.e. inflow to the anticyclone. Also, that at 500 mb and less markedly at 700 mbs, deviations were positive ahead of a trough in the upper air, to the top of the wedge, and negative behind the trough. On one occasion the observed deviations agreed with general expectations from significant features of the general situation.

The geostrophic wind was also measured at 3,000 ft, from the surface isobars, due allowance being made for height and excluding all cases of marked horizontal temperature gradients in the vicinity. The mean value of the moduli of the vector deviations of the wind from the geostrophic was found to be 9.1 kts, and the root mean square of these vector deviations 11.4 kts. The algebraic mean of the difference between the scalar geostrophic wind and the actual wind speed was only 0.5 kts. and the root mean square of the difference was 8.3 kts.

The net-work of upper air observing stations over and near the British Isles is not sufficiently dense to enable the geostrophic wind to be measured with reasonable accuracy at the 700 and 500 mb levels. The direction of the contours can be obtained with fair confidence, however, when their exact spacing, in many

C.198126. /cases,

This report is not to be published and is to be treated CONFIDENTIALLY. Its contents must not be quoted

M.R.P. 392
S.C.11./4.
2nd April, 1948.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

The Angular Deviation of the Wind from the Isobars, at Liverpool

by

E.F. ARNOLD

Summary

An attempt has been made to measure the angle between wind direction and isobar at 3 heights, 3,000 ft, 700 mbs (10,000 ft approximately), and 500 mbs (18,000 ft approximately), at six hourly intervals, above Liverpool, during January and February, 1948. In many occasions the isobars (or contours of an isobaric surface) were not sufficiently well defined to permit of measurement, on others, wind observations were not available; in all the following number of measurements were made:-

3,000 ft	119
700 mb	167
500 mb	166

The isobars or contours were drawn, in each case, without reference to any wind observations.

The accuracy of determination of the direction of the wind and of the isobars was considered in each case, and though the exact accuracy is in some doubt, it is considered that many of the deviations of the wind direction from the isobars which were found, were significant. The means of these deviations were (reckoned positive across the isobar from high to low pressure):-

At 3,000 ft	7°
700 mb	1.5°
500 mb	0.1°

The deviations measured at 3,000 ft were probably not typical, for it seems likely that, even at 3,000 ft the mountains have an appreciable effect on the wind direction. Analysis of the deviations showed no correlation with the stability of the air.

An attempt was made to correlate the deviations at 700 and 500 mb with the general weather situation and development with some apparent success. It was found that, at both levels, there was a negative deviation on the boundary of an anticyclone, i.e. inflow to the anticyclone. Also, that at 500 mb and less markedly at 700 mbs, deviations were positive ahead of a trough in the upper air, to the top of the wedge, and negative behind the trough. On one occasion the observed deviations agreed with general expectations from significant features of the general situation.

The geostrophic wind was also measured at 3,000 ft, from the surface isobars, due allowance being made for height and excluding all cases of marked horizontal temperature gradients in the vicinity. The mean value of the moduli of the vector deviations of the wind from the geostrophic was found to be 9.1 kts, and the root mean square of these vector deviations 11.4 kts. The algebraic mean of the difference between the scalar geostrophic wind and the actual wind speed was only 0.5 kts, and the root mean square of the difference was 8.3 kts.

The net-work of upper air observing stations over and near the British Isles is not sufficiently dense to enable the geostrophic wind to be measured with reasonable accuracy at the 700 and 500 mb levels. The direction of the contours can be obtained with fair confidence, however, when their exact spacing, in many

/cases,

Ind 2t
AF 17/48

cases, is in some doubt. However, neglecting friction, if the actual wind is known and also the direction of the isobars, the geostrophic wind may be calculated when the vertical velocity of the air is small compared with the horizontal and when there is no violent vertical mixing due to thermal instability. The geostrophic wind at Liverpool has been calculated at 700 and 500 mb levels for each occasion with these assumptions. It was found that the mean value of the moduli of the vector deviations of the wind from the geostrophic was 10.0 kts at 700 mb and 12.8 kts at 500 mb. The calculated geostrophic wind at 500 mb was subject to large errors and it is doubtful if the statistics obtained for that height are significant.

1. Introduction

It is well known that the departure of the wind from the geostrophic, (i.e. that required to balance the pressure gradient), is important in the study of moving pressure systems and general dynamic development. (see for example, Sutcliffe 1). It is also important to the forecaster and to the aviator using pressure pattern navigational technique; both use the geostrophic wind as an approximation to the true wind and will require information as to the inaccuracies involved.

An attempt has been made, therefore, to measure the angular deviation of the true wind direction from the geostrophic over a period of two months, January and February, 1946, at Liverpool, at heights of 3,000 ft, 700 mb (approximately 10,000 ft), and 500 mb (approximately 18,000 ft). The speed of the geostrophic wind was also measured at 3,000 ft and compared with the observed wind. At 700 and 500 mb the geostrophic wind speed was deduced theoretically from the measured wind and the observed angular deviation of the wind from the isobars (contours).

2. Data Available

Wind observations made by radar methods using the GL III instrument were available at 6 hourly intervals at Pazakerly, near Liverpool, and as this is about the centre of the British Isles net-work of upper-air observing stations, it was chosen as the site for the investigation. Wind speed and direction to the nearest knot and degree respectively were used.

For the 3,000 ft level the surface isobars were taken as sufficiently close to those for 3,000 ft, all cases of marked temperature change in the horizontal in that area being excluded. (see Section 5(a) i below). About 32 observations of surface pressure within 200 miles of Liverpool were used for drawing the isobars in each case.

For the 700 and 500 mb levels the number of observations varied from 5 to 10 within a radius of 500 miles of Liverpool. The contours of these pressure surfaces were drawn without reference to any wind observations available, and as accurately as possible consistent with the observations. Due consideration was given to continuity both in time and space and where an observation did not fit in with the other observations adjacent to it in space and time, it was neglected. (In very few cases was this necessary). At several places, observations were made every 12 hours, only, and in these cases the means of consecutive readings were used in constructing charts for the intermediate hours. The observing stations and times of observations were:-

Pazakerly, Larkhill, Downham Market, Lerwick, (every 6 hours); Valentia, (00 and 12 hours); Mousehole and Stornoway, (06 and 18 hours); Aldergrove and Leuchars, (06, 12 and 18 hours); and occasionally De Bilt, Brussels, Trapoes, Brest, and, at greater distance, Copenhagen, stations in NW Germany and meteorological reconnaissance aircraft over the Atlantic.

136 17

3. Accuracy of measurement of the angular deviation of the wind from the isobars

The accuracy with which the angle, i , between the wind and the isobars (or contours), can be measured, depends on the accuracy of the wind observations and on the accuracy with which the isobars (or contours) can be drawn. These are considered below.

4. Accuracy of the wind observations.

The method of following a balloon target by radar is probably the most accurate means of measuring the upper winds, in all types of weather, used in this country. The balloon ascends at about 1200 ft per minute and observations of slant range, elevation and azimuth are made every minute and enable the path of the balloon to be plotted and hence the mean wind in each 1200 ft layer to be measured. In practice, the mean of a 3 minute run centred on the particular height in question is taken as the wind appropriate to that level and this is the value reported in code. Near the ground the direction and speed of the wind may be changing rapidly, however, and so the path of the balloon was replotted, up to about 5,000 ft and the best estimate made of the wind direction and speed at the 3,000 ft level.

At heights above 5,000 ft, wind speed and direction change more slowly than nearer the ground and the wind velocity averaged over a three minute interval probably differs from the spot wind at the centre of the layer by something less than the probable error of measurement. (See section (a) (ii) below).

Durst (2) has found that in clear air the wind velocity at a particular height, measured over the period of about one minute, fluctuates about the mean over a period of about one hour, with a standard deviation of the vector difference of between 1/2 and 3 1/2 knots. The wind velocity as measured by radar tracking of a balloon target may thus not be representative of the mean wind excluding the fluctuations over periods of the order of a minute. No account of this has been taken in the following comparison of the directions of the measured wind and the isobars. It will be seen, however, (section 7), that the order of magnitude of the angular deviations of the wind from the isobars found in the present investigation is much greater than that of the short period deviations of the wind from the mean as found by Durst.

The errors which may arise in the use of the GL.III radar vary with the quality of maintenance, the operators and also from set to set, so that it is impossible to assess, accurately, the probable errors of a wind determination. A recent experiment, (Harrison, 3), in which the same balloon target was followed by two radar sets simultaneously, indicates that an average radar set, in a fair state of efficiency and in the hands of experienced operators, should be able to measure slant range, elevation and azimuth to an accuracy such that the respective root mean square casual errors are of the order of 15 yds, 0.1° and 0.1°. (Systematic errors will also occur but are unimportant, usually, in measuring the wind). It is seen therefore that the wind direction may be measured with an accuracy of the order of that indicated in M.R.P. 340. (In that paper the root mean square errors of elevation and azimuth were taken as from 0.1 to 0.25°). The wind speed, for winds greater than 20 kts, can be measured with a root mean square error of the order of 1kt for all ranges covered. (See M.R.P. 340 for method of deduction). For lighter winds systematic errors in elevation may be important and may cause further errors which are impossible to estimate.

(a) Errors in wind measurement at 3,000 ft.

From M.R.P. 340, Table IV, it is seen that the mean wind direction over a layer 1200 ft thick, approximately, can be measured with a root mean square error of the order of 1°.

From the runs followed by balloons, however, it is known that the wind direction frequently changes rapidly with height in the lower layers over Liverpool and there must often be considerable error in estimating the wind direction at 3,000 ft from the tangent to the plotted run of the balloon track, and this error will be much more than that arising from purely casual errors of observation. An attempt has been made to estimate the error involved in the following manner.

36 18

From the plan positions of the balloon in a particular case it is known that the direction of the wind must vary say 20° over a height interval of 1200 ft which includes the level - 3,000 ft - at which the spot wind direction is required; drawing a tangent to a smooth curve between the two points 1200 ft apart in height, at the appropriate point, should give the wind direction at 3,000 ft with an error at most of 20°, and probably much less. If we assume that the chance of equalling or exceeding this 20° error is 1%, then, if the errors are normally distributed, the root mean square error in measuring the wind direction by drawing a tangent to the best looking smooth curve, should be approximately $\frac{1}{3} \times 20°$. This principle has been used to assess the accuracy of each wind direction measurement at 3,000 ft, the root mean square error (in degrees) being taken as 1/3 of the change in wind direction per 1,000 ft. This may be an underestimate of the accuracy, but is probably of the right order and certainly varies in the appropriate way. In most cases this error is greater than the instrumental error of measurement mentioned above and is taken as the main error.

The wind speed does not, in general, vary so much as the direction in the neighbourhood of 3,000 ft, and the average speed over a minute (and therefore over 1200 ft approximately), is probably accurate to 1 or 2 kts in most cases.

(b) Errors in wind measurement at 700 and 500 mb.

The errors in measuring the direction of the wind at 700 and 500 mb have been taken from Table IV of M.R.P. 340 for the heights 10,000 and 18,000 ft. It will be noted that the figures given in this Table are deduced on the assumption that the wind is averaged over one minute intervals; the wind was observed over 3 minute intervals however and the errors in observing this 3 minute average will be 1/3 of those shewn in the Table. An additional inaccuracy is sometimes introduced, however, by taking the 3 minute average wind as applicable to the spot height wind in the centre of the interval and so the original figures as given in Table IV of M.R.P. 340 have been used, though they will often err on the pessimistic side. These errors have root mean square errors of from 1 to 3°, approximately, at 700 mb and from 2 to 5°, approximately, at 500 mb, depending on the distance of the balloon target from the observing site.

5. Inaccuracies in the direction of isobars or contours

The direction of an isobar (or contour) at a particular place will be in error depending on:-

(α) the errors in the pressure readings, (height observations in the case of contours), from which the isobars are drawn.

(β) the distance between the isobars. (With a tight distribution of isobars their direction will be more accurately determinable than for a slack distribution, drawn from the same net of observation points).

(γ) the number of points at which the pressure, (height, in case of contours), is observed for the purpose of drawing isobars.

(δ) the distance between the observing points.

(ε) the curvature of the isobars when drawn since the direction at a particular point is much more accurately determined when the isobars are almost straight than when they are curved.

For a given net of observing stations, therefore, the error in direction of the isobar through a certain point will be a function of (α), (β), and (ε) only. (δ) will depend on the wind speed, roughly, since there is known to be a high correlation between wind and pressure gradient or distance between contours. (ε) is more a matter of drawing than anything else.

In the case of straight isobars then:-

θ_t = root mean square error in measuring the direction of the isobar

$$= f\left(\theta_p, \frac{1}{V}\right) \qquad \text{(1)}$$

where θ_p = root mean square error in measuring pressure or the height of the constant pressure surface.

V = wind speed.

Consider now a straight isobar given by 2 pressure observations (same value) at a distance d apart. Then if G is the horizontal distance between isobars, supposed known, the root mean square error in direction of this isobar, drawn from these two readings, alone, will be:-

$$\theta_t = \sqrt{2}\,\frac{G}{d}\,\theta_p \qquad \text{for isobars at 1mb interval}$$

$$= \frac{C}{\sqrt{d}}\,\theta_p \qquad \text{where C is a constant, since G varies as } 1/V \text{ approximately.}$$

If there are n + 1 observing stations, all of the order d apart and of the order of d from the main observing point No.1, then it is possible to construct $\frac{n(n-1)}{2}$ points, by interpolation, having the same pressure as at No.1 and hence to draw the isobar through point No.1 and this is precisely the operation which is carried out by eye in drawing an isobar. It is to be expected then, that the accuracy of the isobar so drawn will be enhanced about $\sqrt{\frac{n(n-1)}{2}}$ times from that drawn from the two observations only. Of course the distances between the points cannot be the same quantity d for each pair, nor can the $\frac{n(n-1)}{2}$ interpolated points be the same distance d from point No 1, but the argument serves to indicate that for a given net of observing stations the error in the direction of straight isobars will vary as θ_p/V.

For curved isobars the error in drawing the tangent at a point will be a function of 1/V (i.e. approximately as the distance between the isobars), and of the curvature σ_i of the isobar; for the purposes of this work the error was assumed to be a linear function of these two quantities. Thus in general, the mean square error in measuring the direction of an isobar was taken to be of the form:-

$$\theta_t^2 = \left(\frac{A\theta_p}{V}\right)^2 + \left(\frac{B\sigma_i}{V}\right)^2 \qquad \text{(2)}$$

where A and B are constants for the particular net of observing station.

/The

The problem of assessing values for A and B is discussed below, for the three heights considered.

(a) At 3,000 ft.

(1) Use of surface isobars.

If p_3 = atmospheric pressure at 3,000 ft

p_o = " " " " sea level.

then

$$p_3 = p_o - g\int_o^3 \rho \, dz$$ where z is height measured upwards.

$$= p_o - g\int_o^3 \frac{p}{RT} dz \qquad (3)$$

where R is the gas constant for air and T is the absolute temperature. The second term on the right of Eq (3) may be called the correction to be applied to p_o to give p_3 and this correction will thus vary with the horizontal changes in the pressure and temperature fields.

If the temperature field does not vary in the horizontal, ρ will be constant along a surface isobar and hence the isobars at 3,000 ft will be in the same direction as at the surface.

If p remains constant, then variations in ρ due to variations in T are given by:-

$$\frac{\delta\rho}{\rho} = -\frac{\delta T}{T}$$

The variation in ρ between two stations may be considerable if there is an active front between them. (Belasco, (4), quotes 26°F as a mean contrast between Siberian arctic and tropical maritime air, at 3,000 ft, which would give $\delta\rho/\rho = -0.06$ approximately, yielding a correction to p_o differing by about 7 mb between the two air masses). Thus it is not valid to assume that the isobars at 3,000 ft have the same direction as those at the surface if there is an active front in the vicinity.

Considering one air mass only, a horizontal variation of 5°F in the average temperature of the first 3,000 ft, in 200 miles will seldom be exceeded: then $\delta\rho/\rho = .01$, which yields, a correction to p_o which varies by less than 1mb in 200 miles. This correction will only vary the direction of the 3,000 ft isobars from the surface isobars if there is a temperature variation along the isobars, that is, roughly along the wind. In the absence of fronts the temperature variation along the wind is usually much less than 5°F per 200 miles. The effect on the isobars at 3,000 ft is sometimes noticeable, however, on occasions of light winds blowing inland after a night of radiative cooling over land.

It is seen, therefore, that the direction of the isobars at 3,000 ft may be taken from the surface isobars, to a high degree of accuracy, when the temperature variations in the first 3,000 ft are small along the wind.

(ii) Errors in observations of surface pressure.

Systematic errors are assumed detectable when drawing a series of surface pressure charts; thus, for example, the pressure at Holyhead was incompatible with that observed at Penrhos near by and due allowance was made for this on every chart.

Random errors in observing sea level pressure do occur, due to faulty instrument corrections, parallax error in reading the scale etc, and the probable error of such has been estimated (Ashford, (5)), to be 0.14mb in the British Isles.

(iii) Straight isobars.

With isobars corresponding to a 15 kt geostrophic wind this random pressure error, of probable error 0.14mb, would be expected to give a root mean square error of direction of straight isobars, drawn from only two observations 100 miles apart, of 4.5°. (See above at beginning of Section 5.) With the comparatively dense net-work of observations over the British Isles the error of direction of straight isobars must be less than this value, of course. For this work the value $\theta_t = 3°$ for a 15 kt wind, was adopted, (1.5° for a 30 kt wind etc), and this is thought to be a reasonable figure, erring if anything on the pessimistic side.

(iv) Curvature errors.

As seen above, Eq.(2) the error in assessing the direction of the isobar due to the curvature of the isobar may be assumed to be of the form $\theta = c_1/V$. With a curvature of 1/150 nautical miles, it is considered that, with isobars spaced for a 15 kt wind and assuming accurate pressure readings, it would be quite possible to draw the isobars in different ways so that directions varied as much as 6°, thus the root mean square error of direction due to this cause would be of the order of 3°. This would give c_1 a value of 675 and this value was used in the computations. For curvatures less than 1/300 nautical miles the curvature error will be unimportant.

(v) Criterion of accuracy used.

The root mean square error in direction of the isobar to be expected in each case was computed on the basis of (iii) and (iv) above, i.e. an error due to the errors in the basic data (mean level pressures), whose effect varied with the wind strength, and a further error, usually negligible, due to difficulties in drawing curved isobars and measuring the direction of the tangent to the same. (e.g., with a 30 kt wind the root mean square error due to the first was estimated to be 1.5°, due to the second, 0.7° for an isobar curvature of 1/600 nautical miles.)

(b) 700 and 500 mb

(i) Accuracy of measurement of height of a certain pressure level.

Sheppard (6) shows that the error in the height of a standard pressure level due to inaccuracies in the radio-sonde measurements and computations, consists of two parts:-

1. A casual error which has the following probable values:- *

700 mb	500 mb
± 14 ft	± 23 ft

2. A systematic error, of average value:-

700 mb	500 mb	
+ 30 ft	+ 70 ft	Maximum
+ 20 ft	+ 44 ft	Minimum (darkness)

* The figures given by Sheppard are for the 750 mb and 500 mb levels: the figures given here for 700 mb have been interpolated.

The systematic error is due to the lag of the radio-sonde and to radiation effects and will affect all the observations of the net-work used to draw the contours and so should not be important. It should be noted, however, that the Aldergrove observations were made by aircraft and so when comparing this observation with neighbouring radio-sonde observations, the systematic errors of the latter may be important. This applies to one of the observation points only in the British Isles; observations outside the British Isles were made by aircraft or radio-sondes usually of a different pattern from the British type, but being at a distance from Liverpool, their errors can have little effect on the estimation of the contour direction at Wakerly.

Thus the main errors in observing the heights of the 700 and 500 mb surfaces at the most important observation points around Liverpool, were the casual errors as given at (i) above.

(ii) Straight contours.

As seen above at Eq (2), with straight contours the error in direction of the contour should be given by:-

$$\theta_t = \frac{A\theta_p}{V}$$ where A is a constant.

As in (i) (b), if two observations (same value) are used to draw a straight isobar, then

$$\theta_t = \frac{\sqrt{2}}{200} \times \frac{G}{d} \theta_p$$

where the contours are drawn for intervals of 200 ft

G = Horizontal distance between contours, assumed known.

d = distance between the observing points.

θ_p = root mean square error of measuring the pressure height, in feet.

Since 25 mmh corresponds to a geostrophic wind associated with 200 ft contours 280 miles apart, this reduces to:-

$$\theta_t = \frac{\sqrt{2}}{200} \times \frac{25 \times 280}{V} \times \frac{\theta_p}{d}$$ Approximately

$$= 16.4 \frac{\theta_p}{V}$$ degrees, if θ_p is in feet

V in knots

d = 150 miles (the distance between Liverpool and its surrounding observation points approximately)

As there are 5 or more other observation points about Liverpool, the accuracy of the direction of the contours will be considerably better than this value, though as seen above, not by as much as $\sqrt{\frac{5 \times 4}{2}}$ times. Accordingly a compromise between A = 16.4 and $16.4/\sqrt{10}$ was adopted, and using the casual error figures quoted in 3.(b) (i) above, the root mean square errors in measuring the direction of the contours at Liverpool, when straight, were taken as

$$\theta_t = \frac{160}{V} \text{ degrees, at 700 mb}$$

$$= \frac{260}{V} \text{ " at 500 mb, where V is in knots.}$$

No doubt, as in the case of the surface isobars, these figures are rather arbitrary, but they are of the right order of magnitude and vary in the right manner.

(iii) Curvature errors.

With a curvature of 1/350 nautical miles and with the available net of observing stations over the British Isles and adjacent areas, it would be possible to draw the contours in two different ways apparently each quite consistent with the observations yet giving directions at Liverpool differing by as much as 6° , with a wind of 20 kts. (i.e. contours spaced for 20 kt geostrophic wind). The root mean square error in direction due to this difficulty in drawing curved contours is thus likely to be of the order of 3°. This assumption leads to a value of δ in Eq (2) above of 21×10^3 ; this value was used in the computations. It is seen that the curvature error is usually very much smaller than the other error in drawing the contours, discussed in (b) above, and in practice it was found easy to construct a table giving the small correction for curvature to be added to the main error.

(iv) Test of the accuracy attainable in drawing contours with comparatively few observations.

The above methods of estimating the errors involved in measuring the direction of the contours at Liverpool do not rule out the possibility of further errors due to faulty drawing of the contours, personal errors of the computor. Accordingly an experiment was devised and carried out to test the accuracy of drawing, alone. The procedure adopted was as follows.

Six charts were drawn, each with a different pattern of contours, as might be observed over the British Isles; the gradient, or distance between the contours, varied, of course, from chart to chart. The contours were then labelled with appropriate heights with 200 ft differences, and the heights at the observing stations Lerwick, Stornoway, Loughars, Aldergrove, Liverpool, Downham Market, Larkhill, Penzance, Hamburg and a position at 57.5°N 15°W (to represent a weather reconnaissance flight over the Atlantic), were then estimated, as accurately as possible from these contours. The figures so obtained were the basic data for the test. The computor was then given these height observations and he drew contour maps from them and measured the contour directions at Liverpool. The following day, the same observations, altered in absolute magnitude but retaining their relative values, were again given to the computor and again six directions were obtained. At intervals of one or more days the same test was carried out, each time with a set of numbers which looked unfamiliar but which should have resulted in the same pattern of contours. In all six tests were made on each of the six basic charts. The Table I below compares the results with those obtained by measuring the original charts.

/Table I

/36 24

Table I

Results of Chart - Drawing Test

	Chart A	Chart B	Chart C	Chart D	Chart E	Chart F
(1) Direction on basic chart	255.5°	247.5°	252°	5°	75°	257°
(2) Mean direction from 6 tests	257.6°	250.7°	251.0°	1.8°	74.2°	255.2°
(3) Difference (2) - (1)	+ 2.1°	+ 3.2°	- 1.0°	-3.2°	-0.8°	-1.8°
(4) Standard deviation of tests about (1)	2.8°	3.5°	1.8°	5.4°	1.6°	2.1°
(5) Standard deviation of the tests about their mean	2.6°	1.1°	1.5°	0.7°	1.1°	1.7°
(6) gradient of the contours	54 kts	56kts	26kts	46kts	71kts	25kts
(7) Radius of curvature of contours. (n. miles)	∞	300	700	600	600	∞

It is noticeable that there was a tendency to draw the contours in the same way each time (c.f. lines (4) & (5) of Table I); also that the curvature of the contours had no apparent effect on the accuracy, though of course with only 6 trials this was not conclusive. The distance between the contours also does not appear to influence the results and of course this is to be expected since it is only with inaccurate data, (as is found in practice), that this effect is noticeable and in these test cases the data were assumed completely accurate.

It does appear, however, that errors of 2 to 3° are likely from the drawing of the contours alone and this is somewhat higher than was estimated in (c) above.

(v) Criterion of accuracy used

The criterion of accuracy finally adopted was as described in 5.(b) (ii) & (iii) above. Though 5.(b) (iv) has shown that this does not fully take account of the pure drawing inaccuracies, it is thought that the errors under (ii) have been over estimated and that the final combination of (ii) and (iii) should give a rough idea of the errors. Certainly the errors must vary considerably from case to case, (with gradient and curvature), and a sliding scale is preferable to any fixed estimate of the errors. It is worth recording that there was very seldom any difficulty in drawing the contours, which is an indication of the accuracy of the observations.

Summarising, the root mean square error in direction of the contours in each case was taken as a combination of an error due to inaccuracies in the calculated pressure-heights, and a further error, usually negligible, due to difficulties in drawing curved contours and measuring the direction of the tangent to the same. (e.g. for a 30 kt wind, the root mean square error due to the first was estimated to be 5° at 700 mb and 9° at 500 mb and due to the second, 1° with contours having a radius of curvature of 700 nautical miles, the effect of the second on the total error being thus negligible.)

6. Error in measuring the angle between the true wind and the isobar (or contour)

The error in measuring the angle i between the true wind and the isobar (or contour) is of course compounded of the error in measuring the wind (Section 4. above) and the error in drawing the isobars (or contours) and measuring their

direction (section 5, above). In every case the root mean square of the total error was obtained by taking the square root of the sum of the mean square errors due to the two causes.

7. Observed angular deviations of the wind from the isobars

The measurements were made for all occasions in January and February, 1946, for which there were wind observations at Liverpool, (normally 00,06,12 and 18 hours), and for which the wind speed was greater than 12kts at 1,000 ft and roughly 16 kts at 700 and 500 mb. Statistics of the measurements are given below -

(a) At 3,000 ft.

There were 119 measurements in all.

(i) The mean deviation (reckoning flow across the isobars from high to low pressure as positive), was 7.1°

The mean of the estimated root mean square errors in making each of these 119 observations was 3.8° , and the estimated root mean square error of the mean deviation was 0.4°

It is seen that the mean deviation measured is almost certainly significant.

(ii) The mean absolute deviation (regardless of sign) was 10.0°.

(iii) The root mean square of the deviations was 13.9°.

(iv) Frequency of deviations.

The Table II below gives the frequency of occurence of deviations in various ranges, in the four quadrants and also for all directions. The total number of observations in each quadrant is too small to make accurate deductions, but it appears likely that the wind direction has a definite effect on the deviation of the wind across the isobars; whether this is due to the effects of the hills to the west, southwest and east of Liverpool, or whether to the correlation between wind and particular weather types, it is impossible to say for certain, but the former seems much more likely. (See section 8 (a)).

Table II

Frequency of deviation of the wind from the isobars at 3,000 ft

Quadrant	Ranges in degrees								
	< -8	-8 to -4	-4 to 0	0 to 4	4 to 8	> 8	Total no.	Mean deviations	Mean dev. regardless of sign.
E		1	1	1	3	5	11	14.5	16.2
S		1	3	0	7	18	29	12.4	12.9
W	4	6	12	11	13	22	68	5.1	8.4
N	3	2	0	1	2	3	11	2.4	10.2
Total (all directions)	7	10	16	13	25	48	119	7.1	10.0

(v) Significance of the deviations.

Large deviations were frequent and very many were greater than the estimated root mean square error of observation.

/Dividing

Dividing i ny θ_i the root mean square error of measuring i, is a means of reducing the significance of each deviation to the same standard. The Figure I below shows the proportion of occasions on which a particular value of $|i/\theta_i|$ was exceeded, and for comparison, the same curve for a normal error distribution. It will be seen that the two curves are not at all similar. For example, if the inaccuracy of measurement of i has been underestimated, (from sections 4 & 5 above it would appear to have been overestimated, if anything), and θ_i should be doubled in every case, the frequency curve for $|i/\theta_i|$ will still be quite unlike the normal frequency curve.

In fact the observed deviations are not normally distributed as would be expected if they were entirely due to random errors of observation.

It is deduced, therefore, that in many of the cases the deviations measured were significant and represented real differences and cannot be due to errors in observation alone.

(b) At 700 mb

The total number of observations at this level was 167 .

(i) The mean deviation (positive from high to low pressure), was

1.5°

The mean of the estimated root mean square errors in making each of the observations was 4.9°

The root mean square error in deriving the mean deviation was estimated as 0.4°

As the scale of the errors in making the measurements is in some doubt (see Section 3 above), it is doubtful if the mean deviation given above is really significant

(ii) The mean absolute deviation (regardless of sign) was 9.2°.

The estimated root mean square error of this function was, of course, 0.4°.

(iii) The root mean square of the deviations was 12.3°.

For comparison, the root mean square of all the estimated θ_i's (i.e. the root mean square of the root mean square errors) was 5.2°.

(iv) Frequency of deviations.

The Table III below gives the frequency of occurence of deviations in various ranges, for the four quadrants and also for all directions.

Table III

Frequency of deviation of the wind from the isobars at 700 mb

Quadrant	Ranges in degrees								
	< -8	-8 to -4	-4 to 0	0 to 4	4 to 8	> 8	Total no.	Mean deviation	Mean dev. regardless of sign
E	5	1	1	0	1		8	- 10.4	11.4
S		2	0	2	0	9	13	+ 11.1	12.8
W	15	10	11	24	16	33	109	+ 3.8	8.7
N	15	4	7	6	3	2	37	- 6.0	9.0
Total (all directions)	31	19	20	36	21	40	167	+ 1.5	9.2

Again there is a marked dissimilarity of frequencies in the four directions. This may be because of the small number of observations, so that the samples obtained were not representative; or it may be because certain directions were associated with particular weather types, e.g. all the East winds were associated with anticyclones. (See also section 8 below).

It was thought, too, that systematic errors in one or more of the observing stations might explain the differences. For instance, if height observations to the west of Liverpool were consistently too high, the contours would be consistently veered from the true direction with winds with a westerly component and consistently backed from the true direction with winds with an easterly component. To get a rough estimate of the systematic error required to explain, for example, the positive mean deviation of the westerly winds of 5.7°, the following method was adopted. The mean speed of the westerly winds (i.e. winds between SW and NW) was 42 kts, which corresponds with 200 ft contours 146 miles apart for geostrophic motion. (The geostrophic approximation is sufficiently accurate for this rough calculation). If, therefore, the systematic error which is assumed to cause the positive mean deviation, is in one observation 175 miles to the west of Liverpool, (say an interpolated value between those observed at Valentia and Aldergrove), then this height error δh will be at least that given by:-

$$\frac{\delta h}{200} \times \frac{146}{175} = 0.064 \qquad (0.064 \text{ radians} = 3.7^{\circ}).$$

which implies that Valentia, Aldergrove and Liverpool observations alone have been used in the derivation of the contour direction. This gives δh = 15 ft approximately and is a lower limit, for, of course, the other observations are used in drawing the contours and the effect of an error in one will be only partly reflected in the contours because of the smoothing process in drawing. Similar calculations show that to bring the mean deviations of easterly, southerly, and northerly winds to zero would require systematic errors of at least 42, 38 and 21 ft respectively, in appropriate observations.

The radio-sonde instruments used by the various observing stations all came from more or less the same source so that systematic errors were unlikely because of differences in the instruments. Calibration errors might introduce a systematic error into the height observed; this is unlikely in the pressure element as station pressure is read more accurately than to 1/2 mb; a temperature error of 1°F would be required to explain a systematic error in height of the 700 mb surface of 15 ft. (Sheppard, 6), and this is much larger than could occur without detection. It has been seen above (section 5(b) (i)) that radio-sonde observations of the height of the 700 mb surface have systematic errors of + 20 to + 30 ft. Thus the observations at Aldergrove, by aircraft, will tend to have a systematic error, compared with the other observations over the British Isles, of - 25 to - 30 ft; thiswould, of course, tend to decrease measured deviations of westerly winds from the contours, so that systematic errors from this source cannot explain the asymmetry of Table III.

It would appear, therefore, that much of the general character of the deviations shown in Table III is real. (See also section 8).

(v) Significance of the deviations.

As in 7 (a) v, the frequency of i/θ_i has been computed and Figure 2 shows the proportion of occasions on which a particular value of $|i/\theta_i|$ was exceeded.

Again it is obvious that a distortion of the curve by some appropriate factor cannot make it similar to the normal error curve, as there are too many large deviations. In fact, though the values of the errors of observation are known only approximately, there can be little doubt that many of the observations of deviations of the wind direction from the contours were significant.

(c) At 500 mb

The total number of observations at this height was 186.

(i) The mean deviation, (positive from high to low pressure) was + 0.1°. The mean of the estimated root mean square errors in observing each of the deviations was 7.4°, and the estimated root mean square error in deriving the mean deviation was 0.6°. The mean deviation found was thus not significant.

(ii) The mean absolute deviation (regardless of sign) was 9.8°. (The root mean square error in deriving this was again 0.6°.)

(iii) The root mean square of the deviations was 13.2°. For comparison, the root mean square of all the estimated θ_1's (i.e. of the root mean square errors) was 7.8°.

(iv) Frequency of deviations.

The Table IV below gives the frequencies of occurence of various deviations for the four quadrants and also for all directions.

Table IV

Frequency of deviations of the wind from the isobars at 500 mb

Quadrant	Ranges, in degrees <-8	-8 to -4	-4 to 0	0 to 4	4 to 8	>8	Total No.	Mean deviation	Mean dev. regardless of sign.
E	8	3	1	0	0	0	12	- 15.3	15.3
S	1	0	2	2	2	10	17	+ 14.2	15.3
W	15	5	23	17	17	32	109	+ 2.6	8.3
N	18	6	7	10	3	4	48	- 6.6	10.0
Total (all directions)	42	14	33	29	22	46	186	+ 0.1	9.8

As at 700 mb the frequencies are very dissimilar with regard to direction, and the same broad characteristics are apparent at both 700 and 500 mb levels. For the same reasons as in (b) iv above, it is thought that these characteristics are not due to systematic errors but represent real tendencies. For instance the mean speed of the westerly winds at 500 mb was 52 kts and a systematic error in the mean of Valentia and Aldergrove height observations of at least 13 ft (positive) would be required to reduce the mean deviation of the westerlies to zero; this would require a calibration temperature error of about 0.3°F, (Sheppard, 6), which is not only unlikely but also inconsistent with the same quantity calculated for 700 mb, (1°F). The systematic errors required to reduce the mean deviations of the winds in the other directions to zero would be, at least, for East winds 47 ft, for South winds 60 ft, for North winds 30 ft, for distances of 175 miles between observing stations. (See also section 5).

(v) Significance of the deviations.

As in (a) v and (b) v, the frequency of $|\delta/\theta_1|$ has been computed and Figure 3 shows the proportion of occasions on which a particular value of $|\delta/\theta_1|$ was exceeded. As at 700 mb, it is obvious that a distortion of the curve by some appropriate factor cannot make it similar to the normal error curve as there are too many large deviations. In fact, though there is considerable doubt as to the exact magnitude of the errors of observation, there can be little doubt that many of the observations of the deviations of the wind from the isobars were significant.

836 29

6. Correlation of angular deviation of the wind from the isobars with the general weather situation.

(a) At 3,000 ft.

It was quite impossible to find any relation between the angular deviations of the wind from the isobars and the general weather situation e.g. the stability of the air in the first 3,000 to 4,000 ft appeared to have no relation with the angular deviation and as large values (positive or negative) were observed on occasions of extreme stability (anticyclonic) as in very unstable conditions. As suggested in section 7 (a) iv, it seems probable that the surrounding hills are the main influence directing the small deviations of the wind from the isobars.

(b) At 700 and 500 mb

(i) Anticyclonic conditions.

There were 3 periods during the two months when Liverpool was on the edge of a large anticyclone and was contracted by the main frontal systems of the disturbances moving round it. The Table V summarises the main features.

Table V

Angular deviations of the wind from the Isobars - Anticyclonic Conditions

Level	Total no. of observations.	No. of positive deviations	Mean of positive devs.	No of negative devs.	Mean of negative devs.	Mean of all devs.	Wind direction
700 mb	32	9	5.8°	23	-12.6°	- 7.5°	NW to N
500 mb	37	6	4.5°	31	-12.8°	- 10.2°	24 NW to N 13 N to E

It should be noted that both at 700 and 500 mb the wind was mainly between NW and N and, as seen in Tables III and IV, irrespective of the weather situation the deviations were much more frequently negative than positive with northerly winds. In the 700 mb case the mean deviation in anticyclonic cases was less than for all occasions, and thus, even if there is some systematic factor tending to make northerly deviations negative, (thought to be unlikely, see section 7 (b) v), it is still possible to say that there is a definite tendency for negative deviations in anticyclonic conditions at 700 mb.

The same argument may be applied to the observations at 500 mb; the 13 occasions of wind N to E included all the easterly winds observed and noted in Table IV. The mean of the other deviations was -7.4°, and is seen to be less than the mean of all occasions with northerly winds. Hence, even if some systematic error tends to make northerly and easterly deviations negative, anticyclonic conditions still appear to be correlated with negative deviations at 500 mb.

(ii) Travelling disturbances with associated sinusoidal isobars at upper levels

Associated with travelling depressions there are usually marked ridges (associated with the warm air) and troughs (associated with the cold air) in the upper air which move in the general direction of motion of the depressions. From a scrutiny of the angular deviations from the isobars at 500 mb it soon became apparent that these deviations had a rough pattern so that ahead of the trough up to the top of the ridge the deviations were positive, (i.e. across the isobars from high to low pressure) and behind the trough the deviations were negative.

/The

The Table VI below gives statistics for Liverpool of all cases of cyclonic activity with moving sinusoidal patterns associated with them aloft, in the period January and February, 1946. Deviations were labelled significant if they exceeded, in absolute value, the estimated root mean square error of observation.

Table VI

Deviations of the wind from the isobars associated with moving sinusoidal contour patterns.

Level	Significant deviations		Non significant deviations	
	No. agreeing with model	No. disagreeing with model	No. agreeing with model	No. disagreeing with model
700 mb	40	20	13	12
500 mb	41	7	26	12

It was not possible to be certain of the classification in all cases for it sometimes happened that the dominant movement of the patterns changed suddenly; for instance on 7 and 8th February the general motion was from West to East but on 9th an anticyclone built up to west of Ireland behind a southward moving cold front and thereafter the general motion was NW to SE. Some of the failures of the "model" may have been due to this reason but they cannot have been many. In one case (2nd February) a series of positive deviations at 700 and 500 mb which did not fit the model pictured above, was probably associated with the rapid filling of an old depression over North Scotland, when the observed rise in the contours (with time) could not have been due to advection alone and subsidence was therefore inferred with horizontal inflow at upper levels.

A very striking feature of the model was its frequent success in explaining a deviation of odd sign among a series of deviations of opposite sign, and at 500 mb, with well marked ridges and troughs moving steadily across the map at that level, the rule was almost infallible.

It should be pointed out that, as the general direction of motion of pressure systems across the British Isles was West to East, the preponderance of negative deviations with a northerly and positive with a southerly wind, (see Tables III and IV), is explained by this model. Conversely, of course, if systematic errors are responsible for the asymmetrical distribution of deviations with respect to direction, the model has little significance. There were several occasions, however, when the general motion was from SW to NE and the wind never veered beyond West so that they support the model irrespective of any other explanation.

It is possible, of course, to justify the positive and negative deviations in a rough sort of way after the manner of J. Bjerknes (7). For equidistant sinusoidal isobars, if the wind approximates to the gradient wind, an air particle will be subject to acceleration forward of the trough and deceleration behind the trough, if the speed of translation of the system is less than the speed of the wind, as is normally the case. From the equations of motion, therefore, in these circumstances, positive deviations are to be expected forward of, and negative deviations behind the trough.

Alternatively, forward of a trough to the top of a ridge in the upper air is usually a region of ascent as is shewn by the rainfall at the cold and warm fronts and often in the warm sector as well. Since the wind normally increases with height this will mean that a particle of air will, because of its upward motion, be subjected to an acceleration and will therefore have an ageostrophic motion towards low pressure. (Durst, Sutcliffe 8). Behind the trough there is often subsidence in the upper air and the reverse will occur. During January and February, 1946, the wind did increase with height at both the 700 and 500 mb, levels, in the great majority of cases.

9. Comparison of observed wind and geostrophic wind at 3,000 ft

(a) Measurement of geostrophic wind speed.

The surface isobars were drawn on charts of scale 1 : 5×10^6 and the pressure gradient was measured over a distance of 50 to 100 miles, with the usual type of scale, care being taken, when the gradient appeared to vary, to keep Liverpool in the centre of the range. The value so obtained had, of course, to be corrected to apply to 3,000 ft, but it was found that this correction combined with the latitude correction, (the scale being accurate for latitude 55°), was negligible, (2%), compared with other errors of drawing etc.

J, the geostrophic wind, so measured, should be as accurate as the scale permits of reading, and was estimated to be within ±1 kt for winds of 20 kts, ± 2 kts at 40 kts and so on. It is, of course, debateable whether the mean pressure gradient over 50 to 100 miles is an accurate measure of the pressure gradient at the centre of the range, but the J so measured is certainly that used by the forecaster or airman and so the following comparison may be of some use, even if a true measure of the geostrophic wind has not been achieved.

(b) Statistics of the vector $\underline{V} - \underline{J}$

(i) Error in measuring $\underline{V} - \underline{J}$

The error in the scalar magnitude of $\underline{V} - \underline{J}$ is difficult to assess as it depends on i as well as the magnitudes of $\underline{V}$ and $\underline{J}$.

For a given value of V and i, the maximum error in $\underline{V} - \underline{J}$ due to errors in i alone will occur when J = V sec i and will be of magnitude $V\delta i$

where δi is the error experienced in measuring i.

The greatest error in $\underline{V} - \underline{J}$ due to errors in V and J will be when i = 0.

Thus the variance (mean square error) in measuring $\underline{V} - \underline{J}$ will be less than

$$J^2\theta_i^2 \quad + \text{ mean square error in } (V - J)$$

An idea of the order of magnitude of the error can be obtained in this way. The mean J over the period was 34 kts, and the mean θ_i was 3.8° the root mean square error in V − J may be taken as 4 kts, though it may well be smaller. The variance in measuring $\underline{V} - \underline{J}$ in the mean case will thus be less than:-

$(34 \times .07)^2 + (4.0)^2$ kts

i.e. less than $(5 \text{ kts})^2$.

(ii) The mean of $|\underline{V} - \underline{J}|$ was 9.1 kts

(iii) The mean square of $\underline{V} - \underline{J}$ was $(11.9 \text{ kts})^2$

(c) Statistics of the scalar J - V

(i) The mean of J - V over the 119 observations was 0.4 kts. From (a) above and the known accuracy of measurement of wind speed (section 4), each determination of J - V should have a root mean square error of the order of 3 to 4 kts at most and the root mean square error in the mean of J - V, being $1/\sqrt{119}$ of this, will be up to 0.3 to 0.4 kts. The mean of J - V found above may not therefore be significant.

(ii) The mean absolute value of J - V, regardless of sign, was 6.0 kts.

(iii) The root mean square of J - V was 8.3 kts.

/10.

10. Calculation of the geostrophic wind at 700 and 500 mb.

It can be shewn (Bannon, 9) that as a good first approximation the general equations of motion may be written:-

$$\left.\begin{aligned} \frac{du}{dt} - lv &= -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ \frac{dv}{dt} + lu &= -\frac{1}{\rho}\frac{\partial p}{\partial y} \\ 0 &= -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \end{aligned}\right\} \qquad \text{(4)}$$

with the usual notation and where $l = 2\omega \sin\phi$, the Coriolis factor.

These equations are only valid if there are no violent convectional upcurrents but should be a reasonable approximation for other occasions, above 3,000 ft, including those with vertical velocities over a wide area of the order of 10 cm/sec, as are found in active depressions. The vertical velocity, w, is implicit in du/dt and dv/dt, of course, and is small compared with u and v.

The first two equations of Eqn (4) above may be written:-

$$\frac{d\underline{V}_h}{dt} = -l\,\underline{k} \times (\underline{V} - \underline{J}) \qquad \text{(5)}$$

where $\underline{V}_h$ = horizontal component of $\underline{V}$, the velocity vector.

$\underline{k}$ = vertical unit vector

$\underline{J}$ = geostrophic wind.

If σ_h = curvature of the horizontal projection of the trajectory, (positive for cyclonic motion), then:-

$\sigma_h V_h^2$ = acceleration in the horizontal, at right angles to $\underline{V}_h$

$= -l(V_h - J\cos i)$ from Eqn (5)

Hence $V_h = J\cos i - \frac{\sigma_h}{l} V_h^2$

or $J = V_h \sec i\,(1 + \sigma_h V_h/l)$ \qquad (6)

(Petterssen (10) derives a similar equation, by a different method, for purely horizontal motion; here, of course, the motion is near horizontal).

When V_h, i and σ_h are known, therefore, it should be possible to calculate J, and this Eqn (6) has been used in this way to calculate the geostrophic wind at 700 and 500 mb. levels.

(i) Calculation of σ_h the horizontal curvature of the air trajectory.

Blaton's formula

136 33

$$\frac{\partial\psi}{\partial t} = V(\sigma - \sigma_s) \qquad \text{(7)}$$

C.198126. (see for example, Petterssen, (11)). /has

has been used to calculate σ_n. In this equation (7) the vertical component is neglected but this will be a good approximation for any motion in which w is small compared with V.

ψ = angle between $\underline{V}$ and a fixed direction

σ = curvature of the trajectory of an air particle.

σ_s = curvature of the stream line.

Hereafter σ will be used for σ_n.

$\partial\psi/\partial t$ can be calculated from successive observations.

V is measured.

σ_s is more difficult to assess; the following method (due to F.A.Berson, unpublished) has been used to derive it:-

$$\sigma_s = \text{curl}\left(\frac{\underline{V}}{|\underline{V}|}\right)$$ i.e. the curl of unit vector in direction $\underline{V}$.

From Stokes' theorem, the average of σ_s over a triangle will be

$$\int \frac{\underline{V}}{|\underline{V}|} \cdot d\underline{s} \Big/ \text{area of the triangle.}$$

where the $\int$ is taken round the triangle.

If the triangle is taken as that which has the observing stations Larkhill, Downham Market and Fasnakerly as the mid-points of its sides, then the mean of σ_s over the triangle is given by:-

$$\overline{\sigma_s} = \sum d \cos\theta \Big/ \text{area of triangle} \qquad (8)$$

where d is the length of one of the sides and θ the angle the wind makes with it, and the $\sum$ indicates the algebraic sum of such terms for all three sides. This method was used to estimate the curvature of the streamline at Liverpool, care being taken to exclude all cases where there appeared to be a marked change in the general flow pattern in the horizontal which would make the approximation invalid.

(ii) Errors in $\overline{\sigma_s}$

From equation (8) above, the error in measuring $\overline{\sigma_s}$ will depend on the errors in measuring the direction of the wind at the three observing stations of the triangle. It has been seen, (section 4 (b)), that the wind direction at Liverpool had a root mean square error on measurement of 1 to 3° at 700 mb

and 2 to 5° at 500 mb, approximately. The errors of the Larkhill and Downham Market observations were probably larger. However, as D/F methods were used to determine the path of the balloon; Sheppard (6) estimates probable vector errors of about 2 kts at 700 mb and 3 kts at 500 mbs, which with winds greater than 16 kts would mean that the probable errors in direction would be considerably less than 6° and 10° respectively, (taking the vector error always in the most unfavourable direction gives these figures). For all directions of the vector error, the probable errors of direction will be more like 3° and 5° respectively, corresponding to root mean square errors of 4.5° and 7.5° respectively.

The triangle of observing stations is approximately equilateral, of side 125 nautical miles, and this being so, the mean square error in σ_s is given by:-

$$\theta_{\sigma_s}^2 = \frac{d^2}{(\text{area of triangle})^2} \sum \sin^2\alpha\, \theta_\alpha^2$$

where θ_α = root mean square error in measuring α. If we can take all the θ_α's as 0.07 (i.e. 4.5°) at 700 mb and 0.131 (i.e. 7.5°) at 500 mb, this will give an upper bound to θ_{σ_s}. $\sum \sin^2\alpha$ will in general differ little from 3/2, the value for no curvature, since curvatures are small, and hence

$$\theta_{\sigma_s}^2 < \frac{16}{3d^2} \times \frac{3}{2} \times (.078)^2 \quad \text{for 700 mb}$$

$$< \frac{16}{3d^2} \times \frac{3}{2} \times (.131)^2 \quad \text{for 500 mb}$$

i.e. $\theta_{\sigma_s} < 1/600$ (nautical miles)$^{-1}$ at 700 mb

$< 1/400$ " " at 500 mb

As might be expected, the errors in measuring σ_s are comparable with usual values of σ_s.

(iii) Errors in σ

$$\sigma = \sigma_s + \frac{1}{V}\frac{\partial \psi}{\partial t} \quad \text{from Eq (7).}$$

$\frac{1}{V}\frac{\partial \psi}{\partial t}$ is usually very much smaller than σ_s and the main error in σ arises from the error in measuring σ_s.

11. Comparison of calculated geostrophic wind and actual wind at 700 mb.

It was possible to estimate the geostrophic wind at 700 mb on 161 occasions; on the remaining 6 occasions it was impossible to assess the curvature of the trajectory. On 20 of the occasions the air was thermally unstable and the presence of showers and large cumulus clouds indicated that there may have been considerable vertical exchange of air between levels above and below 700 mb and the steps by which the geostrophic wind was deduced may therefore not be valid.

(4) The vector $\underline{V} - \underline{J}$

The magnitude, D, of the vector $\underline{V} - \underline{J}$ is given by

$$D^2 = V^2 \left\{ \left(\sec i \left(1 + 2.4\, \sigma V \right) - \cos i \right)^2 + \sin^2 i \right\}$$

From Eq (6)

if $V \cos i - J = V_1$

we may write the variation in D due to inaccuracies in V, i and σ thus:-

$$D\,\delta D = \delta V \frac{D^2}{V} - 2.4\,\sigma \sec i\, VV_1 \delta V$$

$$- \delta i\, VV_1 \left(\sec i \tan i \left(1 + 2.4\sigma V\right) + \sin i \right) + V^2 \sin i \cos i\, \delta i$$

$$- 2.4\, V^2 \sec i\, V_1 \delta\sigma$$

$$\delta D = \delta V \left(\frac{D}{V} - 2.4\,\sigma \frac{V_1}{D} V \sec i \right) + \delta i \left(\frac{V^2}{D} \sin i \cos i - \frac{V}{D} \left(J \tan i + V \sin i \right) \right) - 2.4\, V^2 \sec i \frac{V_1}{D} \delta\sigma$$

D/V is usually small.

$\frac{V \sin i}{D}$ and $\frac{V_1}{D}$ are always less than or equal to 1.

i was usually small and in almost all cases was less than 30°.

δV is of the order 1 kt, δi of the order 5°, $\delta\sigma$ of the order $\frac{1}{600}$ (nautical miles)$^{-1}$.

It is seen, therefore, that the principal error in D comes from the 3rd term above. For large V the error in D may be considerable. In considering D, the function $2.4\, V^2 \delta_\sigma$ has been taken as representing its root mean square error: it may over-estimate the error in many cases but will seldom if ever under-estimate it.

(i) The mean of $|\underline{V} - \underline{J}|$ for the 141 cases was 9.1 kts.

If the 20 doubtful cases were included, the mean was 10.0 kts.

(ii) The mean square of $\underline{V} - \underline{J}$ was (11.6 kts)^2 for the 141 cases.

Including the 20 doubtful cases, the figure was (13.7 kts)^2.

(iii) The mean variance (mean square error) in measuring $\underline{V} - \underline{J}$, estimated as described above, was less than (6.6 kts)^2 including the doubtful cases, (7.1 kts)^2.

(b) The scalar J - V

As in (a) above, it is apparent that the main error in J - V is that due to inaccuracies in measuring σ, and the error in J - V is roughly the same as in $|\underline{J} - \underline{V}|$

(i) The mean of J - V over the 141 cases was - 0.5 kts. The 20 doubtful cases had a mean of 7.7 kts and the mean of all cases including these, was +.6 kts.

The root mean square error of the mean was roughly 1/12 of that indicated in (iv) below so that the observed mean is probably not significant.

(ii) The mean of J - V regardless of sign was 6.0 kts; including the doubtful cases, it was 6.5 kts.

(iii) The root mean square of J - V was 9.1 kts; 10.9 kts including the doubtful cases.

(iv) As in (a) iii, the root mean square error in measuring J - V was less than 6.6 kts.

12. Comparison of calculated geostrophic wind and actual wind at 500 mb.

The geostrophic wind was estimated in 175 cases; on the remaining 11 occasions it was impossible to assess the curvature of the trajectory. On 14 occasions the air was thermally unstable and the calculated geostrophic winds may have been in error.

At this level the winds were usually stronger than at 700 mb and inaccuracies in measuring the curvature of the air trajectories greater. The accuracy of measuring the geostrophic wind was, accordingly, low and it is doubtful if the following statistics are significant.

(a) The vector $\underline{V} - \underline{J}$

(i) The mean of $|\underline{V} - \underline{J}|$ for the 161 valid cases was 12.3 kts.

Including the doubtful cases the mean was 12.8 kts.

(ii) The mean square of $\underline{V} - \underline{J}$ was (14.5 kts)^2; including the doubtful cases, the figure was (15.4)^2.

(iii) In the same way as in section 8 (a) iii, the mean variance of the errors in measuring $\underline{V} - \underline{J}$ was less than (16.6 kt)^2 ; if doubtful cases were included, the mean was (17.2 kts)^2.

(b) The scalar J - V

(i) The mean of J - V was - 0.8 kts for the 161 cases; including the doubtful cases, the mean was + 1.2 kts. As at 700 mb, these figures are probably not significant. (See iv below).

/(ii)

(iii) The mean of J - V regardless of sign was 8.2 kts; 8.4 kts including the doubtful cases.

(iii) The root mean square of J - V was 11.0 kts; including the doubtful cases, 11.4 kts.

(iv) As in (iii) the mean standard deviation of the errors in J - V was less than 16.6 kts.

13. Conclusions

The main results of the measurements may be summarised as follows:-

(a) At 3,000 ft, at Liverpool, the wind had a mean angular deviation from the isobars of about 7° from high to low pressure. This may have been due to local peculiarities.

No relation was discovered between the angular deviation of the wind from the isobars and the weather situation.

(b) At 700 and 500 mb the mean angular deviation of the wind from the isobars was small. The mean deviation regardless of sign was of the order of 9°, and it is of interest to note that this differs little from the same figure for 3,000 ft.

The deviations on the edge of anticyclones appeared to be significantly negative i.e. wind blowing in towards high pressure.

There was also an apparent systematic deviation across the contours associated with travelling troughs and wedges at these upper levels, positive ahead of a trough and negative behind a trough. These relations with the weather may explain the dissimilarity of the deviation frequencies with regard to direction, but alternatively if there is some systematic error which causes this dissimilarity, this would largely explain the observed correlations of the deviations with the weather: the former appears the more likely, however.

(c) The mean absolute value of the vector deviation of the wind from geostrophic was about 9 kts at 3,000 ft and 700 mb. At 500 mb the value was about 12 kts but the determination was much less accurate than at the lower levels and may not have been significant.

(d) At the 700 and 500 mb levels the inaccuracies in measuring the angle between wind and contour are probably too great to enable this angle to be investigated other than statistically and except when the angle i is obviously large it would be unwise for forecasters to make deductions from an apparent divergence of wind and isobar without other evidence.

References

1.	R.C. Sutcliffe	Q.J. Roy. Met. Soc., (1938), 64, No.276, p.495.
2.	C.S. Durst.	M.O. , 558, The Fine Structure of Wind in the Free Air.
3.	D.N. Harrison	A.R.P. 387, The Errors of GL III Observations.
4.	J.E. Bellasco	S.D.T.M. 89, The Temperature Characteristics of Different Classes of Air over the British Isles.
5.	O.M. Ashford and the staff of M.O.4.,	unpublished.
6.	[illegible]	C.D.T.M. 7 , ... VII. On the Accuracy of M.O. Radio-Sonde Temperature, Height and Wind Observations.
7.	J. Bjerknes and J. Holmboe	American Met. Soc., Journal of Met., Vol 1, No.1, 1944.
8.	C.S. Durst and R.C. Sutcliffe	Q.J. Roy. Met Soc., (1938), 64, No.273, p.82.
9.	J.K. Eawson	M.R.P. 453, A Note on the Insufficiency of the Gradient Wind Approximation in Disturbed Conditions. (Appendix).
10.	S.Petterssen	S.D.T.M. 78, Computation of Winds in the Free Atmosphere.
11.	S.Petterssen	Weather Analysis and Forecasting, 1940, p.224.

M.R.P. 392.

Figure 1.

Frequency that $\left|\frac{\varepsilon}{\theta_c}\right|$ exceeds certain values. — 3,000 ft.

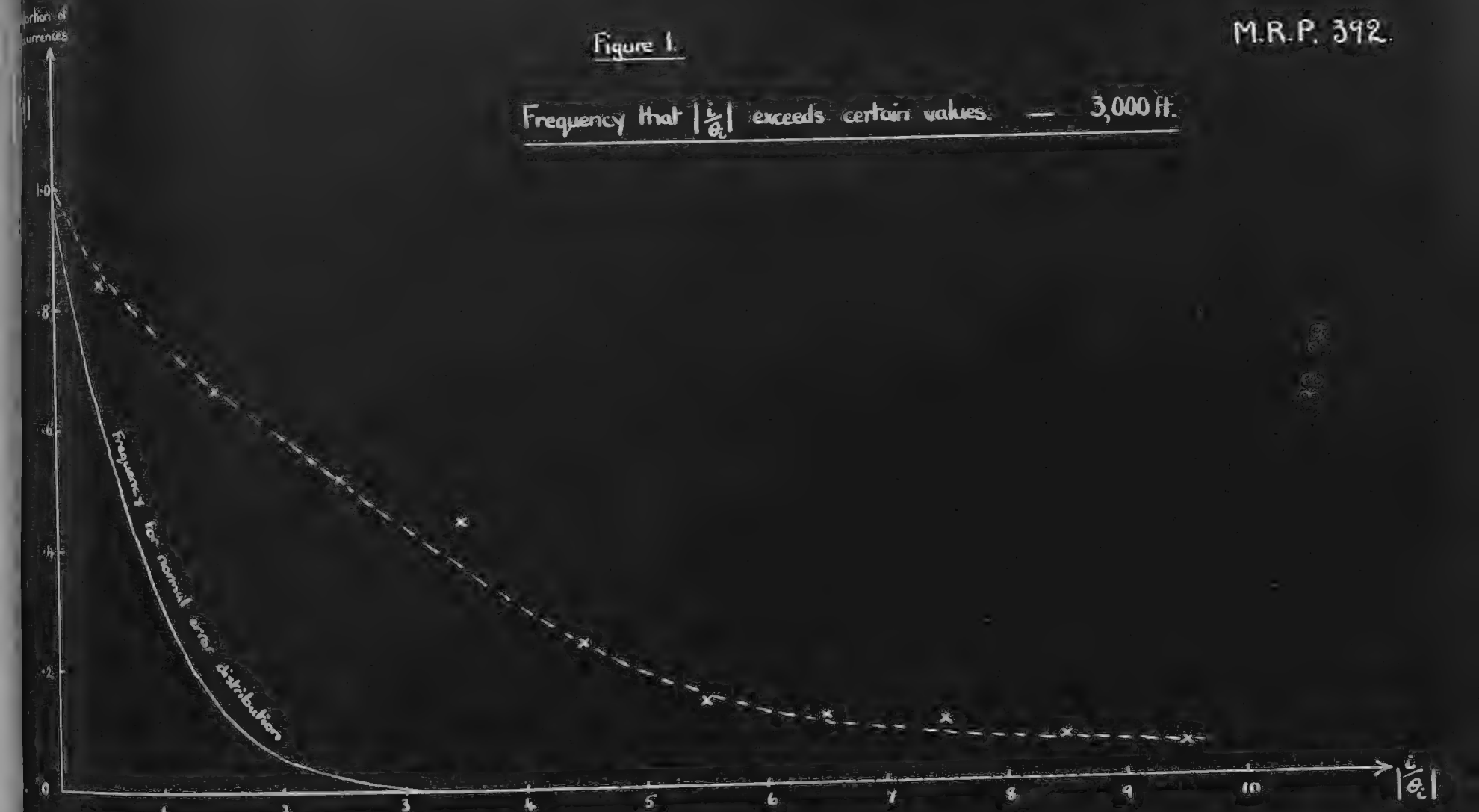


Figure 2.

M.R.P. 392.

Frequency that $\left|\frac{i}{\alpha}\right|$ exceeds certain values. — 700 mb.

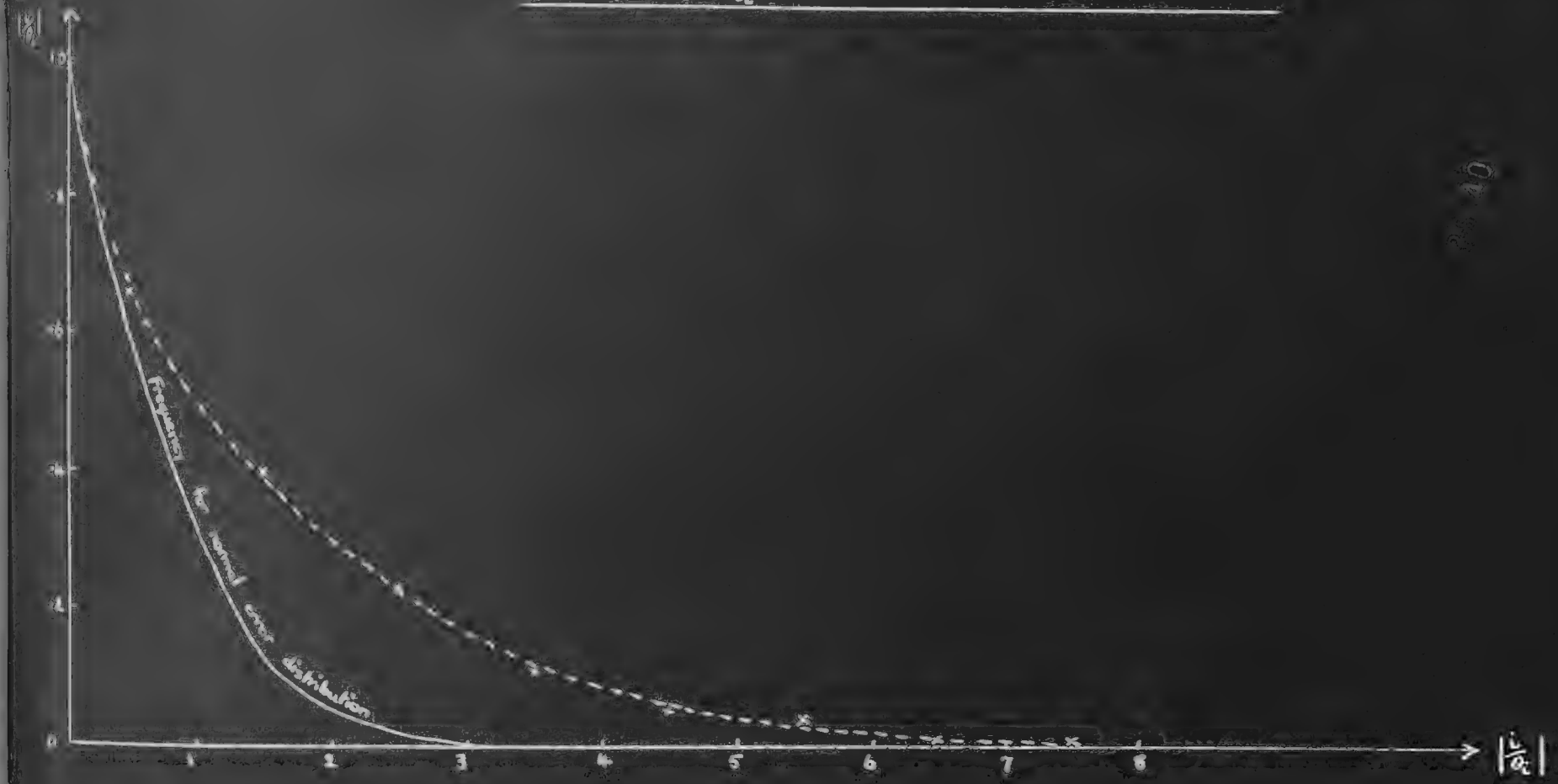
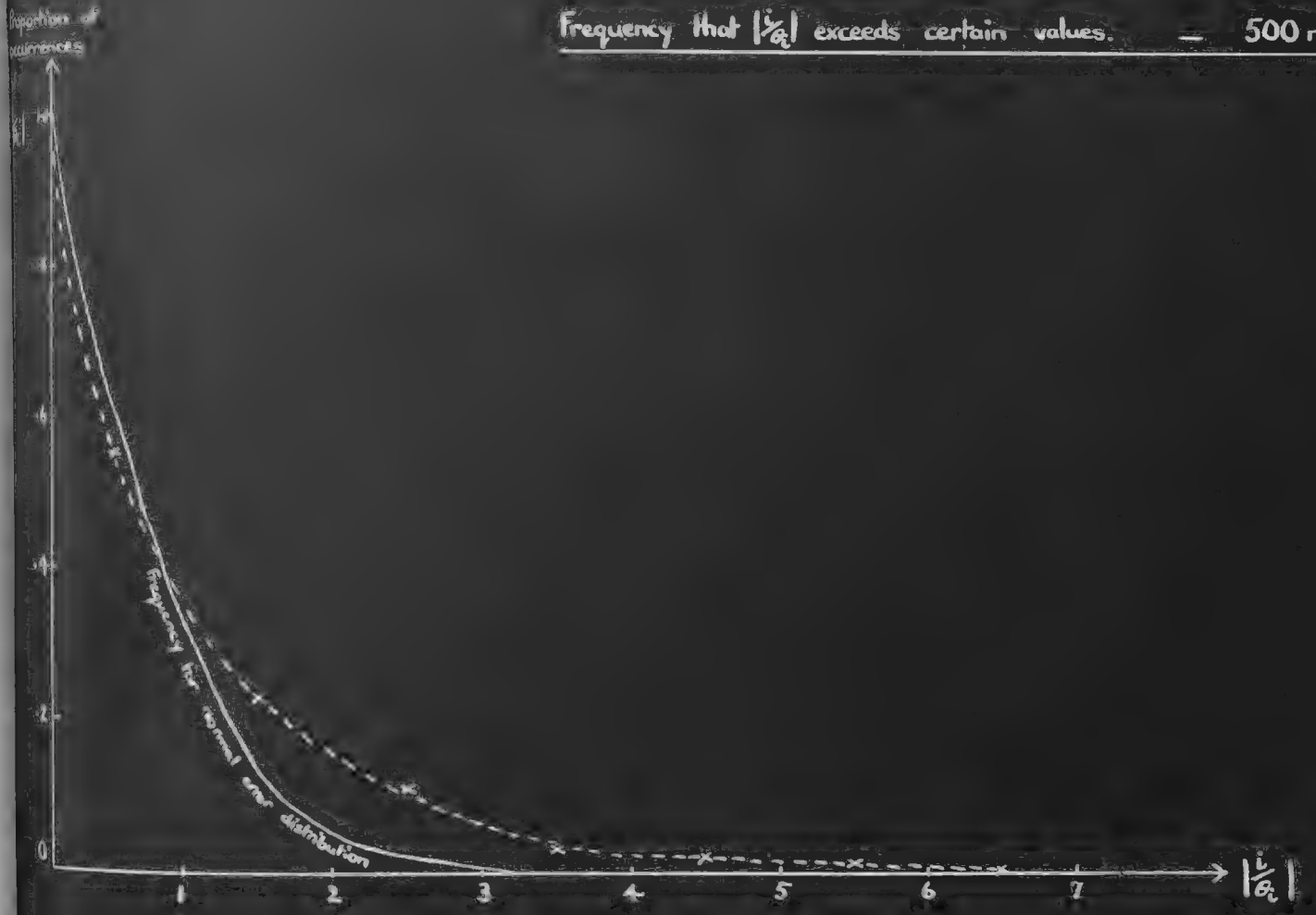


Figure 3.

M.R.P. 392.

Frequency that $|\frac{\Delta}{\sigma_t}|$ exceeds certain values. — 500 mb.



(UNPUBLISHED CARD) UNCLASSIFIED

ATI 101 992 (COPIES OBTAINABLE FROM CADO)

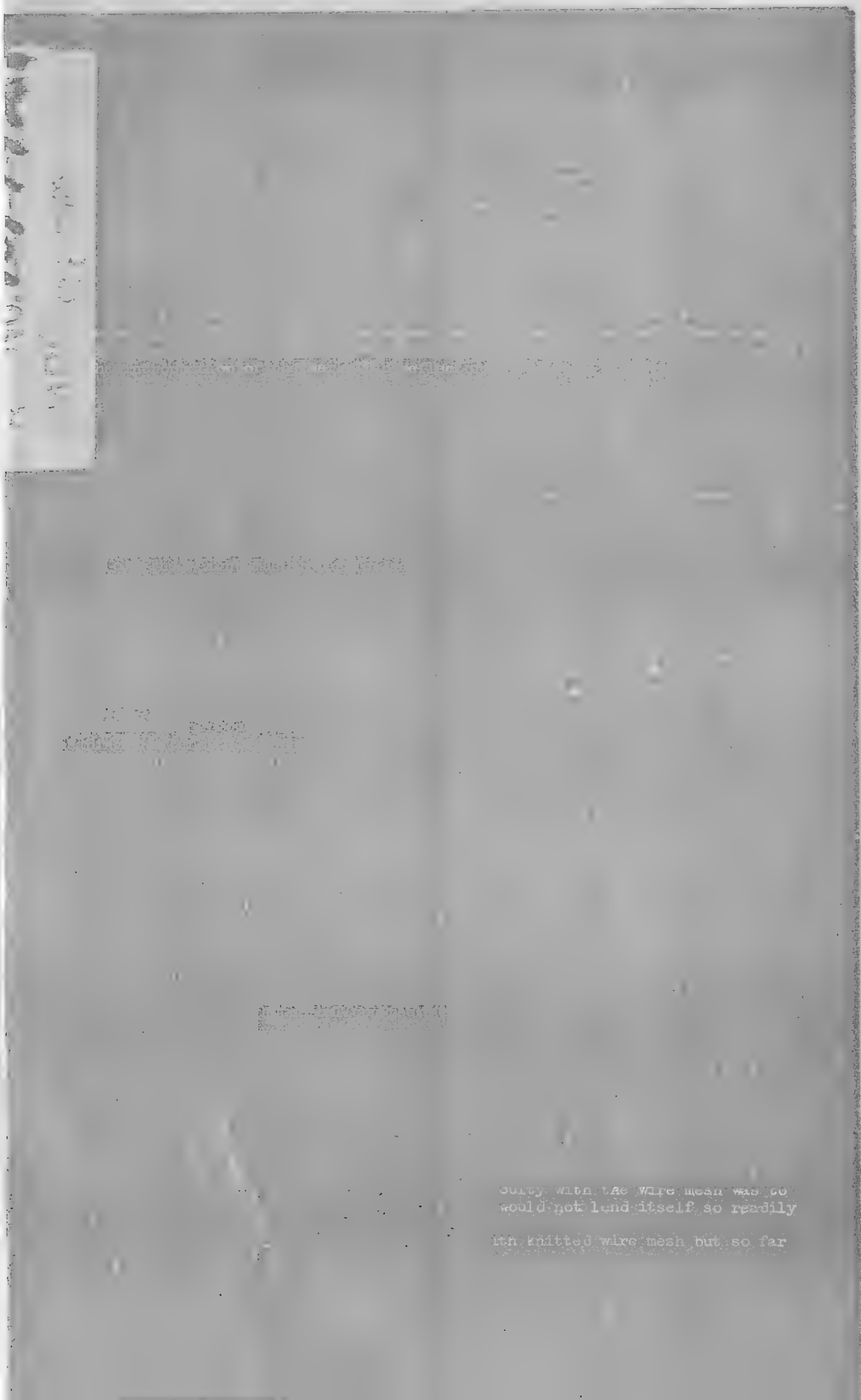
METEOROLOGICAL RESEARCH COMMITTEE, AIR MINISTRY, ENGLAND
(M.R.P. 398)

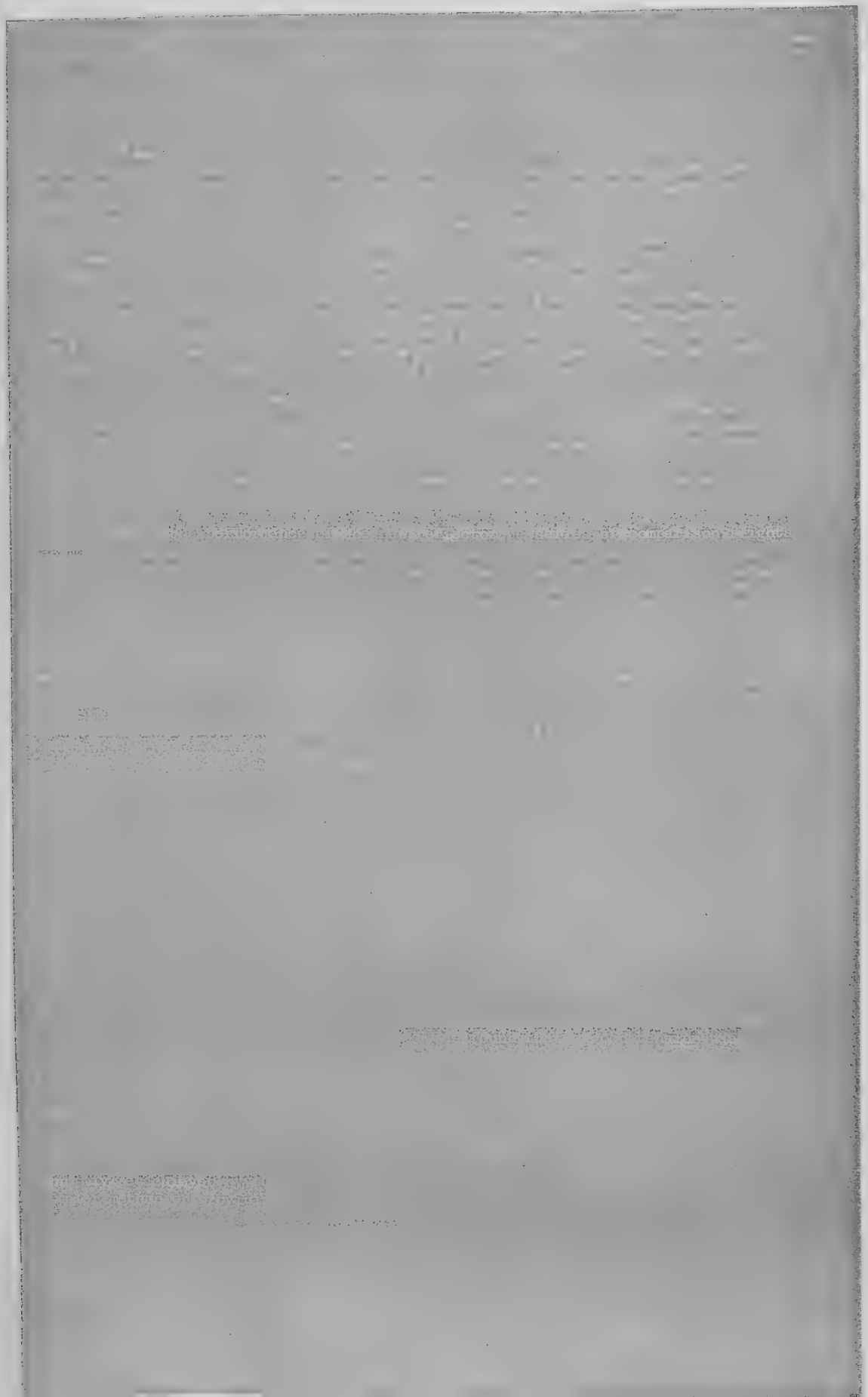
THE DEVELOPMENT OF THE M.O. RADAR REFLECTOR MK IIB

O.M. ASHFORD; H.J. FERRER 6 MAY'48 5PP DIAGR,GRAPHS

ELECTRONICS (3) REFLECTORS, RADAR
RADAR (2)

UNCLASSIFIED





Free Lift (grams)

UNCLASSIFIED

UNCLASSIFIED

MRP's 402, [illegible] 40c

AIR MINISTRY

M.R.P. 401
20th May, 1948.

METEOROLOGICAL RESEARCH COMMITTEE

INDEX OF M.R.P.'s APPROVED FOR GENERAL DISTRIBUTION.

(Appendix No.4. of M.R.P. 200)

M.R.P.	TITLE	AUTHOR
346	Anomalous Sound Reception Experiments	R.J. Murgatroyd
350	Index of M.R.P.'s approved for General Distribution.	
352	The Size Distribution of Rain Drops.	A.C. Best
353	The Estimation of Large Scale Vertical Currents from the Rate of Rainfall.	J.K. Bannon
360	The Upper Atmosphere - Estimated Distribution of Temperature, Pressure and Wind up to the 45 Km level.	A.H.R. Goldie
362	A Preliminary Study of the Vertical Exchange of Moisture near the Ground	F. Pasquill
363	A Tentative Evaluation of Evaporation from Grassland in terms of Temperature Measurements near the Surface.	F. Pasquill
364	Comparison of G.L. IIIB and S.C.R. 658 Wind Finding Equipment.	R.F. Apps
372	Solar Radiation at Kew Observatory	J.M. Stagg
376	The Magnitude of the Casual Variations of Temperature in the Free Air.	R. Frith
377	A Contribution to the study of Convection in the lowest layers of the Atmosphere	G.D. Robinson
383	Note on the Insufficiency of Gradient Wind approximation in Disturbed Conditions.	J.K. Bannon
385	Research Programme	
387	The Errors of G.L. III observations.	D.N. Harrison

G.198612/D.S./5/48.

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

NOV 17 1952

Its
e quoted.

M. R. P. 451.
20th November, 1948.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

INDEX OF M. R. Ps. APPROVED FOR GENERAL DISTRIBUTION
(Appendix No. 5 of M. R. P. 200)

M. R. P.	TITLE	AUTHOR
392	The Angular Deviation of the Wind from the Isobars at Liverpool.	J.K. Bannon
398	The Development of the M.O. Radar Reflector Mk. IIB.	O.M. Ashford & H.J. Ferrer
401	Index of M. R. Ps. approved for General Distribution	
402	Small Scale Temperature and Humidity Patterns in the Free Air.	R. Frith
405	The Speed Correction Coefficient of the M.O. Knife-edge Electrical Resistance Thermometer and its variation with Air Density.	R. Frith & H.C. Shellard
406	Errors in Winds measured with G.L. III Radar Equipment.	J.K. Bannon
409	Upper Winds over the World.	C.S. Durst & C.E.P. Brooks
422	The Accuracy of Mk. II R/S Observations.	D.N. Harrison
424	Eddy Diffusivity and Conductivity near the Ground and the Indirect Evaluation of Natural Evaporation.	F. Pasquill
428	Measures of Atmospheric Circulation.	A.H.R. Goldie
436	Met. Factors in the occurrence of Turbulence at High Altitudes - Interim Report.	J.K. Bannon
437 (Revised)	A Note on a more General Criterion of Turbulence (after Richardson) and its application to the Atmosphere.	J.K. Bannon

224033/PH/3/49/250.

MRP's 284 293, 292, 298, 309

M.R.P. 402
S.C.III/6
18th May, 1948.

contents may not be quoted.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE.

SMALL SCALE TEMPERATURE AND HUMIDITY PATTERNS IN THE FREE AIR.

BY
R. FRITH

1. INTRODUCTION.

The degree of homogeneity of temperature and humidity in the free air is of importance, both as an indication of the degree and type of large scale turbulence present; and also as a factor governing the general stability of the atmosphere.

That irregular variations of temperature, of magnitude 1 to 2 degrees (F), and scale 5 to 10 miles, exist in the upper air was shown in M.R.P. 376; but, beyond this, information appears to be very scanty. We therefore feel that we should make known without delay the results of three recent flights which throw more light on the subject.

Two of these flights, in cloud free air, were deliberately designed to measure the variations of temperature and humidity over an area, and we shall describe these first.

On the third, also in cloud free air, which was made for another purpose, readings of temperature alone were made.

2. THE SPECIAL RUNS.

On these runs, measurements of temperature and humidity were made, during level flight, over an area, relative to the air, of some 30 miles square. This area was traversed 6 times along parallel legs, 30 miles long, and 5 miles apart.

The total time of flight was about 1½ hours, and the fact that a consistent picture of the whole area is obtained indicates that the life of any patterns found can be measured in hours, at least.

During the flights the air position of the aircraft at the beginning and end of each leg was read on an air position indicator. Temperatures were read on a flat plate electrical thermometer mounted beneath a wing; and humidities on a Frost Point hygrometer. The absolute accuracy of the readings is about ± .2°F in temperature and ± 2°F in frost point. All readings taken are shown in the air plots in figures 1 and 3.

3. 12th April, 1948.

The first run was made at 760 mbs. between 1000 and 1200 Z on April 12th. The first leg began at Farnborough and ended 30 miles due west of base. Each leg was north of the previous one, but there was a northerly wind of such strength that the ground positions of the 6 legs were almost identical.

Fig. 1 shows the air plot of this run, and Fig. 2 the tephigrams for Larkhill and Downham Market for 0800 Z on the same day.

It will be seen that we were at the top of, or slightly above, a deep almost isothermal layer.

From Fig. 1 we see that there is a 'trough' of dry air running across the area, with 'lows' apparently spaced some 25 to 30 miles apart. Associated with this 'trough' is a tongue of warm air; but the correlation between dry air and warm air is not very clear.

4. 13th April, 1948.

Fig. 3 shows the air plot of a run made between 1415 and 1635 on April 13th, at a height of 580 mbs. Once again a northerly wind resulted in the ground position of the six legs changing but little, each lying roughly E - W through Farnborough.

/Fig. 4

Fig. 4 shows the tephigrams for Larkhill and Downham Market at 1500 Z.

Although the range of frost points, -26 to -40°F, is smaller, there is again a clear pattern; and with a stronger association of dry air with warm, and moist air with cold.

5. THE THIRD RUNS.

Apart from the two runs just described, on February 9th, 1948, we obtained temperature readings, at a fixed level (700 mbs) over a somewhat larger area.

An air plot is shown in Fig. 5. The ground area covered was roughly Salisbury Plain - Isle of Wight - Farnborough. The flight was from 1100 to 1300 Z.

Fig. 6 shows the tephigram for Larkhill at 1500 Z.

Fig. 5 is remarkable in showing a very strong gradient of temperature to a 'warm pool' in the NE corner.

6. CONCLUSIONS.

It appears that closed temperature and humidity patterns, on a scale measured in tens of miles, exist, with some degree of persistence, in the free atmosphere. There is some similarity between the temperature and humidity isopleths, drier air tending to be warmer than moist.

Temperature variations are probably usually of the order of 1 to 2 degrees (F), but may be much more. The variations in Frost Point may be anything up to 30 degrees (F).

Added later.

Since this note was prepared, four more runs of the type described above have been made, and air plots are shown in Figs. 7 to 10.

Both temperature and humidity readings were taken at close intervals. For ease of reproduction however, only selected ones are shown.

April 19th. Fig. 7 shows a run at 550 mbs. Fig. 8 shows a run at 750 mbs made, as near as could be determined, over an area, relative to the air, vertically below the 550 mb run. The lower area was inside a haze layer, whilst the upper area was well above this layer. It was therefore not to be expected that any similarity would be found between the two.

April 27th. This interesting example shows a very close association between cold and moist air; and between warm and dry air.

April 28th. On this day the South and South-east portions of the area examined was cloudless at all levels; but to the North-west there were increasing amounts of Altocumulus below the aircraft. In the extreme North-west the top of this cloud was above the level of flight so that at the beginning of the fifth leg we passed through a few wisps of cloud; and the whole of the latter part of the sixth leg was in cloud.

These runs support strongly the conclusions to which we came in the final paragraph of this paper.

FIG.I.

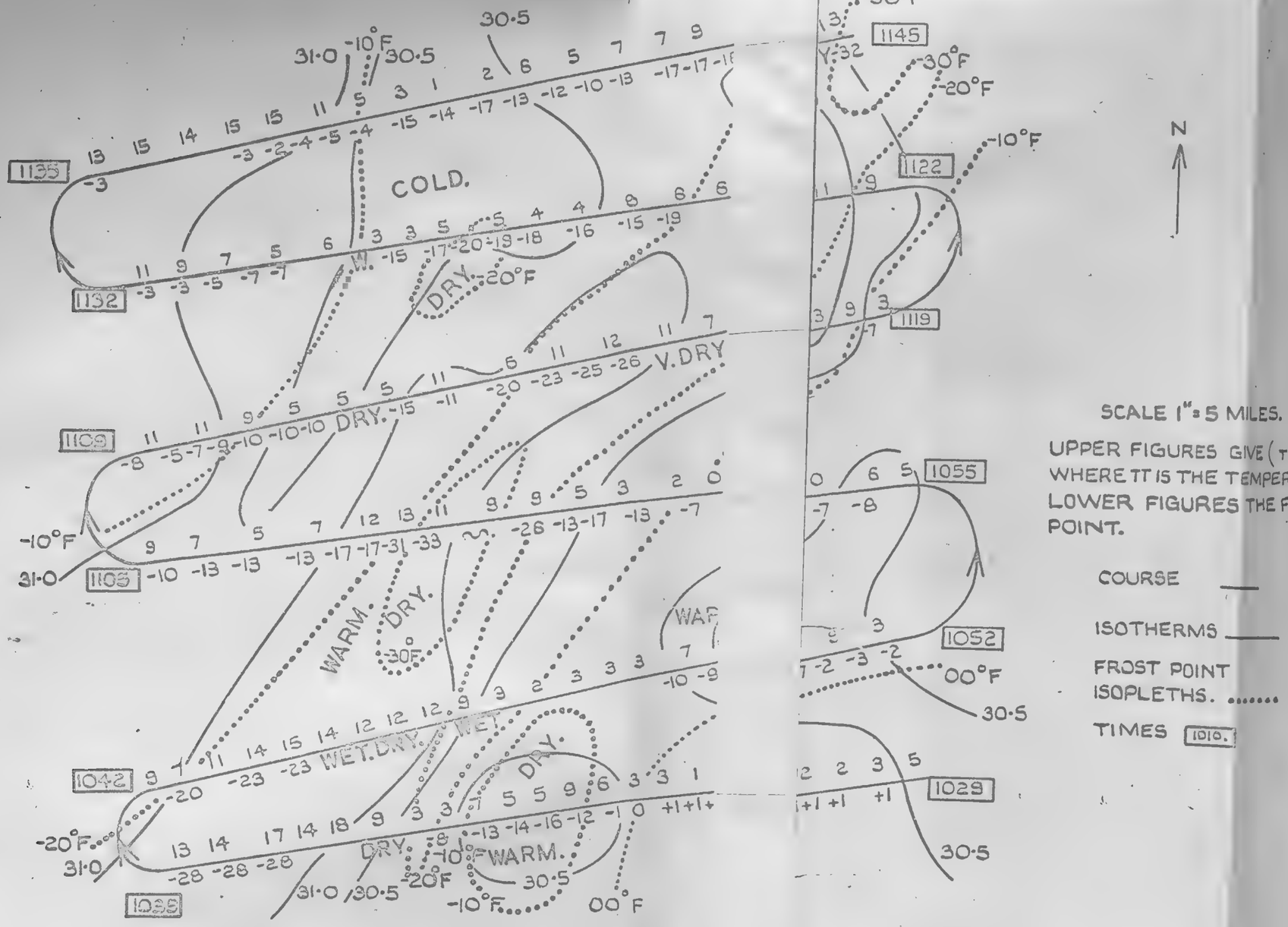


FIG.I. AIRPLOT OF TEMPERATURE AND HUM
AT 760 MBS ON 1
AREA FARNBOROUG
TIME 1,000

TY MEASUREMENTS MADE
I. APRIL 1948.
-SALISBURY PLAIN.
OOZ.

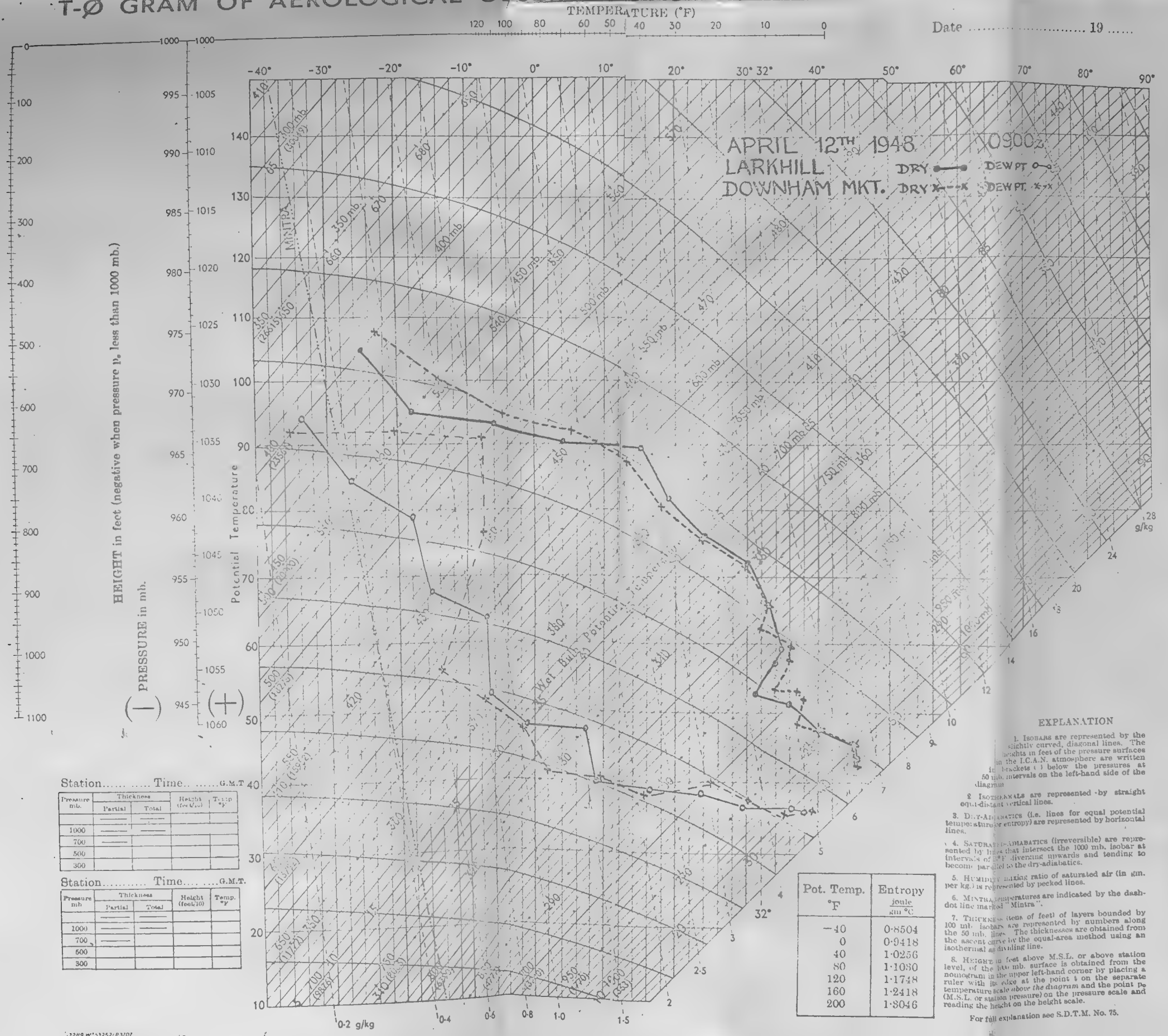
Form 2810 (1946 edition)

AIR MINISTRY, METEOROLOGICAL OFFICE

FIG. 2

T-Ø GRAM OF AEROLOGICAL OBSERVATIONS

Date ..... 19 .....



Station..... ..... Time.. .....G.M.T

Pressure mb.	Thickness		Height (feet/10)	Temp. °F
	Partial	Total		
1000				
700				
500				
300				

Station..... Time..... .....G.M.T.

Pressure mb	Thickness		Height (feet/10)	Temp. °F
	Partial	Total		
1000				
700				
600				
300				

Pot. Temp. °F	Entropy joule gm °C
−40	0·8504
0	0·9418
40	1·0256
80	1·1030
120	1·1748
160	1·2418
200	1·3046

EXPLANATION

1. ISOBARS are represented by the slightly curved, diagonal lines. The heights in feet of the pressure surfaces in the I.C.A.N. atmosphere are written in brackets ( ) below the pressures at 50 mb. intervals on the left-hand side of the diagram.

2. ISOTHERMALS are represented by straight equi-distant vertical lines.

3. DRY-ADIABATICS (i.e. lines for equal potential temperature or entropy) are represented by horizontal lines.

4. SATURATED-ADIABATICS (irreversible) are represented by lines that intersect the 1000 mb. isobar at intervals of 2°F diverging upwards and tending to become parallel to the dry-adiabatics.

5. HUMIDITY MIXING ratio of saturated air (in gm. per kg.) is represented by pecked lines.

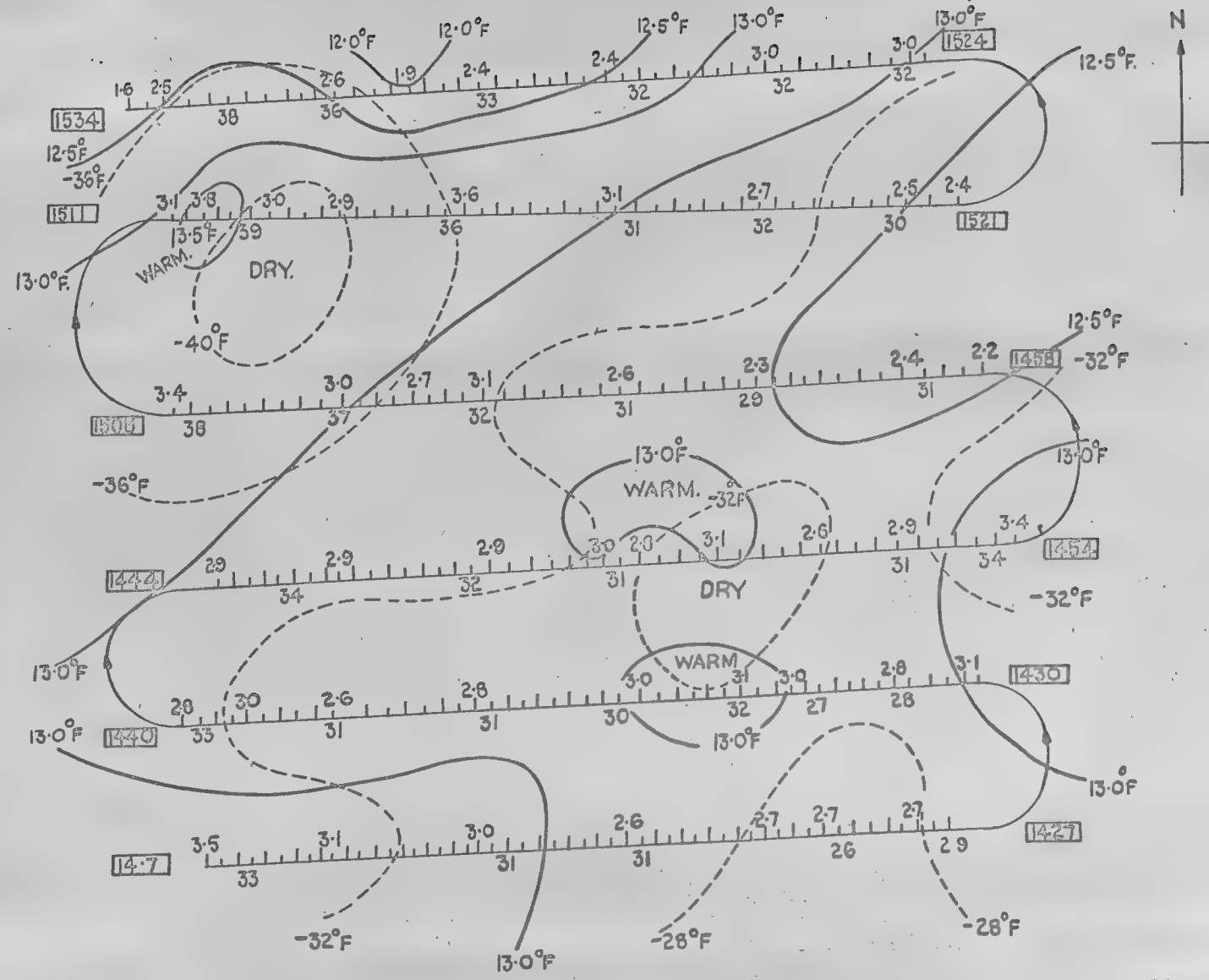
6. MINTRA temperatures are indicated by the dash-dot line marked "Mintra".

7. THICKNESS (tens of feet) of layers bounded by 100 mb. isobars are represented by numbers along the 50 mb. lines. The thicknesses are obtained from the ascent curve by the equal-area method using an isothermal as dividing line.

8. HEIGHT in feet above M.S.L. or above station level, of the 1000 mb. surface is obtained from the nomogram in the upper left-hand corner by placing a ruler with its edge at the point t on the separate temperature scale *above the diagram* and the point p_o (M.S.L. or station pressure) on the pressure scale and reading the height on the height scale.

For full explanation see S.D.T.M. No. 75.

12289 W1533521 P3707
C.B.H. (3014) WL 376571 3d P1866 7500 6/47



SCALE 1" = 5 MILES

UPPER FIGURES (TT-10)
WHERE TT IS TEMPERATURE °F
LOWER FIGURES = FROST POINTS (NEGATIVE)

COURSE ———
ISOTHERMS
FROST-POINT ISOPLETHS.
TIMES. 1454

FIG. 3

FIG.3 AIRPLOT OF TEMPERATURE AND HUMIDITY MEASUREMENTS AT 580 MBS. ON 13TH APRIL 1948.

AREA:- FARNBOROUGH - SALISBURY PLAIN. TIME 1400-1530Z.

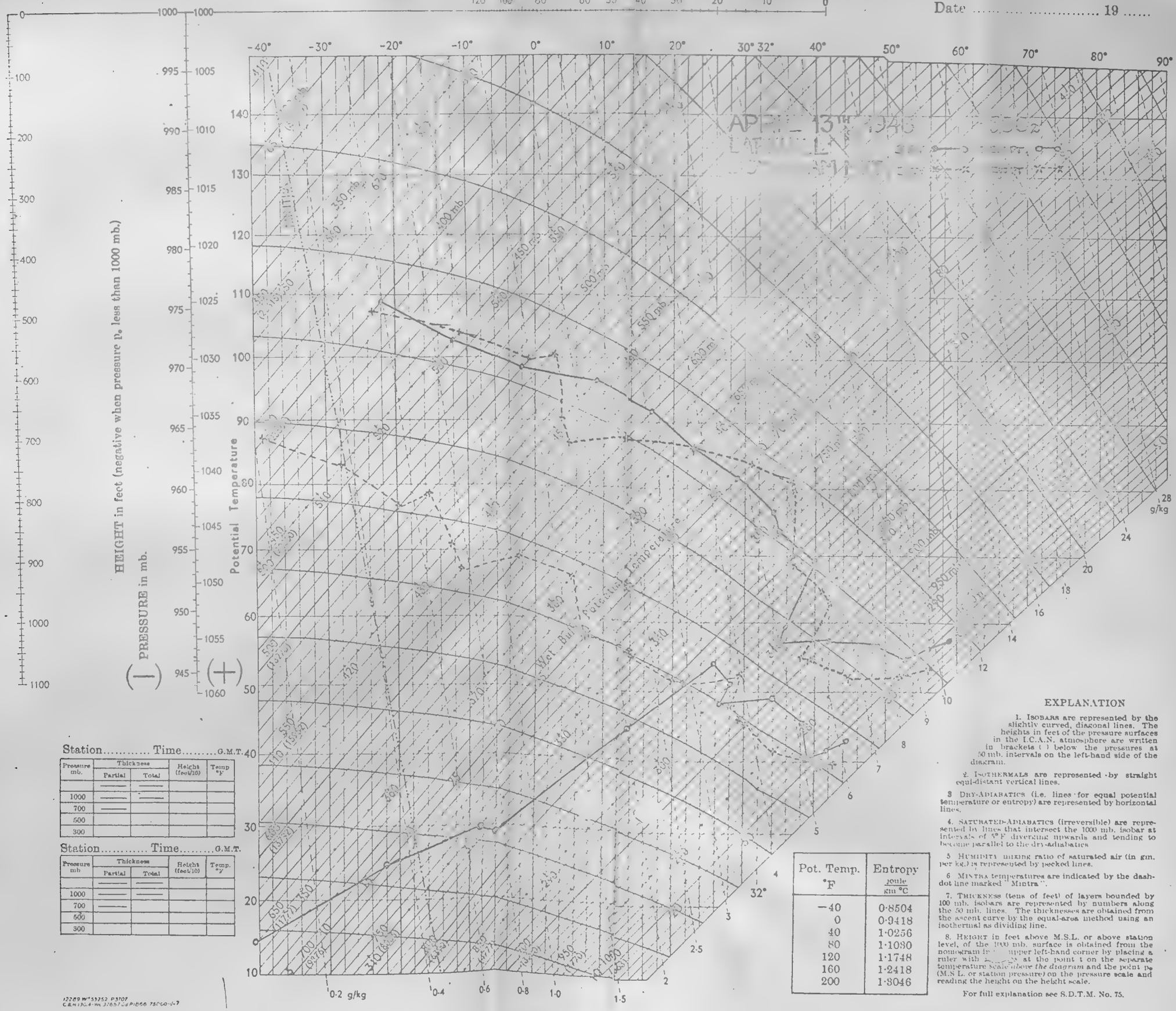
Form 2810 (1946 edition)

AIR MINISTRY, METEOROLOGICAL OFFICE

T-Ø GRAM OF AEROLOGICAL OBSERVATIONS

FIG. 4

Date ..... 19 .....



Station..... Time.....G.M.T.

Pressure mb.	Thickness		Height (feet/10)	Temp °F
	Partial	Total		
1000				
700				
500				
300				

Station..... Time.....G.M.T.

Pressure mb	Thickness		Height (feet/10)	Temp. °F
	Partial	Total		
1000				
700				
500				
300				

Pot. Temp. °F	Entropy $\frac{\text{joule}}{\text{gm °C}}$
−40	0·8504
0	0·9418
40	1·0256
80	1·1030
120	1·1748
160	1·2418
200	1·3046

EXPLANATION

1. ISOBARS are represented by the slightly curved, diagonal lines. The heights in feet of the pressure surfaces in the I.C.A.N. atmosphere are written in brackets ( ) below the pressures at 50 mb. intervals on the left-hand side of the diagram.

2. ISOTHERMALS are represented by straight equi-distant vertical lines.

3. DRY-ADIABATICS (i.e. lines for equal potential temperature or entropy) are represented by horizontal lines.

4. SATURATED-ADIABATICS (irreversible) are represented by lines that intersect the 1000 mb. isobar at intervals of 5°F diverging upwards and tending to become parallel to the dry-adiabatics.

5. HUMIDITY mixing ratio of saturated air (in gm. per kg.) is represented by pecked lines.

6. MINTRA temperatures are indicated by the dash-dot line marked "Mintra".

7. THICKNESS (tens of feet) of layers bounded by 100 mb. isobars are represented by numbers along the 50 mb. lines. The thicknesses are obtained from the ascent curve by the equal-area method using an isothermal as dividing line.

8. HEIGHT in feet above M.S.L. or above station level, of the 1000 mb. surface is obtained from the nomogram by [illegible] upper left-hand corner by placing a ruler with [illegible] at the point t on the separate temperature scale *above the diagram* and the point p_0 (M.S.L. or station pressure) on the pressure scale and reading the height on the height scale.

For full explanation see S.D.T.M. No. 75.

12289 W.S.3252. P.3707
C.&H.13.4-Wt.37657 C.&P.1/56 78(CO)-4-7

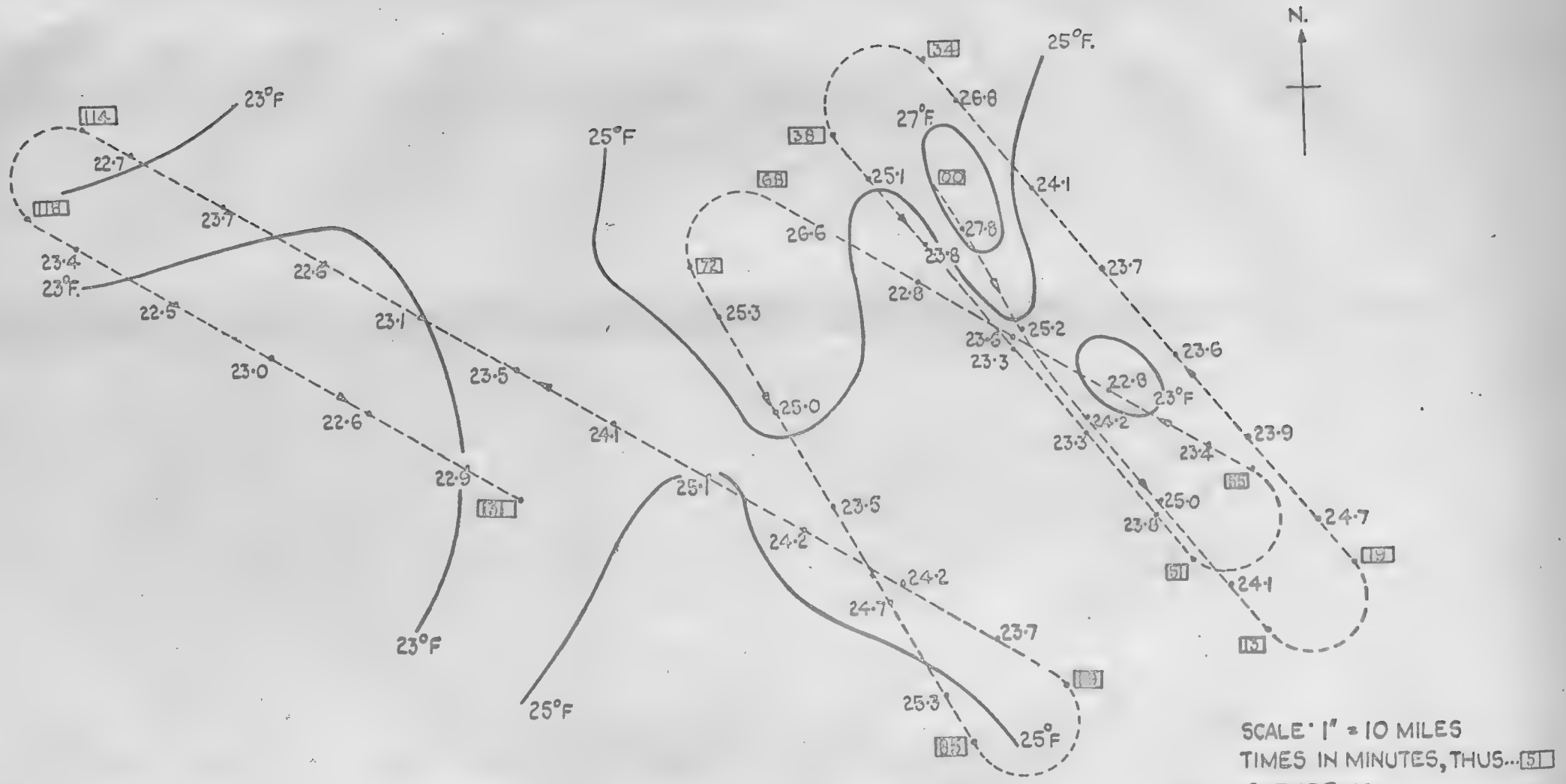


FIG.5 AIR PLOT OF TEMPERATURE READINGS MADE AT 700 MBS. ON 9TH FEB. 1948.
AREA - SALISBURY PLAIN - ISLE OF WIGHT - FARNBOROUGH. TIME 1100-1300Z.

FIG.5

Form 2810 (1946 edition)

T-Ø GRAM OF AEROLOGICAL OBSERVATIONS

AIR MINISTRY, METEOROLOGICAL OFFICE

FIG. 6

TEMPERATURE (°F)

80 60 50 40 30 20 10 0

Date ..... 19 .....

FEBRUARY 9TH 1948.
LARKHILL 1500z

DRY BULB ——— DEW POINT - - -

HEIGHT in feet (negative when pressure p. less than 1000 mb.)

(—) PRESSURE in mb. (+)

Potential Temperature

Wet Bulb Potential Temperature

0·2 g/kg 0·4 0·6 0·8 1·0 ·5

Station..... Time.....G.M.T.

Pressure mb	Thickness Partial	Thickness Total	Height (feet/10)	Temp °F
1000				
700				
500				
300				

Station..... Time.....G.M.T.

Pressure mb	Thickness Partial	Thickness Total	Height (feet/10)	Temp °F
1000				
700				
500				
300				

Pot. Temp. °F	Entropy joule / gm °C
−40	0·8504
0	0·9418
40	1·0256
80	1·1030
120	1·1748
160	1·2418
200	1·3046

EXPLANATION

1. ISOBARS are represented by the slightly curved, horizontal lines. The heights in feet of the pressure surfaces in the I.C.A.N. atmosphere are written in brackets ( ) below the pressures at 50 mb. intervals on the left-hand side of the diagram.

2. ISOTHERMALS are represented by straight equi-distant vertical lines.

3. DRY-ADIABATICS (i.e. lines for equal potential temperature or entropy) are represented by horizontal lines.

4. SATURATED-ADIABATICS (irreversible) are represented by lines that intersect the 1000 mb. isobar at intervals of 5°F. diverging upwards and tending to become parallel to the dry-adiabatics.

5. HUMIDITY MIXING ratio of saturated air (in gm. per kg.) is represented by pecked lines.

6. MINTRA temperatures are indicated by the dash-dot line marked "Mintra".

7. THICKNESS (tens of feet) of layers bounded by 100 mb. isobars are represented by numbers along the 50 mb. line. The thicknesses are obtained from the ascent curve by the equal-area method using an isothermal as dividing line.

8. HEIGHT in feet above M.S.L. or above station level, of the 1000 mb. surface is obtained from the nomogram in the upper left-hand corner by placing a ruler with its edge at the point t on the separate temperature scale *above the diagram* and the point p_0 (M.S.L. or station pressure) on the pressure scale and reading the height on the height scale.

For full explanation see S.D.T.M. No. 75.

12289 W'53252/P3707
C.B.H.3014-WL.37657-D&P11/48-7500 (4)

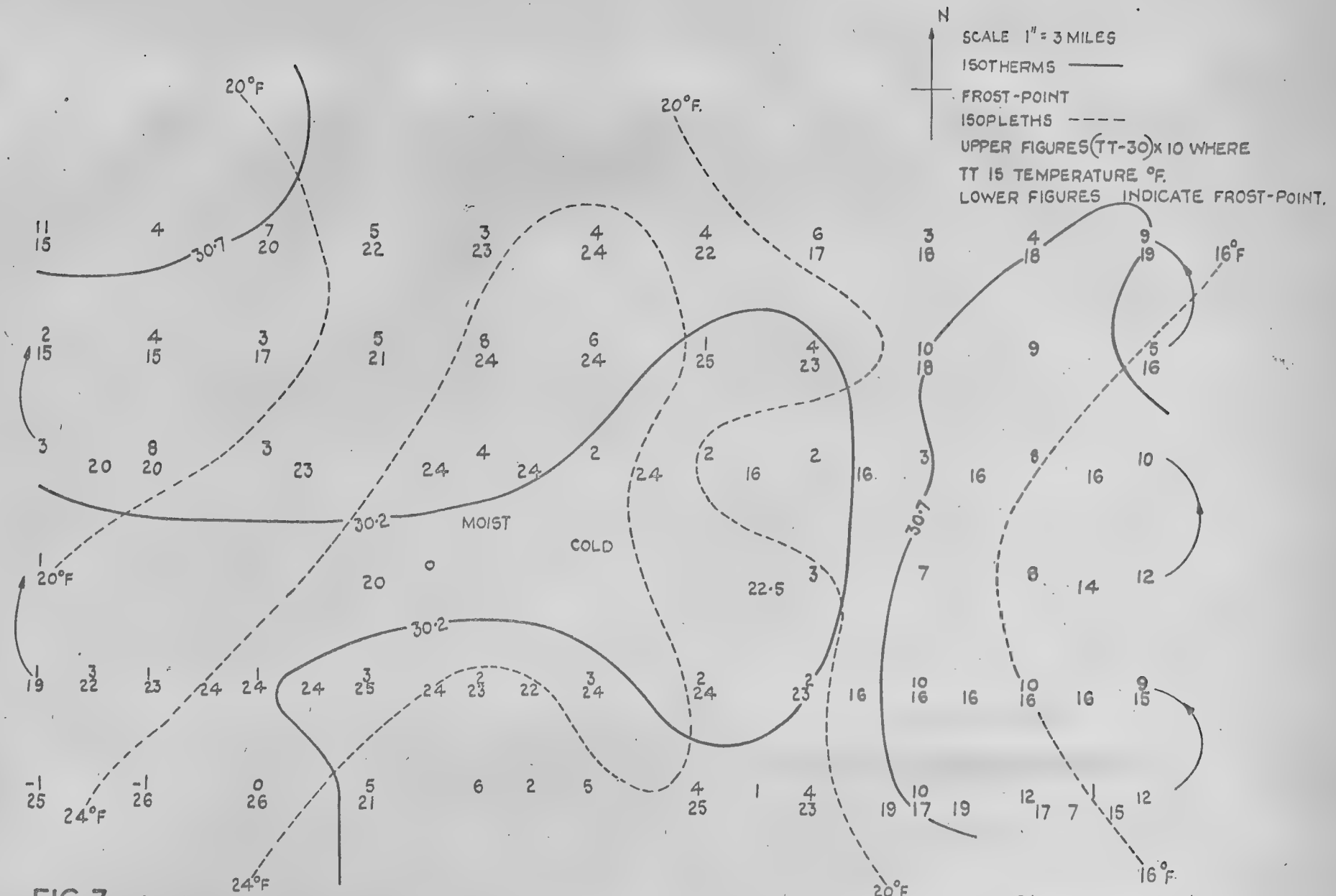


FIG.7 AIRPLOT OF TEMPERATURE AND HUMIDITY READINGS MADE AT 750 MBS. ON 19TH APRIL 1948.
TIME 0930 – 1030 Z. AREA,- ABINGDON – GLOUCESTER.

FIG.7

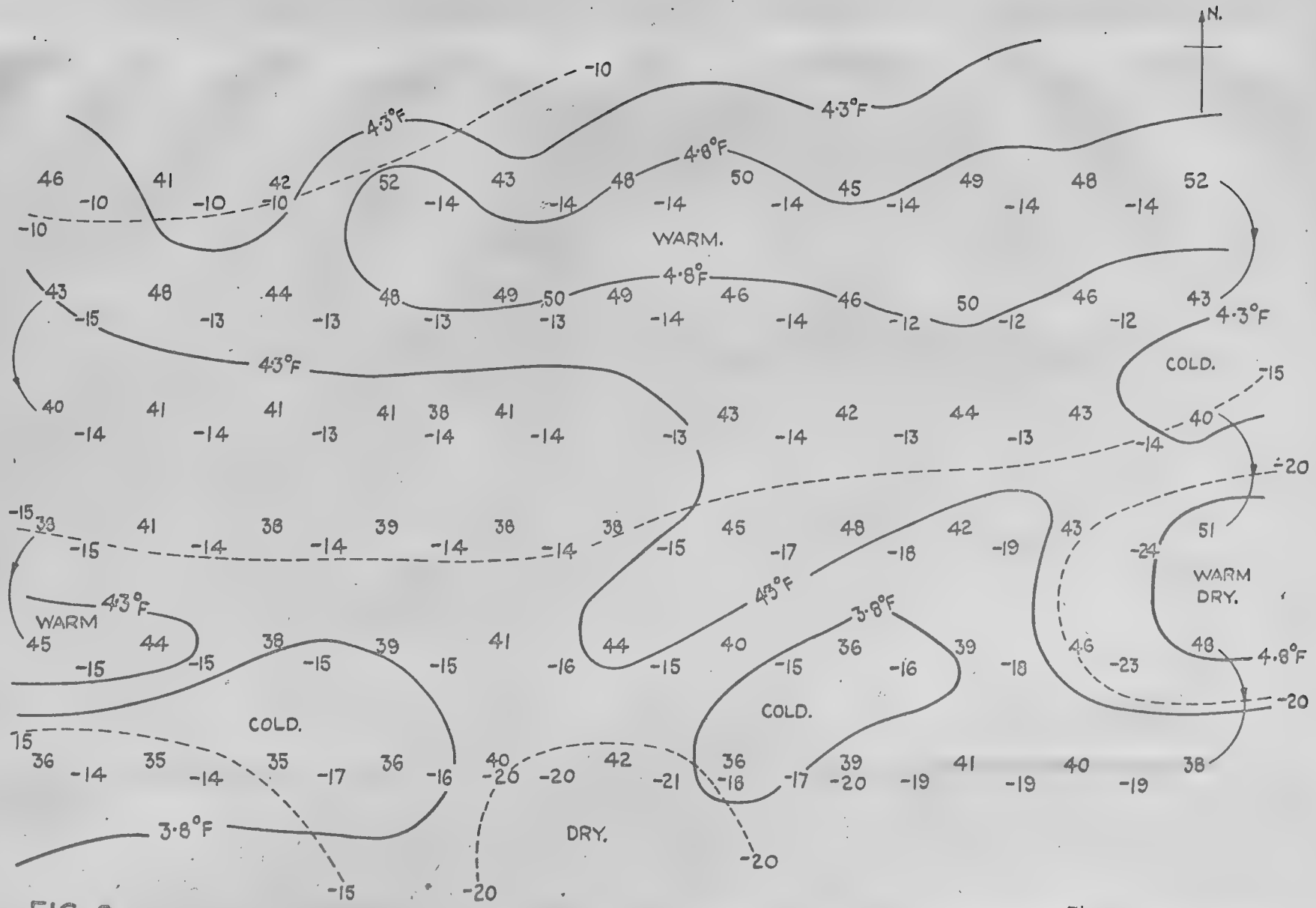


FIG. 8 AIRPLOT OF TEMPERATURE AND HUMIDITY READINGS MADE AT 550 MBS. ON 19TH APRIL 1948.
TIME 1100-1200 Z. AREA, ABINGDON – GLOUCESTER.

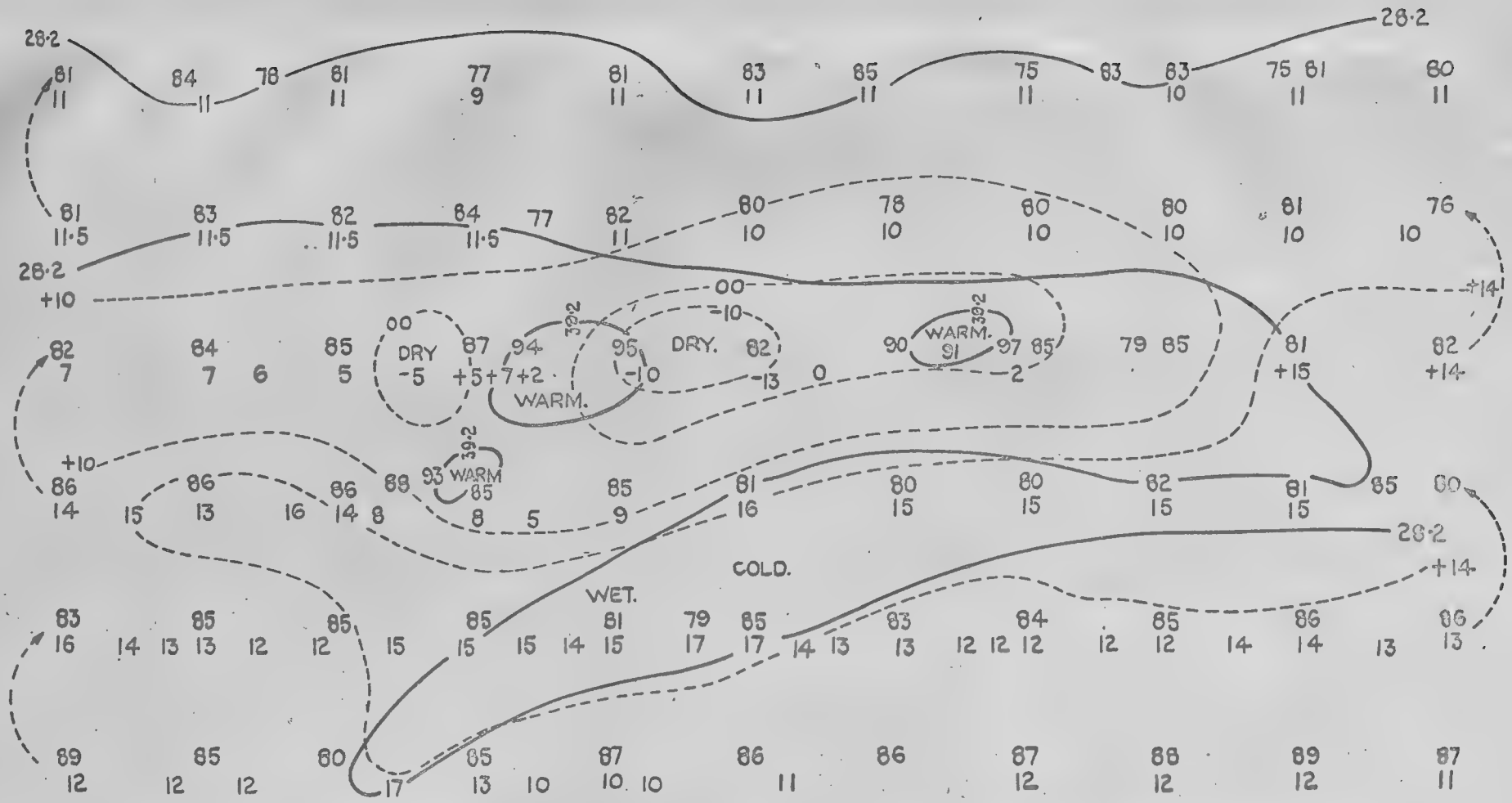


FIG.9. AIRPLOT, OF TEMPERATURE AND HUMIDITY MEASUREMENTS AT 700 MBS. ON 27TH APRIL 1948; TIME 1100 - 1200 Z. AREA:- FARNBOROUGH — SALISBURY PLAIN.

SCALE 1" = 3 MILES. ISOTHERMS —— FROST POINT ISOPLETHS - - - - - -
UPPER FIGURES (TT-20)A10, WHERE TT IS TEMPERATURE IN DEGREES (F.)

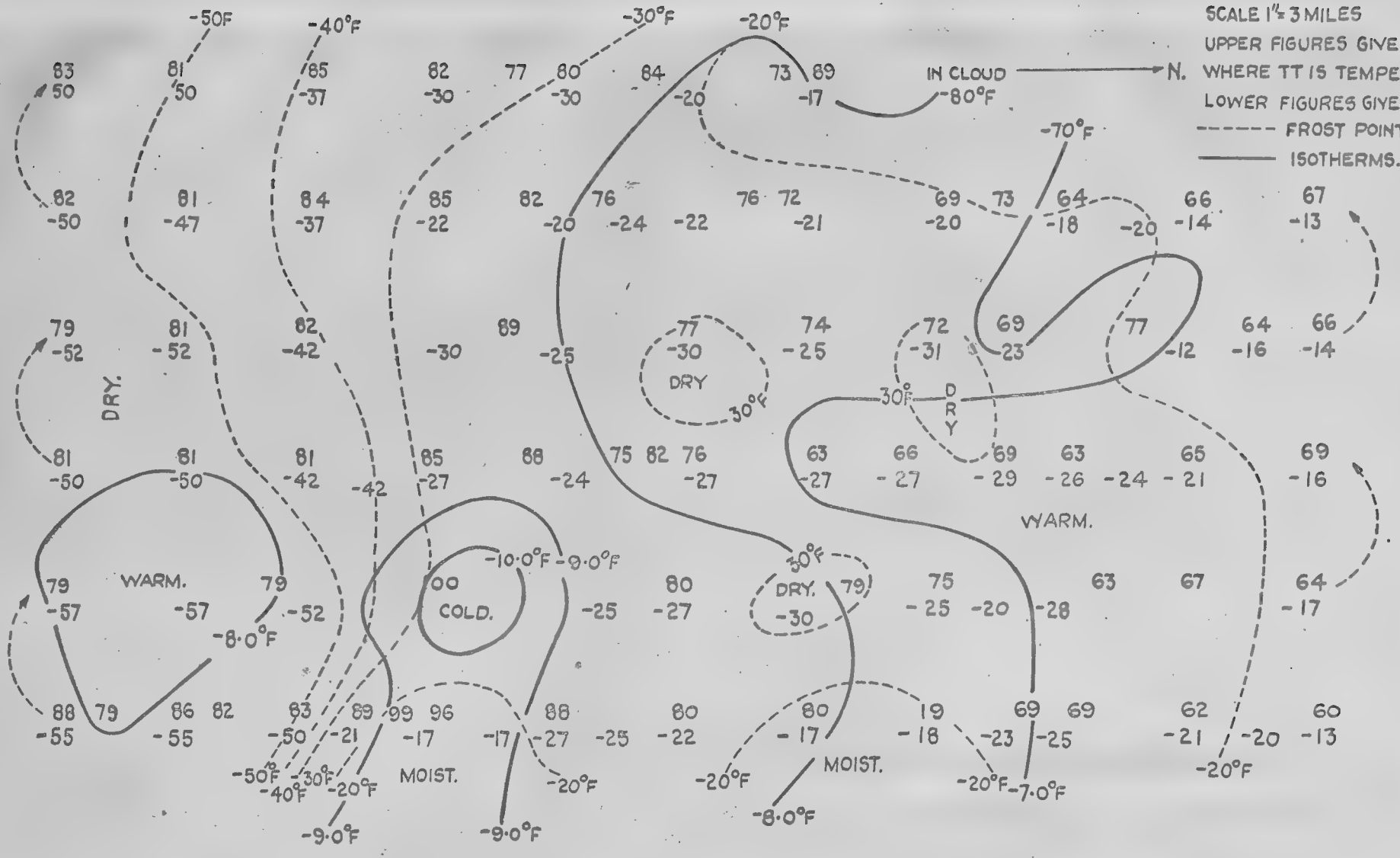


FIG.10

FIG.10 AIRPLOT OF TEMPERATURE AND HUMIDITY AT 500 MBS. ON 28TH APRIL 1948. TIME 1400-1500 Z.
AREA:- ENGLISH CHANNEL, SOUTH OF ISLE OF WIGHT.

This ... contents ...

S.C. 1.76.
27th May, 1948.

AIR MINISTRY NOV 17 1952

METEOROLOGICAL RESEARCH COMMITTEE.

THE SPEED CORRECTION COEFFICIENT OF THE M.O. KNIFE-EDGE ELECTRICAL RESISTANCE THERMOMETER AND ITS VARIATION WITH AIR DENSITY.

By R. Frith and H.C. Shellard.

ATI-172129

I. INTRODUCTION.

1. The correction which has to be subtracted from the temperature indicated by a thermometer mounted on an aircraft in order to obtain the true air temperature is generally taken to be

$$\Delta T = \alpha\left(\frac{V}{100}\right)^2 ,$$

where α is the speed correction coefficient and 'v' is the true air speed. The speed correction coefficient for the M.O. resistance thermometer has a value, according to measurements by A.W. BREWER and G.E.W. HARTLEY (1), of

$$1.26^{\circ}F\left(\frac{\text{m.p.h.}}{100}\right)^{-2} \text{ or } 1.67^{\circ}F\left(\frac{\text{knots}}{100}\right)^{-2} \quad : \text{ (the latter units}$$

will be used throughout this paper). The approximate value of 1.7, as given in Table III p. 16 of M.O.470, has been generally accepted for the instrument when in routine use.

2. It was shown here early in 1947 (2), by comparing the readings of two or three thermometers mounted on the same aircraft, that an α found for one element in one position is not appropriate to another element in another position, the greater part of the difference being due, it is believed, to the different mounting. During this work the suspicion arose that there might also be a variation of α with the height of the aircraft i.e., with air density and possibly also with certain other factors. Much additional evidence has been accumulated from flight tests on both Halifax and Mosquito aircraft and we now consider that we can state with reasonable certainty that the value of α for the resistance thermometer does decrease with increasing height (i.e., decreasing air density).

3. The evidence obtained from our own flights is discussed in sections III and IV, but first we shall, in section II, re-examine and discuss the results presented by M.R.P.247, and show that they also support our case.

4. As further indirect evidence the following is quoted from a letter written in September, 1942 by P.A. SHEPPARD to the Meteorological Officer, H.A.F., Boscombe Down:-

> "It is true that the theory suggests α to be independent of height (air density) but observations at High Altitude Met. Flights would be more consistent with other observations if α decreased with height, and a few test runs gave some indication of this effect."

This would appear to show that evidence did then exist, presumably from a comparison of aircraft and radio-sonde temperature ascents, to support the idea of a decrease of α with height.

II. THE 1944 SPEED RUNS MADE FROM BOSCOMBE DOWN.

1. Table I presents the results of the original flight tests made from Boscombe Down and is extracted from Appendix I to M.R.P.247, the values of α having been converted to

$$^{\circ}F\left(\frac{\text{knots}}{100}\right)^{-2} .$$

(1) M.R.P.247. M.O. Aircraft Resistance Thermometer Element. A.W. Brewer and G.E.W. Hartley.

(2) M.R.P.335. Report on accuracy of speed corrections to air temperature measurements made from aircraft. R. Frith.

Most of the flights were made at low and medium levels but a few runs were made at 20,000 - 22,000 ft.

2. In M.R.P.247 it is stated that "within the error there is no significant difference between any of the different installations", and a mean value for all the installations is therefore evaluated at 1.67. Whilst this statement is true for the mean values for each mounting it appears to have been overlooked that for any one mounting there is a marked tendency for individual measurements of α to decrease with height.

3. In Table II we have rearranged the results into convenient height ranges and have evaluated mean α's for each mounting in each range, for each type of aircraft in each range, and finally for all mountings in each height range.

4. In the case of the Spitfire and the Boston the decrease of α with height is now obvious. The Mosquito values fit less well due mainly to the rather high values of α obtained at medium levels for mounting number 1, but even so, it will be seen that for Mosquito aircraft no value of α obtained above 10,000 ft exceeds 1.55 and only one of the twelve values obtained below 10,000 ft is less than 1.60. Only five of the total of 29 runs were made at heights above 10,000 ft but these 5 give a mean α of 1.55 compared with a mean α of 1.70 from the other 24 runs.

III. SPEED RUNS MADE FROM SOUTH FARNBOROUGH IN HALIFAX AIRCRAFT.

1. It has been shown(3) that in the free air there are very considerable horizontal variations of air temperature within quite short distances and it is therefore necessary to carry out a large number of runs in order to obtain an accurate value of the correction coefficient. It is also desirable to fly so far as possible in the same "patch" of air throughout a run. With these points in mind we adopted the following technique in our early Halifax speed runs.

2. With the aircraft in straight and level flight at the selected height and the lowest possible airspeed, 20 or 25 readings of temperature were taken in rapid succession, usually about 5 to 10 seconds apart. For each temperature reading simultaneous values of height and airspeed were logged, but no readings were taken if height and airspeed differed by more than 30 ft and 3 knots respectively from the values chosen. After a turn through 90° a further similar set of readings was taken with the indicated airspeed approximately 10 knots higher, this being repeated until the highest speed was reached, the turns always being made in the same sense. In this way the course followed the sides of a square and the aircraft remained in approximately the same patch of air throughout. It was usual to repeat the run with decreasing speeds, and in this way 40 to 50 temperature readings corresponding to each airspeed were obtained. After correcting for minor height variations, mean temperature was plotted against $(V/100)^2$, where V = true airspeed and a value of α obtained from the slope of the resulting line. The usual speed range for the Halifax is from 140 to 180 knots indicated so that normally we had 5 points through which to draw the curve; on one or two occasions, however only four speeds were available due to poor engine performance. The limited speed range of the Halifax is a distinct disadvantage for speed run work.

3. The results obtained from five early flights in Halifax ST 817 made in the way described above were rather disappointing for a variety of reasons, the most important being poor aircraft performance, inability to adhere to the 'square' plan caused by the aircraft being carried too far from base by strong upper winds, and the effect of the sun's radiation on the element (not suspected at the time - see para. 7 below). From the two flights which gave fairly good straight lines however, the values of α were 1.72 at 530 mbs and 2.03 at 835 mbs.

(3) M.R.P.376. The magnitude of the Casual Variations of Temperature in the Free Air. R. Frith.

4. At this stage the aircraft became unserviceable and experiments had to be transferred to Halifax ST 796. On this aircraft we had two elements, one beneath the nose on the starboard side, as standard, and the other mounted on an extension beneath the nose on the port side so that it was about one foot away from the aircraft skin.

5. It was decided that in future speed runs the 'square' method would be abandoned in favour of a 'reciprocal course' method. The disadvantages of the former are that too much time is wasted on turns and that it does not effectively eliminate the effects of large scale horizontal temperature variations. The new method was to fly on a course selected beforehand as being approximately along the isotherms at the height chosen, and to take ten readings of temperature etc., for each speed flying successively at 140, 150, 160, 170, 180, 180, 170, 160, 150, and 140 knots, and then after a 180 degree turn, at 180, 170, 160, 150, 140, 140, 150, 160, 170, 180 knots on the reciprocal course. By repeating this process we obtained 80, and sometimes 100, temperature readings for each of the five speeds.

6. Of the first five flights the last three were satisfactory, the first being rejected on account of the very peculiar results obtained which could not be properly explained, but were perhaps due to unusually large horizontal temperature gradients and our failure in this case to stick to an exact reciprocal course, and the second on account of a faulty thermometer and to engine trouble restricting the speed range. The results of the other three are set out in Table III as also are those obtained in Halifax ST 817. It will be seen that both elements have an α which is higher at the lower level.

7. Unfortunately it now became clear from an examination of the differences between the two elements when flying on different courses, that the sun's radiation was having a marked effect not only on the element on the extension but also on that under the nose. It therefore became necessary to install another element underneath the wing where it could not possibly be affected by direct radiation from the sun.

8. Four speed runs have been made to measure α for the wing mounting and the results are included in Table III. Unfortunately it has not yet been possible to obtain a run at low level for this element although a part run at 900 mbs made recently suggested a value of about 2.0. The second half of Table III summarises all the Halifax runs and shows that for all mountings α is higher at the lower levels.

IV. SPEED RUNS MADE FROM SOUTH FARNBOROUGH ON MOSQUITO AIRCRAFT.

1. Speed runs have also been made using Mosquito aircraft. On these aircraft the element is mounted beneath the nose, but we believe that it is adequately shielded from radiation.

2. The method was similar to the 'reciprocal course' method adopted for the later Halifax speed runs. Instead of taking ten readings of temperature in rapid succession at each speed however, it was found more practical to take only one reading, and that when height, airspeed and temperature had all been steady for approximately five seconds. By this means any instrumental lag effects were reduced to a minimum. Two full runs were made in each direction so that eight temperature readings were obtained for each speed. The range of true airspeed in these runs was about 170 to 320 knots at 500 mbs compared with 185 to 240 at the same height on the Halifax.

3. Figures 1 to 4 show the plots of indicated temperatures against $(V/100)^2$ for four such flights at 300, 500, 700 and 900 millibars respectively, on the same aircraft and with the same element and mounting. The lines of best fit calculated by the method of least squares are drawn in.

4. The results of all the speed runs made on Mosquito aircraft to date are set out in Table IV with the mean values at the foot. It is quite clear that α does decrease with increasing height and the results are very consistent. This is brought out in Figure 5 in which the mean values of α are plotted against mean values of P/459+T (proportional to air density). The four points lie on a smooth curve.

/V.

V. CONCLUSIONS.

1. The value of the speed correction coefficient for an M.O. knife edge resistance thermometer decreases with decreasing air density in an apparently regular way.

2. The same effect probably occurs with other types of thermometer except perhaps those exposed inside pitot heads.

3. The magnitude of the variations of α shown is such that an assumption of $\alpha = 1.7^{\circ}F\ (\text{knots}/100)^{-2}$ would introduce errors of at least 0.9, 1.6 and 2.5°F in the corrections applied to air temperatures measured at 300 millibars at true airspeeds of 300, 400 and 500 knots respectively. These errors may be much greater with other mountings, and at higher levels.

4. If accurate speed corrections are to be applied to temperatures indicated by elements mounted on high speed aircraft it will be necessary to carry out speed runs at several heights on the type of aircraft concerned, and perhaps on each individual aircraft. It may even be necessary to redetermine the corrections if the element is changed.

5. Since practically the same indicated speed range is used in the runs at each height, it follows that the mean true air speeds increase with height. Thus if the speed correction law were not of the form $\alpha\ (V/100)^2$ we would expect to find values of α varying with true airspeed (i.e., with height). However we have examined whether this can be the explanation of the variation found and have come to the conclusion that any departure from the true square law which may exist is, at the speeds with which we are concerned, very small.

TABLE I.

AIRCRAFT & MOUNTING	DATE	HEIGHT (FEET)	T_o (°F)	α $\left(\frac{°F(\text{knots})^{-2}}{100}\right)$	MEAN FOR MOUNTING
Spitfire	12.7.44	2,000	53.9	1.74	
MD 190	16.7.44	1,000	62.1	1.70	1.68
	3.8.44	22,000	-10.9	1.60	
Spitfire	13.8.44	2,000	57.5	1.75	
MD 114	15.9.44	1,000	59.9	1.70	1.70
	16.9.44	4,000	42.3	1.78	
	25.9.44	6,000	36.1	1.55	
Mosquito	5.10.44	2,000	42.5	1.55	
PF 391	7.10.44	10,000	27.5	1.76	
Mounting 1.	15.10.44	2,500	41.7	1.76	1.65
	16.10.44	20,000	-20.0	1.54	
Mosquito	5.10.44	2,000	44.8	1.60	
PF 391	7.10.44	10,000	30.2	1.68	
Mounting 2.	15.10.44	2,500	43.9	1.75	1.64
	16.10.44	20,000	-17.7	1.52	
Mosquito	6.11.44	15,000	4.1	1.55	
MM 174	6.11.44	9,500	16.6	1.71	
Mounting 1.	9.11.44	8,000	6.8	1.81	1.71
	10.11.44	8,000	14.2	1.79	
Mosquito	6.11.44	15,000	6.6	1.55	
MM 174	6.11.44	9,500	19.7	1.60	
Mounting 3.	9.11.44	8,000	9.7	1.73	1.64
	10.11.44	8,000	17.2	1.70	
Boston	27.11.44	3,000	31.2	1.72	
BZ 315	22.10.44	6,000	27.2	1.72	
	29.11.44	3,000	31.5	1.76	
	24.10.44	10,000	26.2	1.56	
	29.11.44	3,000	31.2	1.70	1.70
	28.10.44	9,000	14.9	1.72	

TABLE II

HEIGHT (feet) / AIRCRAFT & MOUNTING	1,000 to 3,000	4,000 to 6,000	8,000 to 10,000	15,000	20,000 to 22,000
Spitfire MD 190	1.74 1.70	- -	- -	- -	1.60 -
Spitfire MD 114	1.75 1.70	1.78 1.55	- -	- -	- -
Mosquito PF 391 Mounting 1.	1.55 1.76	- -	1.76 -	- -	1.54 -
Mosquito MM 174 Mounting 1.	- - -	- - -	1.71 1.81 1.79	1.55 - -	- - -
Mosquito PF 391 Mounting 2.	1.60 1.75	- -	1.68 -	- -	1.52 -
Mosquito MM 174 Mounting 3.	- - -	- - -	1.60 1.73 1.70	1.55 - -	- - -
Boston BZ 315	1.72 1.76 1.70	1.72 - -	1.56 1.72 -	- - -	- - -
Mean α for:					
Spitfire	1.72	1.67	-	-	1.60
Mosquito Mounting 1.	1.65	-	1.77	1.55	1.54
Mosquito Mounting 2.	1.67	-	1.68	-	1.52
Mosquito Mounting 3.	-	-	1.68	1.55	-
Boston	1.73	1.72	1.64	-	-
All Aircraft and mountings	1.70	1.68	1.71	1.55	1.55

TABLE III

RESULTS OF SPEED RUNS ON HALIFAX AIRCRAFT.

DATE	AIRCRAFT	VALUE OF α	HEIGHT	MOUNTING
17.12.47	ST 817	1.72	527 mbs	Beneath nose
17.12.47	"	2.03	835	" "
24.2.48	ST 796	1.76	929	" "
24.2.48	"	1.91	929	On extension beneath
26.2.48	"	1.79	505	nose
27.2.48	"	1.72	505	Beneath nose
27.2.48	"	1.75	505	On extension beneath
16.3.48	"	1.71	505	nose
16.3.48	"	1.66	505	Beneath wing
23.3.48	"	1.59	508	" "
23.4.48	"	1.57	503	" "
29.4.48	"	1.45	504	" "

MEAN VALUES OF α OBTAINED

MOUNTING	LOW LEVEL	HIGH LEVEL
Beneath nose of ST 817	2.03	1.72
Beneath nose of ST 796	1.76	1.72
On extension beneath nose ST 796	1.91	1.75
Beneath wing ST 796	-	1.57
All mountings	1.90	1.66

N.B. In each case the value of α given was evaluated by the method of least squares.

TABLE IV

RESULTS OF SPEED RUNS ON MOSQUITO AIRCRAFT.

DATE	AIRCRAFT	VALUE OF α	HEIGHT
10.3.48	RG 248	1.60	300 mbs
11.3.48	"	1.76	700
11.3.48	"	1.67	500
12.3.48	"	1.78	900
22.3.48	"	1.68[x]	500
24.3.48	"	1.80	900
24.3.48	"	1.73	500
25.3.48	"	1.74	700
14.4.48	VL 621	1.58	300
23.4.48	"	1.61	300
26.4.48	"	1.52	300

MEAN VALUES OF α

AIRCRAFT \ HEIGHT	900	700	500	300
RG 248	1.79	1.75	1.69	1.60
VL 621	-	-	-	1.57
Both	1.79	1.75	1.69	1.58

[x] This measurement of α was obtained from the results of a 'double run' with Halifax 796.

G.198638(o)/D.S./5/48.

FIG. 1.

RUN AT 300 MBS. IN MOSQUITO RG. 248 ON 10/3/48.

BEST COMPUTED LINE DRAWN IN: $\alpha = 1.60$

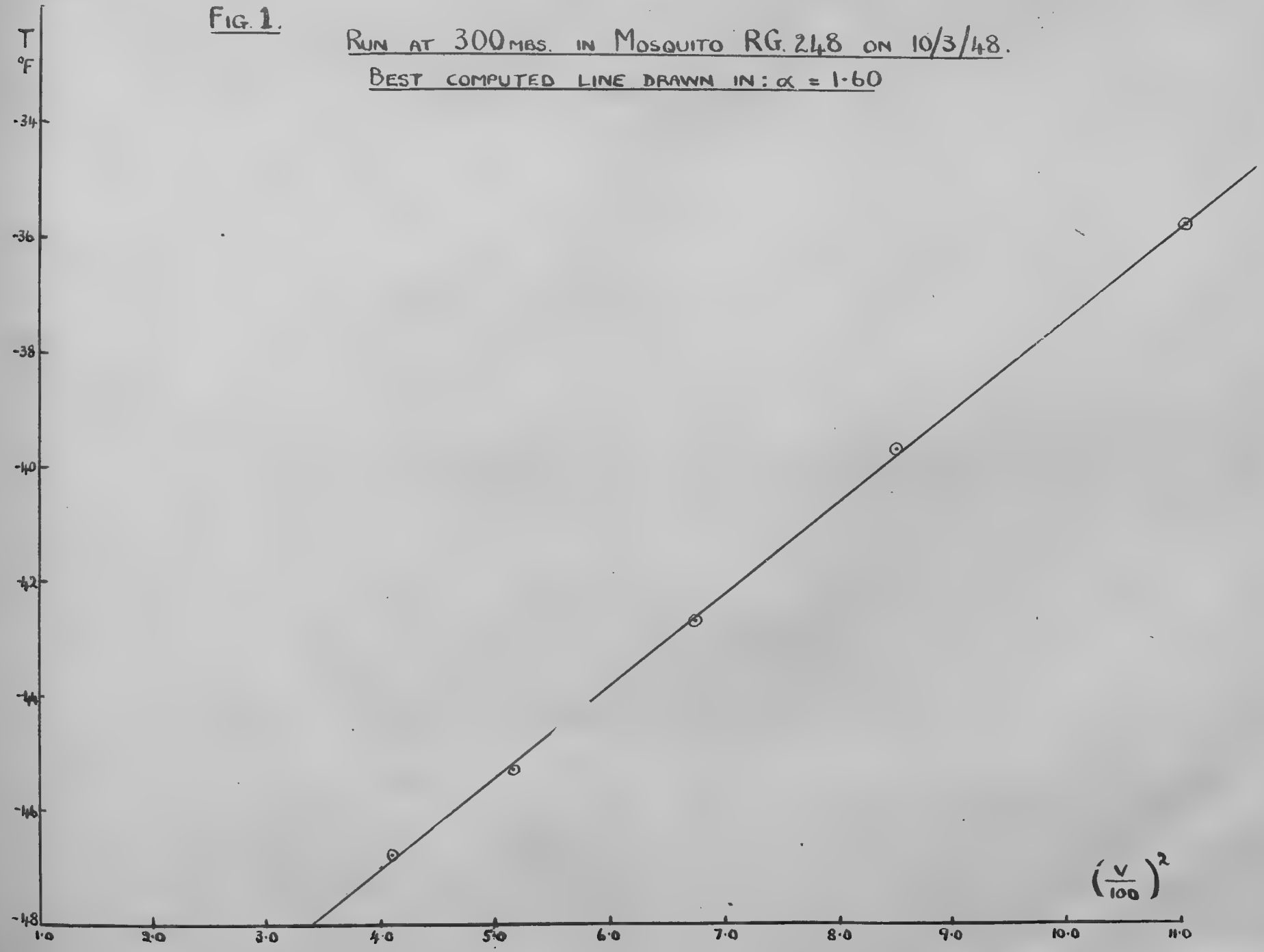


FIG. 2.

RUN AT 500 MBS. IN MOSQUITO RG. 248 ON 11/3/48.

BEST COMPUTED LINE DRAWN IN: $\alpha = 1.67$

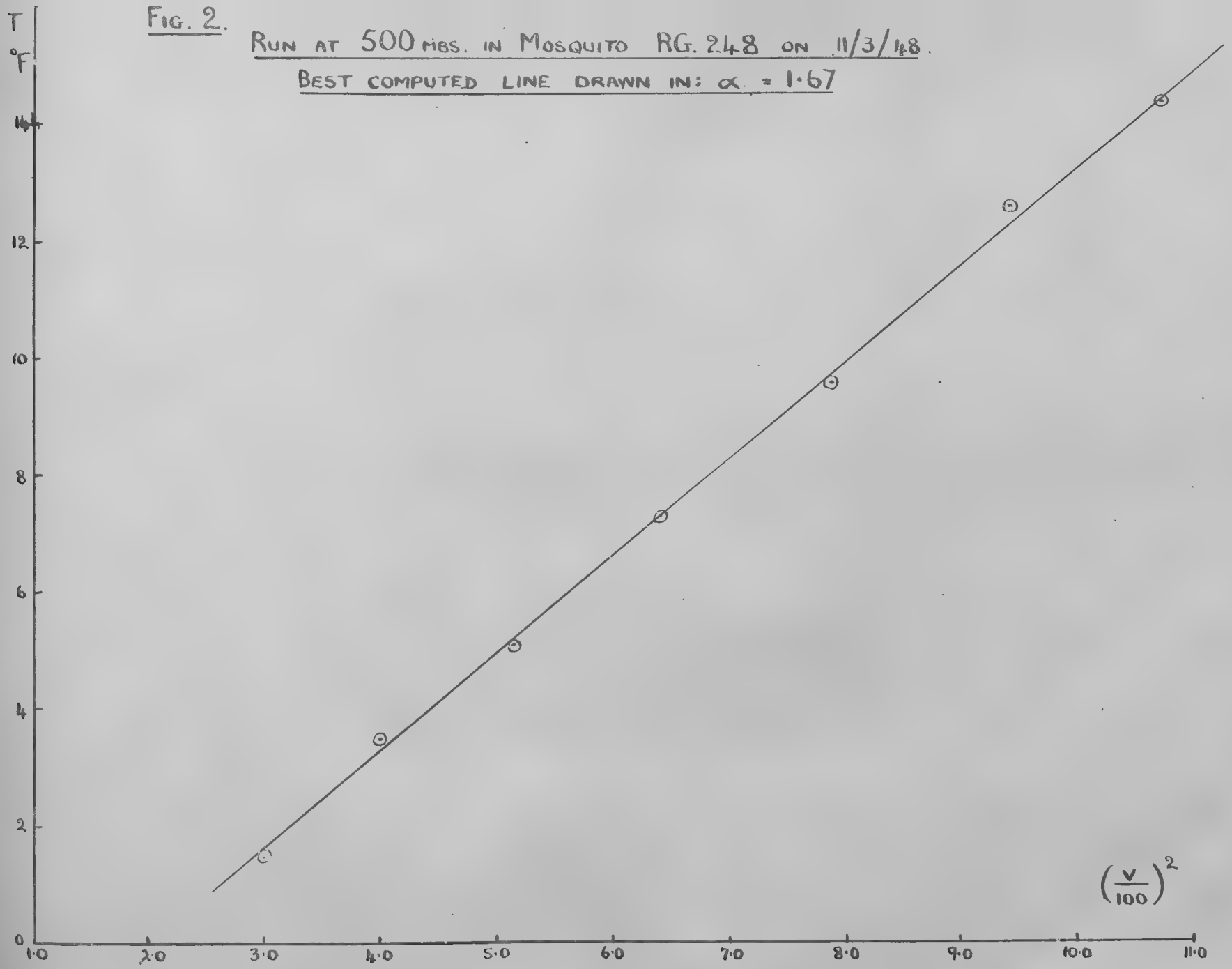


FIG. 3.

RUN AT 700 MBS. IN MOSQUITO RG 248 ON 11/3/48

BEST COMPUTED LINE DRAWN IN: $\alpha = 1.76$

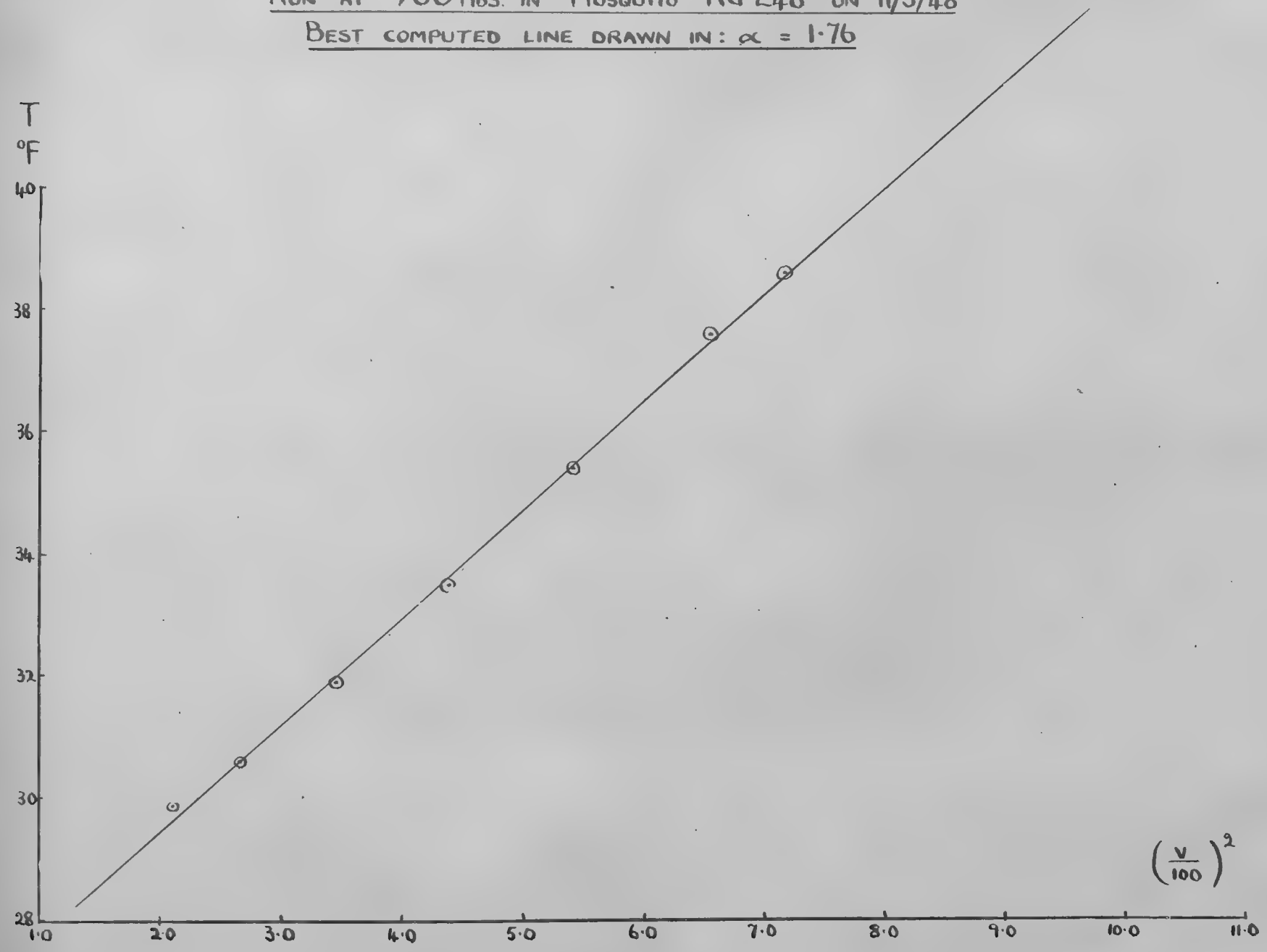


FIG. 4.

RUN AT 910 MBS. IN MOSQUITO RG. 248 ON 12/3/48.

BEST COMPUTED LINE DRAWN IN: α = 1·78

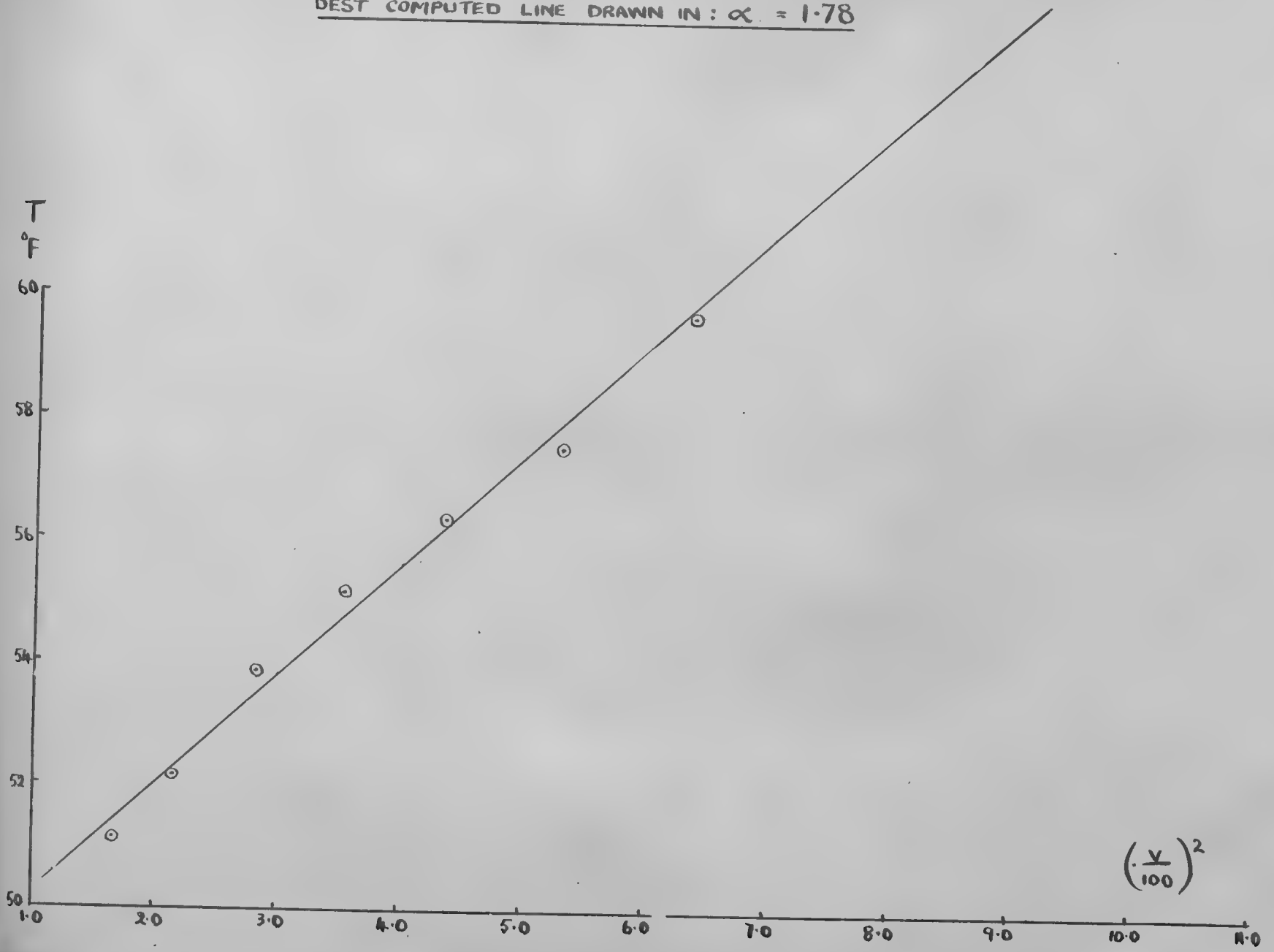
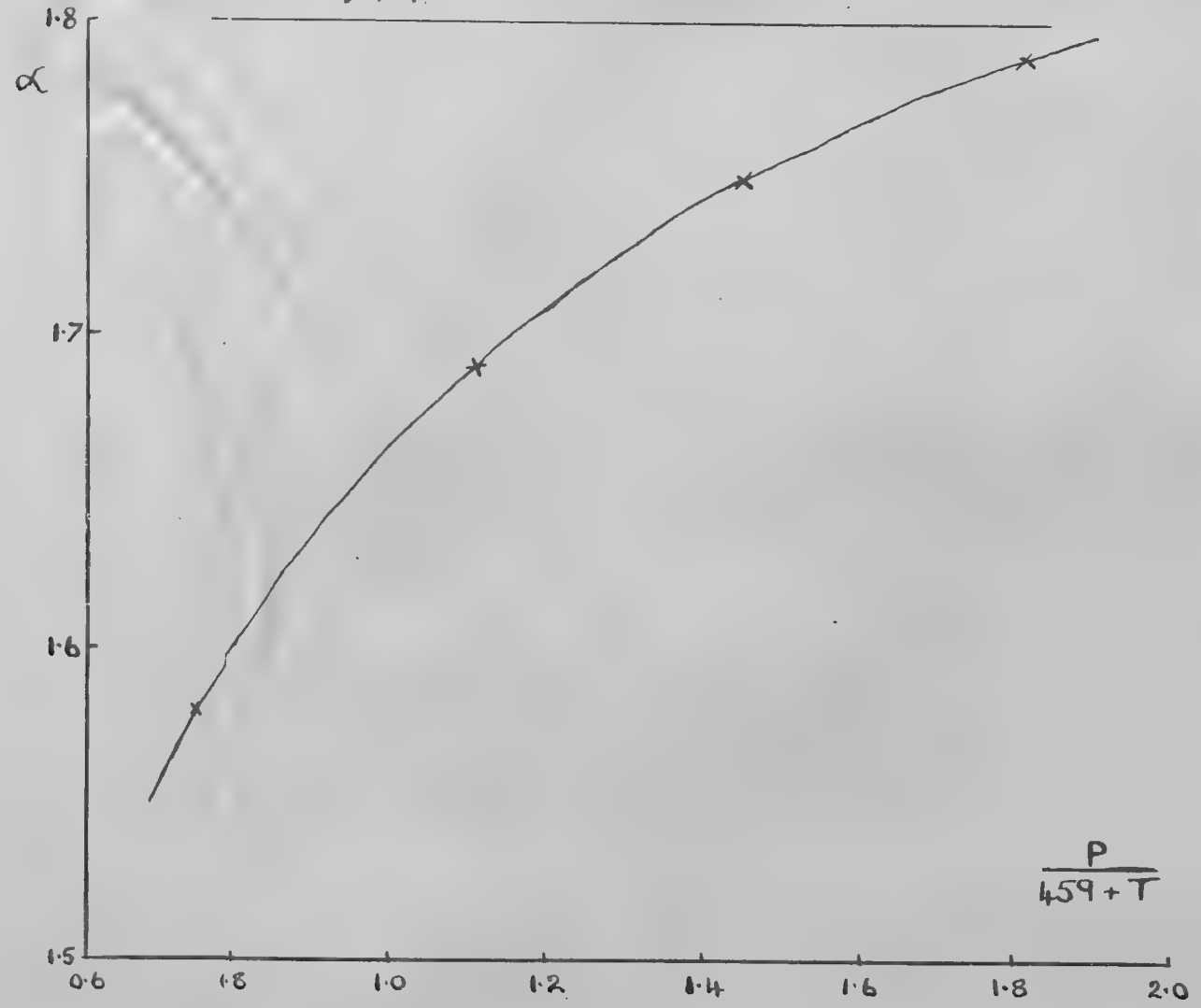


FIG. 5.

MEAN 'α' FOR M.O. RESISTANCE THERMOMETER MOUNTED ON MOSQUITO AIRCRAFT, PLOTTED AGAINST $\frac{P}{459+T}$ (PROPORTIONAL TO AIR DENSITY).



contents must not be quoted.

ATI 112130

M.R.P. No. 406

S.C. 1/7

NOV 17 1952

AIR MINISTRY

29th May, 1948

METEOROLOGICAL RESEARCH COMMITTEE

Errors in Winds measured with GL III Radar Equipment

by

J.K.Bannon

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

Introduction and Summary

M.R.P. 387 gives estimates of the errors in making observations on a moving target, (a balloon), with a GL III radar.

The connection between the basic errors of the radar and the errors in the computed wind is, of course, merely a matter of geometry, and the purpose of the present note is to show the order of magnitude of the errors in computed wind which may result from the radar errors mentioned above. To do this the root mean square error in the radar observations of range, elevation and bearing have been deduced from M.R.P. 387 ; these quantities θ_r, θ_E and θ_B are given in section 1 below.

The three quantities θ_r , θ_E and θ_B are the fundamental casual errors of observation. At any observation they lead to errors in the computed plan position of the balloon and also to errors in the computed height. The errors in the computed plan position have much the greatest effect on the wind computed as the horizontal distance moved between two consecutive positions. The errors in computed height affect the computed wind velocity only when the wind itself varies with height and are, in fact, negligible. Formulae have been deduced showing the root mean square errors in the wind as a vector quantity, in wind direction and in wind speed, all as functions of the fundamental errors θ_r , θ_E and θ_B .

Regarding the radar set as a scientific instrument it is natural to consider the errors as functions of the distance from the set. The forecaster who uses computed winds, however, considers them only as functions of height. Height is related to distance from the radar set if the mean wind up to the height of observation and also the rate of ascent of the balloon are known. Accordingly the root mean square errors in the wind have also been computed as a function of height.

The rate of ascent of the balloon and target has been taken as 1,200 ft per minute, the normal rate of ascent for determining winds by radar in this country. For small variations from this rate the root mean square of the vector error as a function of slant range and elevation remains unaltered, of course ; as a function of height it changes approximately in proportion, i.e. for a 10% increase in rate of ascent the vector error is decreased approximately 10%.

The period between observations has been taken as one minute. If a longer period, say x minutes, is taken, then the errors in determining the mean wind in that layer will be 1/x of those tabulated.

1. Errors of observation with GL III Radar

From the mean deviations of the range, azimuth and elevation as given in Table I of M.R.P. 387, the root mean square errors in observing the range, azimuth and elevation of a balloon target are easily calculated, (assuming normal distribution of the casual errors), as given in Table I below. (Approximate values).

Table I

Root mean square errors in range, bearing and elevation

θ_r (range)	θ_B (bearing)	θ_E (elevation)
16 yards	0.1°	0.1°
	= .0017 radians	= .0017 radians

It should be noted that these root mean square errors refer only to the casual errors of observation ; as pointed out in M.R.P. 387 , systematic errors do occur in all the variables but , except in conditions of light winds when the elevation is large, these systematic errors will not affect the calculated wind to any great extent which depends more on differences between consecutive observations.

It must also be emphasised that the performance of a radar set for wind finding may vary considerably according to the manner in which it is maintained and also according to the skill of the operators. The error parameters above should, therefore, be considered as being of the right order of magnitude rather than as being strictly applicable to any radar observations and the same qualifications will apply to the figures obtained in the following sections for the errors in the derived wind.

2. Effect of Observational Errors on Computed Wind Velocity

Errors in th measurement of the average wind through a shallow layer will arise in two ways :-

(a) through errors in the observed plan positions of the two adjacent points of the path of the balloon from which the horizontal wind velocity is calculated.

(b) through errors in the observed height of the layer.

It is convenient to treat these errors separately and it will be seen that those arising from (b) are usually very small and quite negligible.

3. Plan Errors

Let A and B in Figure I be the plan of two points of a balloon ascent and A^1, B^1 their observed positions.

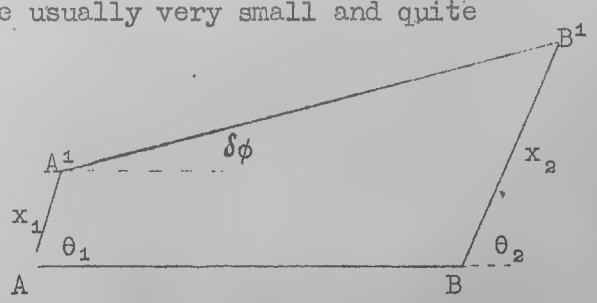


Fig.1

Let $AA^1 = x_1$, $BB^1 = x_2$ and let AA^1 and BB^1 make angles θ_1 and θ_2, respectively, with AB. x_1 and x_2 are assumed small compared with AB.

Also let T = time interval between A and B and V be the true wind speed from A to B.

/Then

Then the error in measuring the direction of the wind is $\delta\phi$, given by

$$TV\,\delta\phi = x_2 \sin\theta_2 - x_1 \sin\theta_1$$

and the error in measuring the speed of the wind is δV , given by

$$T\,\delta V = x_2 \cos\theta_2 - x_1 \cos\theta_1$$

Since the observations made at A and B are independent and the sources of errors of these observations will alter but slowly with height, it is easily seen that if θ_ϕ^2 and θ_v^2 are the mean square errors of direction and wind speed respectively, the following equations hold :-

$$\left.\begin{aligned} T^2V^2\theta_\phi^2 &= 2 \times \text{Mean square error of fix at right angles to the wind.} \\ T^2\theta_v^2 &= 2 \times \text{Mean square error of fix in the direction of the wind.} \end{aligned}\right\} \quad \ldots\ldots (1)$$

Let O in Figure 2 represent the radar and A the horizontal projection of the balloon at a specified time when the wind velocity makes an angle α with OA.

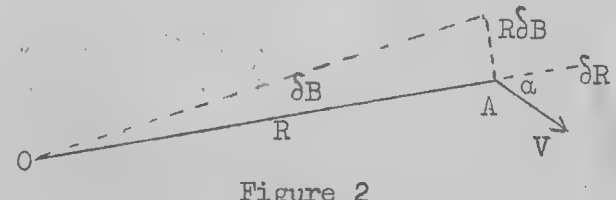


Figure 2

Then if δB is the bearing error and δR the error in observing the horizontal range, R , of A, the square of the error of fix at right angles to V is

$$= \delta R^2 \sin^2\alpha + R^2\,\delta B^2 \cos^2\alpha$$

and the square error of fix along the direction of V

$$= \delta R^2 \cos^2\alpha + R^2\,\delta B^2 \sin^2\alpha$$

and hence, since there is normally little change in α between consecutive observations, we have from Eq.(1) :-

$$\left.\begin{aligned} T^2V^2\theta_\phi^2 &= 2\,(\,\theta_R^2 \sin^2\alpha + R^2\,\theta_B^2 \cos^2\alpha\,) \\ T^2\theta_v^2 &= 2\,(\,\theta_R^2 \cos^2\alpha + R^2\,\theta_B^2 \sin^2\alpha\,) \end{aligned}\right\} \quad \ldots\ldots (2)$$

where θ_R = root mean square error of R.

Since $R = r \cos E$

it is easily seen that

$$\theta_R^2 = \theta_r^2 \cos^2E + r^2 \sin^2E\,\theta_E^2 \quad \ldots\ldots (3)$$

4. Vector Error in Wind - Plan Errors

If OA represents the velocity V at a certain point, and OA^1 the measured velocity, where $OA^1 = V^1$ and the angle $A^1OA = \delta\phi$, then

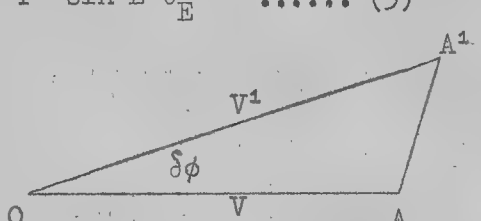


Figure 3

$$AA^{1^2} = V^2 + V^{1^2} - 2VV^1\,(\,1 - \delta\phi^2/2\,)$$

neglecting $\delta\phi^4$ etc.

$$= \delta V^2 + V^2\,\delta\phi^2$$

where $\delta V = V^1 - V$.

/Thus

Thus the vector error (represented by AA^1 in the above case), will have a variance or mean square error given by :-

$$\theta^2 = \theta_v^2 + V^2 \theta_\phi^2 \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

Hence from Equation (2),

$$\theta^2 = 2 (\theta_R^2 + R^2 \theta_B^2) \times (0.0296)^2 \text{ kts}^2$$

when R and θ_R are in yards and T = 1 minute.

$$= (0.0418)^2 (\theta_R^2 + R^2 \theta_B^2)$$

Hence from Eq.(3)

$$\theta^2 = (0.0418)^2 \left\{ (\theta_r \cos E)^2 + r^2 \cos^2 E \, \theta_B^2 + r^2 \sin^2 E \, \theta_E^2 \right\}$$

$$= (0.67 \cos E)^2 + (7.12 \times 10^{-5} r)^2 \quad \ldots\ldots\ldots \quad (5)$$

from Table I.

If the balloon ascends at 1,200 ft per minute,

$$\tan E = 11.83 / \bar{V}$$

where $\bar{V}$ = magnitude of the mean vector wind up to the height in question, in knots,

and we can write

$$\theta^2 = \frac{(0.67)^2}{1 + (11.85/\bar{V})^2} + (7.12 \times 10^{-5} h)^2 \times \left\{ 1 + (8.43 \times 10^{-2} \bar{V})^2 \right\} \ldots (6)$$

where h = height in question, in yards.

Table II has been computed from Equation (5) and gives the root mean square of the vector error in wind for various elevations and ranges up to 70,000 yds. (To the nearest knot).

Table III has been computed from Equation (6) and gives the root mean square of the vector error in wind for various heights and mean wind velocities. Figures to the right of and below the line in the table refer to slant ranges from the radar set of more than 70,000 yards. This Table will be most convenient for use when the wind observation is tabulated against height or pressure as, for example, in the Daily Weather Report ; the mean vector wind can usually be determined roughly by inspection.

Table II

Root Mean Square Vector Error as a Function of Slant Range and Elevation

to the nearest Knot

	Slant range in thousands of yards							
	5	10	20	30	40	50	60	70
All elevations	Less than 1	1	2	2	3	4	4	5

Note that, to the accuracy of 1 knot, the variation of E does not affect the result.

Table III

Root Mean Square Vector Error as a Function of Height and Mean Wind

to the nearest Knot

Mean wind (kts)	Height in thousands of feet: 3	10	20	30	40	50	60
10	Less than 1 knot	1	1	1	1	2	2
20		1	1	2	2	2	3
30		1	1	2	3	3	4
40		1	2	3	3	4	5
50		1	2	3	4	5	6
60		1	3	4	5	6	
70		2	3	4	6	7	
80		2	3	5	7		
90		2	4	5	7		
100		2	4	6			

5. Height Errors

If θ_h is the root mean square error in measuring a height, then the root mean square error in measuring the mean height of a layer will be

$$\frac{\sqrt{2}}{2}\,\theta_h$$

assuming that θ_h varies but little through the layer.

The root mean square errors in direction $(\theta_{\phi h})$ and speed (θ_{vh}) due to errors in height will be given by :-

$$\theta_{\phi h} = \frac{\sqrt{2}}{2}\,\frac{\partial \phi}{\partial z}\,\theta_h$$

$$\theta_{vh} = \frac{\sqrt{2}}{2}\,\frac{\partial V}{\partial z}\,\theta_h$$

where z is height measured upwards and ϕ is the wind direction, and V the wind speed.

Since $h = r \sin E$

then $\theta_h^2 = \theta_r^2 \sin^2 E + r^2 \cos^2 E\, \theta_E^2$

Thus θ_h will attain large values only when $r \cos E$ is large, and, in fact, the important cases are covered by :-

$$\theta_h \doteqdot r \cos E\, \theta_E$$

/Hence

Hence

$$\theta_{\phi h} = \frac{\sqrt{2}}{2} \frac{\partial \phi}{\partial z} r \cos E \, \theta_E$$

$$\theta_{vh} = \frac{\sqrt{2}}{2} \frac{\partial V}{\partial z} r \cos E \, \theta_E$$

The mean square of the vector error in wind due to height errors alone will thus be :-

$$\theta_h^2 = \theta_{vh}^2 + V^2 \theta_{\phi h}^2$$

$$= \frac{1}{2} \left\{ \left(\frac{\partial V}{\partial z}\right)^2 + V^2 \left(\frac{\partial \phi}{\partial z}\right)^2 \right\} r^2 \cos^2 E \, \theta_E^2 \quad \ldots\ldots (7)$$

Except in the first 2,000 ft or so of the atmosphere, $\partial\phi/\partial z$ seldom exceeds 10° per 1,000 ft and $\partial V/\partial z$, 10 kts per 1,000 ft, and their respective values are usually much less.

If we take $\partial\phi/\partial z = 10^\circ/1{,}000$ ft $= 5.24 \times 10^{-4}$ radians/yard

$\partial V/\partial z = 10$ kt/1,000 ft $= 3 \times 10^{-2}$ kts/yard

then the second term in Eq.(7) above will be of importance for $V \geq 30$ kts. With $V = 60$ kts and at extreme range (70,000 yards) and with low elevation

$$\theta_h = 3.5 \text{ kts.}$$

From Table II it is seen that this would increase the total root mean square of the vector error of the wind determination from 5 kts to 6 kts, (i.e. $\sqrt{(5^2 + 3.5^2)}$). In fact, in almost all cases the height error affects the root mean square of the vector error in the wind measurement by much less than 1 knot and is thus negligible.

6. Error in Wind Direction

From Eq.(2) it is seen that the plan errors in wind direction are determined by

$$\theta_\phi^2 = \frac{2}{T^2V^2}(\theta_R^2 \sin^2\alpha + R^2\theta_B^2\cos^2\alpha)$$

$$= \frac{2}{T^2V^2}\left\{(\theta_r^2\cos^2 E + r^2\sin^2 E \, \theta_E^2)\sin^2\alpha + r^2\cos^2 E \, \theta_B^2\cos^2\alpha \right\}$$

and from Section 5 we know this to be the main error and that errors due to errors in the measured height of the balloon are in almost all cases negligible.

For a large number of ascents in this country the angle α is small. A rough calculation made at Shoeburyness from the results of radar wind finding ascents there showed that the mean value of α, regardless of sign, was 10°, so that in 90% cases, $|\alpha|$ will most probably be less than 20°. (If the distribution of α is normal, 90% cases will be covered by $|\alpha| < 17^\circ$).

The term in θ_B will thus be by far the most important in a large number of cases and to get an idea of the order of magnitude of θ_ϕ we may take

$$\theta_\phi = \frac{\sqrt{2}}{TV} r \cos E \, \theta_B \qquad \text{(this will be a good approximation for } r > 5{,}000 \text{ yds and } \alpha < 20^\circ \text{)}$$

$$\text{or} = 4.18 \times 10^{-3} \frac{r \cos E}{V} \text{ degrees} \quad \ldots\ldots\ldots\ldots (8)$$

/when

when r is in yards and V in knots.

This may be expressed in terms of height and mean velocity, assuming a rate of ascent of 1,200 ft per minute) thus :-

$$\theta_\phi = 35.3 \times 10^{-5} h \frac{\bar{V}}{V} \text{ degrees} \quad \ldots\ldots\ldots (9)$$

where h is in yards and V and $\bar{V}$ in knots.

From these two equations, (8) and (9), θ_ϕ may be calculated either in terms of the slant range and elevation or, (applicable when the wind observations are tabulated according to height), in terms of height and mean velocity up to that height.

Table IV gives values of $4.18 \times 10^{-3} r/V$ for varying r and V and thus gives , (see Eq.(8)), a useful upper limit to θ_ϕ which will also be quite a good approximation (within 13%) when E is less than 30°.

Table V, calculated from Eq.(9), gives values of θ_ϕ for various values of h and $\bar{V}/V$; $\bar{V}/V$ can be estimated roughly by inspection in most cases.

Table IV

Upper Limit to Wind Direction Root Mean Square Error in terms of r and V in Degrees (α small)

V in knots	Range in thousands of yards 5	10	20	30	40	50	60	70
10	2	4	8	13				
20	1	2	4	6	8			
40	1	1	2	3	4	5	6	7
60		1	1	2	3	3	4	5
80			1	2	2	3	3	4
100			1	1	2	2	3	3

(Low velocities and long ranges are unlikely to occur together and so part of the upper right hand side of the table has been omitted).

Table V

Root Mean Square Error in Wind Direction in terms of Height and $\bar{V}/V$ in Degrees (α small)

$\bar{V}/V$	Height in thousands of feet 3	10	20	30	40	50	60
1.2	Less than .5	1	3	4	6	7	9
1.0		1	2	4	5	6	7
.8		1	2	3	4	5	6
.6		1	1	2	3	3	4
.4		1	1	1	2	2	4

When α is not small, the approximations of Eq.(8) and (9) are not valid but the angular errors are in fact less than for the case α small except when E approaches $\pi/2$. Tables IV and V thus give upper limits to the errors in wind direction except for high elevations.

7. Error in Wind Speed

From Eq. (2) it is seen that the plan errors in wind speed are determined by

$$\theta_v^2 = \frac{2}{T^2}\left(\theta_R^2 \cos^2\alpha + R^2\theta_B^2\sin^2\alpha\right)$$

$$= \frac{2}{T^2}\left\{(16^2\cos^2 E + r^2\theta_E^2\sin^2 E)\cos^2\alpha + r^2\cos^2 E \sin^2\alpha\, \theta_B^2\right\}$$

$$= (4.18 \times 10^{-2})^2\left\{(16^2\cos^2 E + r^2\theta_E^2\sin^2 E)\cos^2\alpha + r^2\cos^2 E \sin^2\alpha\, \theta_B^2\right\} \text{ knots}$$

if r is in yards, and of course taking T as one minute.

As seen from Section 5, the plan error will be the only important error, that due to height inaccuracies being negligible.

It is not possible to treat the cases where α is small in the same way as for wind direction as the various terms in the above expression for θ_v are often of the same order of magnitude. The root mean square of the vector error of wind measurement may, however, be used as an upper limit to θ_v.

8. Conclusion

As stated in Section 1, the basic errors of the radar instrument vary with the skill of the operators and the state of efficiency of the machine itself. The figures quoted in the preceeding sections must therefore be interpreted only as giving the order of magnitude of the errors in wind determined by the usual radar method and the manner in which these errors vary with range of the target from the radar set etc.

Table III is probably the most instructive; from it it is seen that the order of the root mean square of the vector error in determining the mean wind in a layer approximately 1,200 ft thick is not more than 2 knot up to a height of 10,000 ft, seldom exceeds 3 knots at 20,000 ft and is never more than 5 knots.

M.R.P. 409
S.C. II/6

...LY
...not be quoted.

5th June, 1948.

Air Ministry

Unpublished
Contents must not be quoted.

5th June, 1948.

Air Ministry

Meteorological Research Committee

Upper Winds over the World

by

[illegible]

(Note: This memorandum describes the present position of work on item II 15 b of M.R. .300 and has been prepared following minute 7 of the 61st meeting of the Synoptic and Forecast Sub-Committee)

Early in 1945 it was realised that as soon as Civil Aviation began to redevelop and particularly as flight was contemplated to take place eventually in the Stratosphere a demand for winds up to 40,000 feet would become insistent which could not be directly met by the scattered observations which were then available.

2. It appeared that the only way in which the problem could be dealt with was by generalising from the known facts combined with a considerable amount of scientific interpolation.

0494736

3. If a seasonal wind rose could be expressed by two parameters, the vector mean wind and the standard deviation it would then be possible to plot maps of these two parameters for points where they could be derived from measurements and then by interpolation and analogy the intervening gaps could be filled.

4. The first step was to discover whether the wind distributions given by wind roses conformed to a normal law. This was discussed in a paper printed in the Journal of the Royal Meteorological Society. +

5. Having obtained reasonable agreement the investigation was pursued by constructing maps of the normal height of the surfaces represented by 700 mb., 500 mb., 300 mb., 200 mb. and 100 mb. from which by the geostrophic balance the vector mean winds could be determined in extra tropical latitudes. For the inter-tropical regions streamlines were drawn by the use of all data available.

6. The plotting of the standard vector deviations was effected by calculation wherever there were data by comparison with the standard vector deviation of surface winds over the sea and by the use of certain empirical rules the most useful of which is that in general the standard vector deviation of wind over any specified point at any height in the troposphere is inversely proportional to the air density.

7. The description of the investigation and the charts (except those for 100 mb.) have been manifolded as S.D.T.M.118 and consists of six parts:

Part I Frequency distributions of winds at a point in the free air (Published also in Q.J.R. Met.S.)

Part II The use of pilot balloon observations in the construction of wind roses.

Part III Seasonal charts of the mean height of the tropopause (Manifolded as M.R. 2334)

36 42

Part IV Seasonal charts of mean heights of isobaric surfaces.

Part V Seasonal charts of standard vector deviation of wind at standard pressure levels.

Part VI Seasonal charts of streamlines and of isopleths of vector mean speeds at standard pressure levels in the tropics.

+ Brunt D., Durst C.S. and Gutruthers N. Upper Winds over the World Part I Q.J.R.Met.S. Jan. 1946 72 pp 55 to 73.

/The

Bailey
Page 9

8. The preparation of the charts has involved much discussion between members of the Climatological Branch and the Investigations Branch, and they may be considered the result of the cogitations and revisions of 5 or 6 independent workers. With Parts IV, V and VI are included written descriptions of how the data have been treated and what assumptions have been made in the drawing of the charts, in order that others may criticise and improve on the depiction wherever possible.

9. The charts can only be considered to be tentative first drafts of the final maps which may be drawn when the network of radiowinds has been completed over the Empire. Co-operation is being sought from the dominions and from foreign countries, and to this end copies of the charts for the 500 mb. and 700 mb. levels were sent to each dominion, some colonies and various foreign countries with an invitation to supply further data and to make criticisms of the distributions shown for their relevant parts of the world. In response to this the Meteorologie Nationale (France) has sent complete wind summaries for Madagascar, the United States have offered to supply all the wind data and temperature data collected by them during the war (these cover many parts of the world outside the United States), Australia has sent valuable criticisms, South Africa have contributed normal heights of 500 mb. and 700 mb. levels and HongKong and New Zealand have also given data. India has recently forwarded their revision of the 500 and 700 mb. contours and standard vector deviations.

10. It is hoped that [illegible] concerning the maps may be received during the course of the next few years.

11. A limited number of copies of Part II to IV including the full set of charts will be available for inspection at the meeting of the M.R.C. on 24th June, 1948.

36 43

NOV 17 1952

13th August, 1948.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

The Accuracy of MK. II Radio-sonde Observations

by D.N. Harrison

AFI-17286

Summary

Soundings with two radio-sondes carried by the same balloon enable an estimate to be made of (a) random errors of observation and (b) instrumental errors which are systematic for a given sounding but random as between one sounding and another. In the case of instrumental errors good agreement is found with earlier data. It is concluded that the "probable errors" are as follows:*

	P		T	U
	Low levels mb	At 200 mb mb	°F	%
Errors of observation	2	1	0.3	1
Instrumental errors	1-2	5	0.7	3

1. Introduction

With the purpose of assessing the instrumental errors of the MK. II radio-sonde a series of ascents was made in the summer of 1947 at Penzance and Larkhill in which two transmitters were carried by the same balloon. The transmitters were tuned to different frequencies, and observations were made with independent sets of ground equipment but with a common time scale. A pair of simultaneous values of pressure, temperature and humidity gives the difference between the total (instrumental plus observational) error for the two transmitters.

In the Penzance ascents (Nos. 1, 2 and 3) one transmitter was suspended immediately beneath the other at a distance of 6 m (the length of the aerial). In the Larkhill ascents the two transmitters were held at the same level about 1.2 m apart by means of spreaders. The usual control readings were made, skilled observers were employed and during the ascent every signal was recorded. The rate of ascent (surface to 400 mb) was about 5.6 - 6.1 m/s, except at ascent No.4 (6.6 m/s) and No.5 (4.6 m/s).

The method of computation was as follows. Frequency readings to the nearest $\frac{1}{2}$ c/s were obtained for each exact minute by linear interpolation between consecutive recorded points, without smoothing. This is the nearest approach that could be made to simultaneity of observation. P, T and U were computed in the usual way, about 250 pairs of values of each being obtained, and the differences between the values for the two transmitters were tabulated. Since these differences originate from frequency errors, they depend on the sensitivity of the meteorological units. The sensitivity of the T unit is about 1 c/s per 1°F - slightly less at the high temperature end and slightly more at the low temperature end. The sensitivity of the U unit is very variable, but the mean value is about 1 c/s per 1% relative humidity. The mean T and U differences may therefore be regarded as frequency differences. In the case of the P unit the sensitivity increases with falling pressure by a factor of more than 2 over the range; the P differences were therefore multiplied by the sensitivity and are expressed in cycles per second. These differences were plotted against the mean of the two pressure readings. A specimen (ascent No.4) is shown in the figure.

Such a comparison provides no information about those errors which are the same for the two transmitters, such as temperature errors due to lag or to solar radiation, but it is possible to separate errors which are random between one reading and another from those which, though different for different transmitters, are systematic, or only change slowly, for a given transmitter. This is done in sections 2 and 3.

/It

* The "90% error", or value which is only exceeded on 10% of occasions, is 2.4 times the p.e.

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILE

It will be assumed that the normal statistical rules are obeyed, so that the "mean deviation" of the difference between two observations is $\sqrt{2}$ times the m.-d. of a single observation and that the "probable error" is 5/6 of the m.-d.

2. Random errors

We start with the difference between a pair of simultaneous readings. The change in this value between consecutive readings is due to random errors, and the mean change without regard to sign (c) is a measure of the mean deviation (d) of a single observation as recorded (not interpolated). This latter is the quantity which we require to deduce. It will be assumed (see Appendix) that d=0.6c and that the "probable error" =0.5c. The values of c are given in Table I. The following points are to be noted:-

(i) Pressure. The mean value is practically the same at all levels, the mean of all being 0.43 c/s. This is compounded of normally distributed errors and errors due to the limitation of reading to the nearest $\frac{1}{2}$ c/s and $\frac{1}{20}$ minute which are uniformly distributed and are not negligible, so that the final distribution cannot be normal. We may say, however, that the m.d. of a single observation is 0.26 c/s and the p.e. 0.22 c/s (the second figures are not significant). 0.22 c/s corresponds to 1.9 mb at 1000 mb and to 0.8 mb from 200 mb onward.

(ii) Temperature. The mean value does not show any significant systematic change during an ascent, and the mean of all is 0.55°F or 0.55 c/s. The reason why this is larger than for pressure is that owing to temperature fluctuations of short period (i.e. comparable with the interval between readings) an interpolated value is subject to a small error, which is not the same for the two transmitters owing to the non-simultaneity of the observations; whereas this is not present in the pressure readings, since the pressure changes smoothly. The m.d. of a single observation is 0.33°F, and the p.e. is 0.27°F.

(iii) Relative humidity. The errors are much larger than those of T in the lower levels, owing to the larger fluctuations and perhaps to a difference in lag between specimens of gold beater's skin. The mean difference of 2.1% corresponds to a mean deviation for a single observation of 1.3% and a p.e. of 1.1%. In the upper levels, where the lag is very great and the readings practically constant, the mean, expressed as a frequency, is about the same as for P and T.

3. Errors systematic for a given radio-sonde.

The algebraic mean value of the difference over a number of consecutive readings is a measure of these errors. These values are shown in Table II, the layers being chosen so as to contain about three to five readings. They are generally systematic over large portions of an ascent, and indicate the difference to be expected between two transmitters arising from (a) errors of calibration, (b) changes in the instruments since calibration, (c) errors in control readings (these should be negligible), (d) varying battery E.M.F., (e) errors in P due to the method of correcting for temperature, (f) errors in U due to the effect of temperature on the armature.

Table III shows the mean of ascents 2, 3, 4, 5, and 7 without regard to sign, for each layer up to 200 mb, the pressure values being expressed in c/s and in mb. These form a nearly homogeneous series, and, although the number of ascents is too small to give a truly representative mean*, it will be of interest to compare

/them

* On ascent No.3 one transmitter gave pressures so low that in routine practice the sounding would have been classed as anomalous and not reported. There is nothing in the records to indicate the origin of the very low pressure readings, which are responsible for the large positive differences in Table II. The other transmitter had an intermittent fault between 750 mb. and 500 mb., but there is no reason to doubt the accuracy of the subsequent readings. If this ascent were excluded, the pressure differences of Table III would be reduced by half in the upper levels.

them with earlier data. It will be assumed that the mean difference between two instruments is $\sqrt{2}$ times the m.d. of a single instrument.

(i) Pressure. The mean differences imply a mean deviation for a single radio-sonde increasing from a little over 1 mb at low levels to 6 mb at 250-200 mb (p.e. 1 mb - 5 mb). From comparisons of radio-sonde (MK.I) with radar observations in 1944 (MRP 203), mean deviations of 4 mb at low levels and 6 mb between 6 Km and 19 Km were found (p.e. 3mb - 5mb): these figures are comparable, since casual errors were reduced by smoothing and the deviations were taken from the mean (systematic) error at each height; the mean deviations may still, however, contain a contribution due to radar height errors, so that the true p.e. is probably less. E.G. Dymond,[1] from trials with two P units on one transmitter, found the p.e. of a single determination to be 1.5 mb at 1 Km, increasing to 4.5 mb at 13 Km (170 mb); these figures represent errors due to the application of temperature corrections and errors of calibration, and are therefore again comparable with Table III.

(ii) Temperature. Between 900 mb and 250 mb the mean difference is practically constant at about 1.1°F, which implies a m.d. for a single radio-sonde of 0.8°F and a p.e. of 0.7°F. The ascents (Table II) are fairly consistent, so this is probably a satisfactory mean value, and it is in good general agreement with figures given by Dymond (loc.cit.p.661) for the average difference between two T units (one in the U position) carried by the same transmitter. Dymond's results, which were based on six flights, are reproduced in Table IV; the mean difference from 800 mb to 180 mb is 1.1°F.

(iii) Relative humidity. Owing to the inhomogeneity of the data, no importance can be attached to the variations between one level and another. There is no evidence of a systematic change. The mean difference of U of 5% implies a m.d. of about 4% and a p.e. of 3% for a single radio-sonde, but much larger errors are liable to occur. At times the difference between two instruments follows rather closely the changes of humidity, which suggests an error of calibration or a difference in speed of response, but this is by no means consistent. Dymond (MRP 206) found a mean difference of 4% in relative humidity between two U units on the same transmitter (p.e. 2%) after correcting for a systematic difference due to one U unit being in the T position.

4. Conclusion

Observational errors (timing, setting, reading and plotting) account for a p.e. of 0.22 c/s in a single determination of P, T or U. For P this corresponds to 2 mb at the surface and less than 1 mb at 200 mb. In the case of T, and still more in U, the observational error is increased by short-period fluctuations which introduce an error of sampling into the temperature measured at a given instant. The total p.e. of a single determination of temperature is 0.3°F at all levels up to 200 mb. For U the p.e. is 2% at low levels, falling to 0.4% at high levels. These are probably limiting values below which the errors cannot be reduced with existing equipment.

Instrumental errors in the radio-sonde account for a p.e. in P of 1-2mb at low levels and 4-5 mb at 200 mb, and of 0.7°F in T and 2-3% in U at all levels up to 200 mb. These errors are largely systematic for a sounding, but random between one sounding and another. They are the type of errors which affect, for instance, horizontal temperature gradients. There is a possibility of reducing these errors by improvements in the meteorological elements. The data have been examined with a view to discovering a relationship between transmitter errors and control corrections or P unit correction factors, but no such relationship seems to exist.

[1] "The Kew Radio-sonde", Proc. Phys. Soc. LIX, p.663 (1947). All Dymond's experiments were done with MK.I transmitters, but they are probably equally representative of the MK.II as far as errors of the type now considered are concerned. (Systematic errors may be different). Two units on the same transmitter would be expected to show slightly smaller differences than two separate transmitters.

APPENDIX

The error of an interpolated value

If p and q are two observations, the interpolated value at a fraction x of the interval between them is

$$r = (1-x)\,p + xq$$

If s is the standard deviation of p and q, the variance of r for a given value of x is

$$(1-x)^2 s^2 + x^2 s^2 = (1-2x + 2x^2)\, s^2$$

All values of x from 0 to 1 are equally probable, and therefore the mean variance of r is

$$t^2 = s^2 \int_0^1 (1-2x + 2x^2)\, dx = \frac{2}{3} s^2$$

Therefore $t = 0.82\ s$

The s.d. of the difference between the readings of two instruments is $\sqrt{2}t$, and the s.d. of the change in this difference between two readings is $u = 2t = 1.64\ s$.

Therefore

$$s = 0.6\ u$$

It is assumed that d and c (section 2) are proportional to s and u respectively.

Table I - Mean change between consecutive readings of difference between two transmitters (random errors)

Ascent No.	1			2			3			4			5			6			7			Mean		
Pressure mb.	P c/s	T °F	U %	P c/s	T °F	U %	P c/s	T °F	U %	P c/s	T °F	U %	P c/s	T °F	U %	P c/s	T °F	U %	P c/s	T °F	U %	P c/s	T °F	U %
Surface - 700	-	.6	2.6	.3	.5	3.3	.4	.4	0.9	.4	.9	1.8	.4	.3	1.7	.4	.7	2.3	.4	.5	2.2	.4	.6	2.1
700 - 400	-	-	1.9	.3	.4	1.4	.4	.1	0.8	.5	.2	0.5	.5	.5	3.4	.3	.5	2.5	.4	.7	2.7	.4	.4	1.9
400 - 250	-	-	1.1	.6	1.1	0.5	.4	.6	0.5	.3	.4	0.8	.7	.4	1.2	-	-	-	.5	.4	-	.5	.6	0.8
250	-	-	-	.2	.9	0.3	.5	.5	0.4	.6	.6	0.5	.5	.6	1.1	-	-	-	.5	.6	0.6	.5	.7	0.6
Mean	-	.6	1.8	.3	.7	1.4	.4	.5	0.5	.5	.5	0.8	.5	.4	2.2	.4	.6	2.5	.5	.6	1.5	.43	.55	1.52

Table II - Mean differences for layers

Ascent No.	1			2			3			4			5			6			7		
Pressure mb.	P c/s	T °F	U %	P c/s	T °F	U %	P c/s	T °F	U %	P c/s	T °F	U %	P c/s	T °F	U %	P c/s	T °F	U %	P c/s	T °F	U %
Surface - 900	-	0	0	-1½	- ½	-3½	0	0	-2½	0	0	- ½	0	+ ½	- 8	0	+ ½	-4½	0	-1½	0
900 - 800	-	0	-1	-1	-1	-	0	0	-1	0	+ ½	- ½	0	+ ½	- 8	0	+ ½	-1	0	-2½	+1
800 - 700	-	-1	-3½	0	0	-1½	-	-	-	+ ½	0	0	0	+1	-12	0	0	- ½	- ½	-3	+8½
700 - 600	-	-1	-2	0	- ½	-2⅓	-	-	-	- ½	- ½	0	+1	+1	0	0	- ½	+5½	-1½	-3	+16
600 - 500	-	+ ½	-4	0	0	-	-	-	-	- ½	0	+ ½	+1½	+1½	- 7	0	-1	+6½	-1½	-3	+16
500 - 400			0	0	+ ½	-	+3	-1	+7½	0	+ ½	+1½	+ ½	+1½	- 2	- ½	-1½	+9½	-1	-3	+15½
400 - 350			+1	+ ½	+ ½	-	+4	-1	+7½	0	+1½	+1	0	0	+ 2	- ½	-1	+15	- ½	-2	-
350 - 300			-2	+1	- ½	-	+4	+ ½	+7½	+ ½	+1½	+2½	- ½	-1	+ 2				- ½	-2	-
300 - 250			-2½	0	-1½	-2	+4½	0	+6	+ ½	+ ½	+4½	-1	-2	+ 2				-1	-2	+17½
250 - 200				0	-2	-3½	+6	0	+7	+2	+ ½	+4½	-1½	-3½	+ 3				-1	-2	+18½
200 - 160				+1	- ½	-3½	+7½	+1	+8	+3	+ ½	+3							-1	-2	+18
160 - 120							+8½	+1	+7	+2½	-	+2							- ½	-2	+17
120 - 100							+9	+1	+6½										0	-2	+16½
100 - 80							+9	+ ½	+6½										- ½	-2	+15
80 - 60							+9½	-1	-												

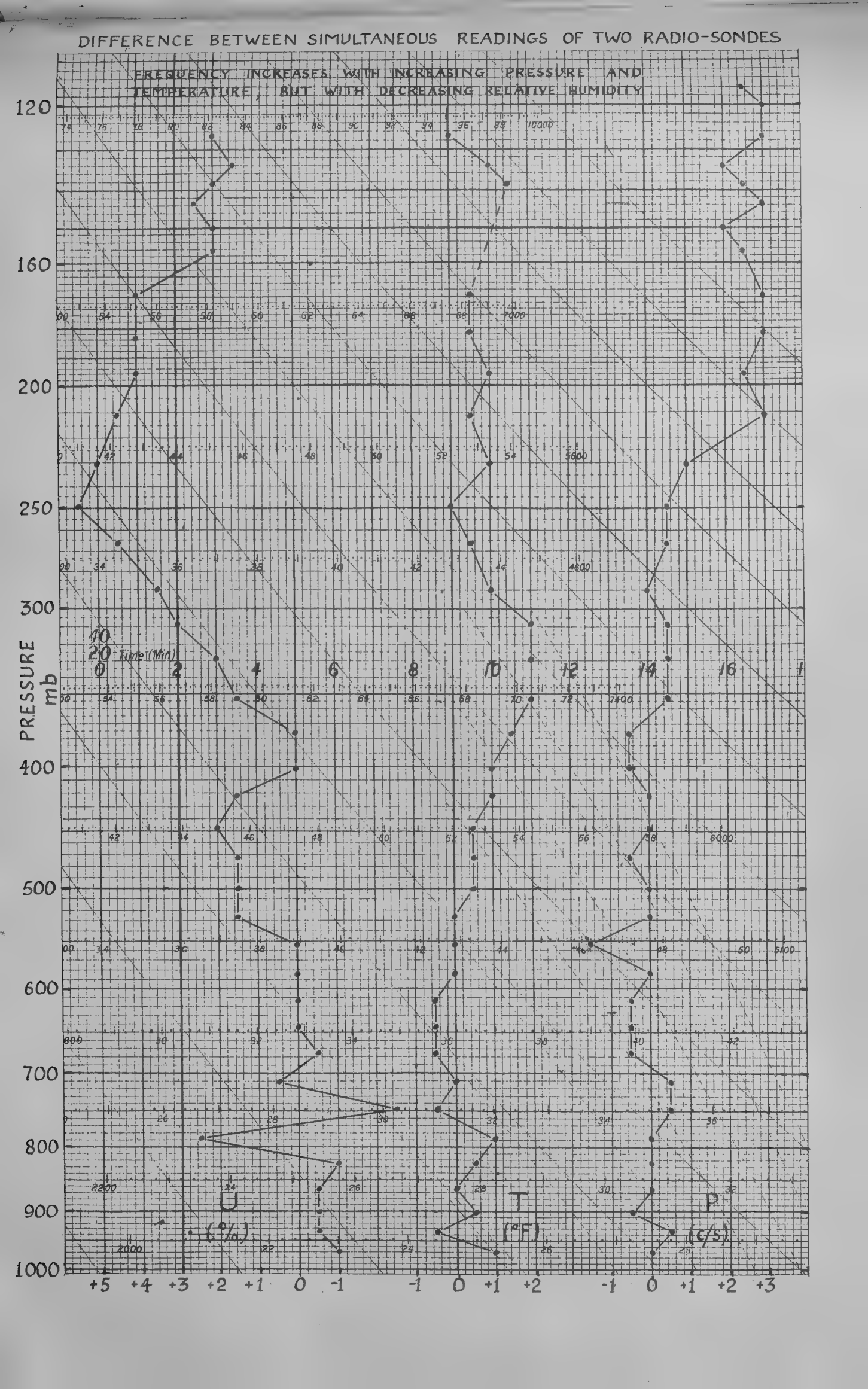
Table III - Mean differences (from Table II, Ascents No.2,3,4,5,7)

Pressure mb.	P c/s	P mb	T °F	U %
Surface - 900	0.3	2	0.5	2.9
900 - 800	0.2	1	0.9	2.6
800 - 700	0.3	2	1.0	5.5
700 - 600	0.7	4	1.3	4.6
600 - 500	0.9	5	1.1	7.8
500 - 400	0.9	4	1.3	6.6
400 - 350	1.0	5	1.0	3.5
350 - 300	1.3	6	1.1	4.0
300 - 250	1.4	6	1.2	6.4
250 - 200	2.1	8	1.6	7.3

Table IV

Dymond

Pressure mb.	T °F
800	0.8
600	0.9
470	1.2
350	1.8
250	1.0
180	1.1


DIFFERENCE BETWEEN SIMULTANEOUS READINGS OF TWO RADIO-SONDES
FREQUENCY INCREASES WITH INCREASING PRESSURE AND TEMPERATURE, BUT WITH DECREASING RELATIVE HUMIDITY
PRESSURE mb
120
160
200
250
300
400
500
600
700
800
900
1000
40
20
Time (Min)
0
2
4
6
8
10
12
14
16
U
(%)
T
(°F)
P
(c/s)
+5 +4 +3 +2 +1 0 -1
-1 0 +1 +2
-1 0 +1 +2 +3

This report is published and is to be treated CONFIDENTIALLY. Its contents must not be quoted.

M.R.P. 424
S.C.III/10
23rd August, 1948

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

AD-52376

EDDY DIFFUSIVITY AND CONDUCTIVITY NEAR THE GROUND AND THE INDIRECT EVALUATION OF NATURAL EVAPORATION

NOV 17 1952

BY

F. PASQUILL

AIR WEATHER SERVICE
TECHNICAL LIBRARY

SUMMARY

A description is given of an experimental study of factors involved in the vertical exchange of water vapour and heat over a clayland pasture under short grass, in fine spring weather. The measurements were carried out over periods of one hour and included the vertical profiles of absolute humidity, wind speed and air temperature up to a height of 2 metres, the soil temperature profile down to 16 inches and incoming and reflected solar radiation. Measurements were also made of the water loss from isolated sections of the surface layers of soil and qualitative and quantitative arguments advanced for the adoption of these measurements as a reasonable approximation to the natural evaporation from the ground. Soil samples were taken to provide data on the thermal properties of the soil. Finally, the net flux of long-wave radiation was computed, by a recently established empirical method, from the present surface layer observations and the data provided by routine upper air soundings.

From the experimental data assembled absolute values of the eddy diffusivity and conductivity are directly obtained and the following features established

(a) In conditions closely approaching neutral equilibrium (small vertical gradient of potential temperature) the eddy diffusivity is identical with the eddy viscosity, the latter being derivable from the explicitly established logarithmic law relating wind profile and surface drag.

(b) With thermal stratification of the surface air layers the eddy diffusivity increases or decreases with the development of instability or stability respectively, and a close approach to a unique relation is demonstrated between the eddy diffusivity parameter $K_v/z^2 \frac{\partial u}{\partial z}$ and the Richardson Number. The magnitude of the variation of eddy diffusivity is such that the influence of atmospheric stability cannot reasonably be neglected, in the interests of simplicity, unless consideration is restricted to heights below ½ metre.

(c) The Rossby and Montgomery, and Holzman, formulae for the effects of atmospheric stability on the wind profile do not provide a convincing explanation of the present results. In unstable conditions a striking agreement is found between the observed eddy diffusivity and the eddy viscosity computed from the empirically founded wind profile law due to Deacon, though in stable conditions a considerable discrepancy appears.

(d) The heat exchange measurements and computed values of the long-wave radiation component are shown to balance satisfactorily in neutral conditions, and the component unaccounted for in other conditions is naturally attributed to turbulent transport. The values so derived for the eddy conductivity are found to be virtually equal to the eddy diffusivity in stable conditions but systematically higher in unstable conditions.

A practical implication of the present results is that short term natural rate of evaporation from a land surface may be evaluated indirectly by the "hydrodynamical method", using appropriate wind profile laws, with satisfactory accuracy, in near-neutral or unstable conditions. The minimum measurements required are absolute humidity at two heights and wind velocity at two or three heights according as the vegetation is short or long. In stable conditions the errors would be considerable unless consideration is restricted to heights below ½ metre. The customary application of "heat balance" considerations, in the ideal conditions of clear sky, leads to a satisfactory specification of evaporation in stable conditions but in unstable conditions a considerable over-estimation seems possible.

I N D E X

	Paragraphs	Page
INTRODUCTION	1 - 2	1
EXPERIMENTAL DETAILS	3 - 11	1
Humidity, temperature and wind profile measurements	4 - 5	1
Measurement of factors required for Heat Balance Considerations	6 - 9	2 - 3
Determination of Water Loss from the Ground	10 - 11	3 - 4
INTERPRETATION OF THE EVAPORATION MEASUREMENTS	12 - 15	4 - 5
REDUCTION OF TEMPERATURE, HUMIDITY AND WIND PROFILES	16 - 21	5 - 7
EDDY DIFFUSIVITY AND EVAPORATION IN CONDITIONS OF NEUTRAL EQUILIBRIUM	22 - 27	7 - 10
THE INFLUENCE OF ATMOSPHERIC STABILITY	28 - 37	10 - 14
HEAT BALANCE ANALYSIS AND DERIVATION OF EDDY CONDUCTIVITY	38 - 52	14 - 20
Evaluation of Heat Absorbed in Soil	39 - 41	15 - 16
Estimation of Long-Wave Radiation Term	42	16
Discussion of Heat Exchange Factors and Derivation of Eddy Conductivity	43 - 52	16 - 20
REFERENCES		

EDDY DIFFUSIVITY AND CONDUCTIVITY NEAR THE GROUND AND THE INDIRECT EVALUATION OF NATURAL EVAPORATION

INTRODUCTION

An earlier investigation, described in MRP 362, gave preliminary data on the vertical distribution of water vapour in the first two metres above grassland and its relation to the rate of loss of water from the ground, the latter factor being prescribed only in order of magnitude from measurements of the change in water content of the soil over a period of several days. In brief, it was shown that as a first approximation natural evaporation is consistent with an eddy diffusivity equal to the eddy viscosity computed from the Prandtl wind profile law for an isothermal atmosphere, but the nature of the investigation precluded any critical examination of the effect on diffusion in the atmosphere of the diurnal variation of atmospheric conditions.

2. The experiments described in the present paper represent an extension of the earlier work in the sense that in addition to measuring the vertical gradients of humidity, temperature and wind speed in the first two metres, an attempt was made to prescribe approximately the corresponding vertical flux of water vapour, and on some occasions the vertical flux of heat, over periods of about one hour. The flux of water vapour, in effect the rate of evaporation from the ground, was obtained from measurements of the water loss from samples of the surface layers of the soil which, though necessarily isolated from surrounding and underlying soil, were otherwise exposed naturally. For the estimation of the turbulent flux of heat measurements were made of incoming and reflected solar radiation and of the heat absorbed or extracted from the soil, while the net effect of terrestrial and atmospheric radiation was computed from the temperature and humidity structure of the atmosphere. The combined gradient and flux measurements specify the magnitudes of the eddy diffusion coefficients for water vapour and heat. These are then examined in relation to the factors determining the diffusive action of the atmosphere, and with reference to the practical problem of prescribing natural rate of evaporation in terms of other meteorological factors.

EXPERIMENTAL DETAILS

3. The site employed was the same clayland pasture as in the preliminary investigation. During the present measurements, which extended over the period 10th to 29th March 1948, the ground surface was unusually dry for the time of the year and the vegetation cover correspondingly backward. Bare soil was visible over a substantial proportion of the area and the grass was mainly 2 cms. long with only occasional withered tufts up to 5 - 7 cms. and showed no appreciable lengthening over the whole period. The observations considered here were made during the prolonged spells of fine or fair weather which then occurred and in wind directions which were fortunately most favourable for uninterrupted exposure, that is, in directions excluding NNE to ESE. The length of level uniform terrain upwind of the instruments was always at least 150 yards, which is accepted as adequate for the establishment of horizontal homogeneity in the vertical profiles up to a height of 2 metres (see, for example, Deacon (1948) on the distance required for the setting up of a characteristic velocity profile). Rainfall over the whole period was slight, approximately 9 mms. being registered during the gap in observations from 11th to 17th March and 0.5 mms. on the night of 28/29th March.

Humidity, temperature and wind profile measurements

4. The instrumental equipment employed for the measurements of temperature, humidity and wind profiles was almost identical with that described previously and reference need only be made to the few aspects in which there were differences. The thermocouple psychrometer housings were slightly wider, due to the aspiration tube being led from the end of the housing instead of from the side (see Figure 1). This resulted in a slight improvement of the ventilation rates obtainable (from 2.2 to 2.4 m./sec. past the thermometer bulbs) and enabled the housings to be rotated so as to keep the wet bulb chambers on the underside when the mast

carrying them was set in a horizontal position for control readings. In order to facilitate the latter process the psychrometer mast was not stood directly on the ground but was mounted at approximately the mid point on a second mast set securely in the ground, in such a fashion that it could be readily swung from the normal vertical position to a horizontal position.

5. The psychrometer mast, carrying psychrometers at 200, 150, 100, 50, 37½ and 25 cms., was set up in line across wind with masts carrying six sensitive cup anemometers at corresponding heights. The three galvanometers used previously were replaced by a single more sensitive and more responsive instrument which indicated temperature differences to 0.01°F and with which a complete set of five dry bulb temperature differences and five wet bulb temperature differences (from 200 cms. to each height in turn) could be read in 2 minutes. Absolute values of the dry and wet bulb temperatures at 200 cms. were read directly to 0.1°F on an auxiliary galvanometer which was switched into circuit with the appropriate 200 cm. element and a reference element in an ice bath. In the first few experiments of the present series 20 such sets of readings were taken in the standard observation period of 1 hour, the anemometers being in operation over the same period. At the end of the run the psychrometer mast was swung into a horizontal position across wind and five sets of readings taken so as to provide zero corrections. At the same time all anemometers were mounted at a height of 2 m. and compared over a period of 20 minutes or so. In these control observations there was reason to suspect that the indicated inequalities in psychrometer performance were frequently partly spurious and due to lateral wind eddying around the closely disposed instruments, causing a given psychrometer to receive air which had been in contact with the irradiated surface of an adjacent instrument. A modified control procedure was accordingly adopted in all subsequent observations. In this the observation period was divided into two parts with a 12 minute interval, and for the second part the psychrometers and anemometers were systematically interchanged, the 200 cm. instrument with that at 100 cm., 150 cm. with 37½ cm. and 50 cm. with 25 cm. On the basis of previous measurements of thermal lag it was known that the interval of 12 minutes would be adequate for adjustment of the reversed psychrometers to their new environments. This procedure complicated the observational and analytical work but on the whole was thought to provide a more dependable instrumental control.

[Note. Confirmatory calibration of the differential circuits several months later indicated a 5% reduction in galvanometer sensitivity. Since in any case the full extent of this deterioration was unlikely to be operative during the measurements discussed, no attempt has been made to allow for it and the initial calibration has been applied throughout.]

Measurement of factors required for Heat Balance considerations

6. Measurements of incoming solar radiation (direct and diffuse) were made by means of a solarimeter provided by Kew Observatory and calibrated there against standard Gorcynksi or Angström instruments. This solarimeter was mounted on a mast at a height of approximately 2 m. above the ground so that it could not be affected by reflected radiation from other instruments, and the thermopile current was indicated by a robust pointer galvanometer stood on the ground nearby. The reflected component of solar radiation was observed merely by inverting the instrument and the zero obtained at intervals by placing over it an opaque cover. One reading of incoming radiation and one of the reflected component were taken during each 3 minutes and it was verified that the interval between the two readings, which was always at least one minute, was long enough for the galvanometer to attain its appropriate deflection. Furthermore, the radiation measurements were made only in clear weather, so that individual readings of the incoming and reflected components did not show a pronounced variation, such as would occur with scattered or broken cloud, and the frequency of reading adopted could thus be expected to give representative mean values.

7. Soil temperatures were measured by means of a multi-point soil thermometer, the design and performance of which will be described in detail elsewhere. It suffices to mention here that the 'thermometer' consists of a non-conducting cylindrical stem ¾" in diameter, with copper strips set in various positions flush with the outer surface. Fine thermocouple wires, carried in the inside of the stem, terminate in single thermojunctions attached to the inner side of each copper strip. The essential principle of the instrument is that it may be installed in the soil, merely by driving it in, with a minimum disturbance of soil

conditions together with the maintenance of adequate contact between the soil and the temperature sensitive surfaces. The wiring and switching arrangement was such that the temperature differences between the thermojunction at a depth of 16 inches and those at 12, 8, 6, 4, 2, 1, ½ and approximately 0 inches were read in succession on the galvanometer described above, to 0.05°F. An additional reading, corresponding to the difference between the 16 inch junction and a junction in a water bath at known temperature, was also taken so as to specify the absolute values of the temperatures. A complete set of such readings was carried out in the minute following the 2 minutes of reading the dry bulb and wet bulb temperature differences.

8. In order to obtain data on the heat capacity per unit volume of the soil, samples were taken with an 'undisturbed core' sampler. This consisted of a steel tube carrying at the top a steel cap and at the bottom a cutting rim tapered on the outside and inset slightly on the inside, the cutting diameter being 4.5 cms. A sampler of similar design, which was used to extract soil cores for evaporation measurements, is illustrated in Figure 2a. On driving the sampler vertically into the ground compression and distortion of the soil occurs outside the tube, while a relatively undisturbed core of soil is left inside the tube, from which it may be subsequently removed intact. During the present experiments a sample down to a depth of 8 inches was taken at a single point, about a yard away from the soil thermometer, on three occasions corresponding to the general periods during which heat balance calculations were carried out. Point to point variations of the water content of the soil are known to be considerable, and an accurate representative measure of water content over an area of ground could only be achieved from a large number of samples. This was quite beyond the scope of the present investigation, and in view of the fact that in any case the temperature measurements were also made at a single point, it seemed reasonable to rely on the single point samples, while making subsequently an appraisal of the possible error so involved on the basis of anticipated spatial variations of water content. The samples so obtained were cut into suitable sections, weighed immediately, and subsequently dried in an oven at a standard temperature of 105°C, until a steady weight was reached, from which the water content of the soil was directly obtained. It should perhaps be explained that although the temperature measurements were carried to a depth of 16 inches the soil sampling was taken down to 8 inches only, as on the basis of previous temperature measurements it was anticipated that for present purposes interest would be largely confined to this layer.

9. For subsequent heat balance considerations a knowledge was also required of the radiative temperature of the ground. It was shown by Robinson (1947) that to a reasonable degree of approximation the radiative temperature of the ground covered by short grass was indicated by a spirit thermometer lying on the ground (actually a Meteorological Office pattern grass minimum thermometer) even when the sun was at appreciable elevation. Since it was impracticable to include in the present measurements a direct measurement of long wave radiation the spirit thermometer method was employed. The thermometer was exposed on the ground adjacent to the other instruments and a reading taken in each of the three minute periods, this again being considered of adequate frequency in clear conditions when violent oscillations of the indicated temperature did not occur.

Determination of Water Loss from the Ground

10. Undisturbed cores of soil 4 inches deep were extracted by means of a sampler similar to that already described but of cutting diameter 3 15/16 inches, (see Figure 2a). These were inserted in close fitting containers made of Tufnol tube of internal diameter 4 inches and wall thickness ¼ inch, so that the soil surface was flush with the rim. Corresponding but slightly larger containers were embedded in the ground with the rims flush with the surrounding soil surface, so forming clean watertight receptacles for the soil containers. The arrangement employed is illustrated in Figure 2b. The effect was thus to isolate a section of the surface layer of the ground from surrounding and underlying soil without markedly disturbing the conditions existing therein, while leaving the actual surface exposed as in the natural state.

11. Prior to a series of experiments a number of 'soil evaporimeters' of this type were prepared and set out about 15 - 20 yards from the anticipated position of the temperature and wind instruments, on a semicircle on the anticipated upwind side of the instruments. Immediately preceding the commencement

of temperature, humidity, wind speed and radiation measurements the soil containers were removed in turn, quickly weighed on an Avery indicating balance set up in the nearby instrument hut, and immediately returned to the appropriate positions in the ground. This procedure was repeated approximately one hour later after the termination of the other observations. The procedure was such that for two minutes of the total period between initial and final weighings the evaporimeter was abnormally exposed and in order to provide a correction for this, one of the evaporimeters was subjected to an additional abnormal exposure of the same duration. The weights were read to 0.01 gm. and the losses observed ranged from 0.1 - 2.5 gm., the control corrections being in the region of 5%. Previous tests had established that weighings with the present balance could be repeated to 0.03 gm. or less.

INTERPRETATION OF THE EVAPORATION MEASUREMENTS

12. The direct determination of the true rate of evaporation from the ground surface is clearly a matter of considerable difficulty, though fortunately the present clayland site offered favourable circumstances in view of the absence of a water table. The result of this latter feature is that in normal circumstances, the ground having become thoroughly wet during the winter, it then dries out progressively from the surface downward during the spring and summer. Qualitatively this feature was unmistakably evident from the appearance of the soil cores extracted at various stages of the present experiments. It follows therefore that there may be certain stages in the drying history of such ground when isolation of a certain depth of soil would not substantially affect the rate of loss of water from it for some period. Such a stage possibly occurs in the early spring, before the surface layers dry out and before root growth penetrates deeply, thus drawing water from deeper layers.

13. In the present measurements the optimum dimensions of isolated soil cores consistent with maximum depth of isolation, capacity and accuracy of balances available and anticipated order of the rate of evaporation, were in the region of 4 inches diameter and depth. These dimensions were accordingly adopted for the main soil evaporimeters. Two features suggested qualitatively that the isolation imposed at a depth of four inches was probably unimportant in the present experiments. In the course of extracting soil cores it was observed that the root growth rarely penetrated below 3 inches so that transpiration would be unaffected. Furthermore, as mentioned previously, it was unmistakably evident that appreciable drying had occurred in the top 2 inches of previously undisturbed ground, whereas there was no visual indication of drying at deeper levels. However, such evidence could not be regarded as conclusive and a quantitative test of some sort was clearly necessary. An approach towards this was provided in the following fashion. Additional soil containers were made to take soil cores of the same surface area but of depths 2 inches and 3 inches respectively and on all occasions one or both of these was used in conjunction with the normal 4 inch evaporimeters. If the vertical isolation exerted substantial influence on the water loss from the core it would be expected that the effects would show up to an increasing degree with the decrease of isolation depth and with the passage of time.

14. Details of water losses observed during each observation are listed in Table 1. The observations have been grouped into sets in which the same soil cores were employed for a number of consecutive observations, these sets being designated by letters A, B, C and D, a subdivision indicated by a dashed letter being made where the soil cores were employed for two days in succession (C' and D'). The data listed include the actual loss from one of the evaporimeters (No. 5) and the relative losses from the remainder. For Group A, evaporimeters Nos. 1 and 2 and the 2" and 3" auxiliary evaporimeters were installed the day before, whereas Nos. 3, 4 and 5 had been in position for eight days, during which period each had lost about 25 gm., which corresponded to approximately 10% of the original water content. For Group C the installation was carried out two days preceding, while for Groups B and D it was carried out on the same morning.

15. Group A, in which the maximum number of evaporimeters was employed, provides the most critical test of performance in view of the wide range of evaporimeter age also involved. In any one observation the losses indicated by individual evaporimeters vary substantially, though in the whole group of

observations there is apparently a fairly consistent relation between individual evaporimeters. We may therefore conveniently examine these results in terms of the mean relative losses for the whole group, and these are reproduced below.

Evaporimeter			No.1	No.2	No.3	No.4	No.5
Evaporimeter Depth	2"	3"	4"	4"	4"	4"	4"
Evaporimeter "age"		1 day				7 days	
Mean relative loss	1.9	1.3	1.4	1.9	1.5	2.0	1.0

From these figures no systematic effect of either evaporimeter depth or evaporimeter age is evident and it seems legitimate to conclude that the differences exhibited are primarily due to a genuine point to point variation in evaporation, arising from heterogeneity in soil moisture, vegetation cover and local exposure. From inspection of the rest of the observations, for which in each group evaporimeter age was constant, there is again no indication of systematic effect of evaporimeter depth. On the argument already put forward these features strongly suggest that in the soil situation then existing the soil isolation imposed had no important effect on the rate of loss of water from the surface, and that the losses observed with the 4 inch evaporimeters may be assumed to give a reasonable approximation to the true rate of evaporation.

REDUCTION OF DATA ON HUMIDITY, TEMPERATURE AND WIND PROFILES

16. Mean values of wind speed, temperature and computed absolute humidity at each of the six heights are given for each observation in Table 2. Each figure represents a mean value over a period of one hour and in the case of temperature and humidity the mean is derived from 20 readings in Observations 6 to 14, 16 readings in Observations 15 to 30. The figures are given to an accuracy consistent with the nominal accuracy of reading, though with regard to temperature and humidity it should be noted that only the differences from one height to another, and not the absolute values, are nominally accurate to 0.01°F and 0.01 gm/cu.m. respectively. Since, however, the interest lies primarily in the gradients of these factors, the differences, and not the absolute values, are of paramount importance. In appraising the real accuracy of these figures the control measures adopted should be kept in mind. The first procedure, in which psychrometers and anemometers were compared at the same height, was employed in Observations 6 to 14; the second and more reliable procedure of systematic interchanging being adopted for the rest of the measurements. In general, therefore, the greatest weight should be attached to Observations 15 to 30, particularly as regards the differences over the intervals 200–100, 150–37½ and 50–25 cms., since for these intervals it may be reasonably assumed that systematic instrumental inequalities not covered by the calibrations are automatically eliminated by the interchange process.

17. Interest first naturally turns to the forms of the vertical profiles, and in this connection it may be recalled that the preliminary investigation demonstrated a close approach to a variation with the logarithm of the height, with only a slight indication of a departure from this in the case of the wind profile in conditions of thermal stratification. The present measurements provide a more critical test of this feature, partly owing to the improvement in accuracy which is anticipated from the control measures and partly to the wider range of atmospheric stability which resulted from the fine weather and, at times, low wind speed. In its present context the term atmospheric stability embraces the combined effects of vertical temperature gradient $\frac{\partial T}{\partial z}$ and wind shear $\frac{\partial u}{\partial z}$ as expressed in the now familiar Richardson Number.

$$\frac{g}{T} \quad \frac{\left(\frac{\partial T}{\partial z} + \Gamma\right)}{\left(\frac{\partial u}{\partial z}\right)^2}$$

where T is in °A and Γ is the dry adiabatic lapse rate.

Values of the vertical gradients of temperature and wind speed (evaluated by methods discussed subsequently) are reproduced in Table 3 together with resulting values of the Richardson Number. As an illustration of the general features of the present vertical profiles, three small groups of observations have been selected as representative of moderate instability, near "neutral" conditions and moderate stability, the mean values of the Richardson No. at 75 cms. being -0.056, -0.005 and +0.067 respectively. The selection has been made from Observations 15-30, which, being subject to a more satisfactory control (see previous paragraph) than the earlier observations, should provide a more critical demonstration of the forms of the profiles. The mean profiles for each property and each group are shown graphically in Figure 3(a), a common representation of all three properties, wind speed, temperature and humidity, being achieved by plotting against the logarithm of the height, the values of the parameter

$$\frac{S_{25} - S_z}{S_{25} - S_{50}}$$

S_z representing wind speed, absolute humidity or temperature at height z cm.

18. The following features are evident in Figure 3. In "neutral" conditions both wind speed and absolute humidity show a close approach to a linear relation with the logarithm of the height. The remaining profiles of wind speed and humidity, and the two profiles of temperature, exhibit a systematic departure from linearity, such that in unstable conditions the S/Log z curve is concave upwards and in stable conditions convex upwards. This is wholly consistent with profile results quoted by Sheppard (1947) and by Deacon (1948). The departure of individual points from straight lines or smooth curves provides some indication of the general accuracy of the measurements and, as might be expected, greatest consistency has evidently been achieved in the wind speed and dry bulb temperature measurements. It may also be noted here that over a surface covered with tall vegetation the linear relation in neutral conditions is only satisfied by applying a 'zero displacement' correction, which physically is usually attributed to the fact that the boundary from which turbulent transport is effective is at some height above the actual soil surface, according to the height and density of vegetation. In the present results the "neutral" profiles suggest that this zero displacement is negligible, which is in keeping with the vegetation conditions described in paragraph 3.

19. In the following analysis considerable importance is attached to the form of the vertical wind profile and a more detailed demonstration of this feature is of some interest at this stage. Wind profiles corresponding to the individual observations in the more accurate group (Nos. 15 - 30) are shown in Figure 3 (b), the parameter $\frac{U_{25} - U_z}{U_{25} - U_{50}}$ being plotted against height on a logarithmic scale. The profiles are here arranged in the order of the Richardson Numbers at 75 cm., and in order to emphasise the variation of the profile with the latter number straight lines have been drawn through the points at 25 and 50 cm. With the exception of a single observation (No. 30) these wind profiles exhibit, in a more detailed and systematic fashion, the variation with Richardson Number already indicated by the three selected groups above. The essential feature is that when the Richardson Number is numerically small, i.e. in conditions approaching neutral equilibrium, there is a linear relation between u and log z. In unstable conditions (Richardson Number negative) u increases less rapidly than log z while in stable conditions it increases more rapidly than log z. Analytical interpretations of these profiles will be discussed in subsequent stages of this paper.

20. As in the preliminary investigation the above profile data afford a basis for discussing the variation with height of the eddy coefficients of diffusion, though such data alone cannot give any direct indication of the absolute magnitudes of these coefficients. However, the present observations, in including measurements of the approximate vertical transport of water vapour and heat over periods of one hour, provide a basis for deriving short term absolute values of the appropriate eddy coefficients of diffusion. For such calculations, and for the evaluation of Richardson Numbers, the actual vertical gradients of temperature, wind speed and humidity are required, and these have been computed from the data of Table 2. In general two procedures have been followed, according to the general consistency of the profile data. For

Observations 6 to 14 smooth curves were drawn through the individual S/log z profiles and tangents drawn to these curves at approximately the mid-point (actually 75 cm.) i.e. well away from the extremities of the curve so that experimental scatter and personal error in drawing would be unlikely to lead to unreproducible results.

21. For Observations 15 to 30 the greater reliability attached to the differences over the various height intervals involved in the interchanging process permitted a reproducible arithmetic derivation of gradients at a number of heights. The basis of this is that except in cases of extreme curvature of the log height curve, the variation with height of the property S may be represented approximately by the equation

$$S = a + b \log (z + c) ,$$

whence

$$\frac{\partial S}{\partial z} = \frac{b}{(z + c)} ,$$

where a, b, c are constants, and may be derived from values of the property S at three heights. Actually c was found to vary systematically with height, so the method adopted was to obtain values of c appropriate to the lower and higher sections of the profile, using the differences 200 - 100, 150 - 37½ and 150 - 37½, 50 - 25 respectively. These values of c were used to deduce $\frac{\partial S}{\partial z}$ at 150 and 37½ cm. respectively, while a mean of the two values was used in computing $\frac{\partial S}{\partial z}$ at 75 cms. The values so obtained are reproduced in Table 3, u, χ and T referring to wind speed, absolute humidity and absolute temperature respectively. Values of the Richardson Number, as already mentioned, are also tabulated there. A few omissions occur in Groups C, C', D and D' when gradients could not be derived with confidence, due either to inefficient performance of the 100 cm. wet bulb or to apparently spurious features in some part of the profile.

EDDY DIFFUSIVITY AND EVAPORATION IN CONDITIONS OF NEUTRAL EQUILIBRIUM

22. The practical aim of the present study was to investigate methods of computing the natural rate of evaporation from other, preferably more easily measured, quantities. One such method, which has been applied to evaporation from the sea (Sverdrup, 1936, and others) and from the land (Thornthwaite and Holzman, 1942) is based on certain treatments of diffusion of matter in the naturally turbulent lower atmosphere. Such treatments rely without exception on the assumption that the transport of matter and momentum occur by identical processes, and that the corresponding diffusion coefficients, eddy diffusivity and eddy viscosity, are equal in magnitude, the latter coefficient being expressible in terms of the mean pattern of flow and determinable from the relatively easily measured wind profile. The data presented here enable a direct test to be made of the validity of the above assumption, and it is proposed to discuss first the simpler case in which the atmosphere is in a state of neutral equilibrium. In specifying the eddy viscosity the deficiencies of existing theoretical approaches may be avoided, following Calder (1947), by appealing directly to the well established law for aerodynamically rough flow through pipes and over flat plates and to the evidence for the validity of this law in the surface layers of the atmosphere. The laboratory law, written in a form convenient for meteorological application, is

$$u_z = \frac{1}{k} \sqrt{\frac{\tau}{\rho}} \log_e \frac{z}{z_o} , \qquad \text{.....(1)}$$

where

u_z is the mean wind speed at height z,

τ the horizontal shearing stress, taken to be constant with height in the surface layers.

ρ the air density,

z_o the roughness parameter of the surface,

k von Karman's constant, the value of which for pipe flow is 0.40.

The eddy viscosity consistent with the above equation is

$$K_m = \frac{u_1 k^2 z}{\log_e \frac{z_1}{z_o}} , \qquad \ldots\ldots\ldots(2)$$

or

$$K_m = k^2 z^2 \frac{\partial u}{\partial z} \qquad \ldots\ldots\ldots(3)$$

23. The applicability of the functional form of equation (1) to the lower layers of the atmosphere in conditions of neutral equilibrium (i.e. zero vertical gradient of potential temperature) is now thoroughly established and the profile data already discussed here provide additional confirmation of this feature for a short grass surface. However, the feature which is of most critical importance in the present connection is the explicit validity of equation (1) in atmospheric flow. This was established recently by Sheppard (1947), who measured the drag of the earth's surface (i.e. the horizontal shearing stress) in near neutral conditions and substituting in equation (1) obtained values for k close to 0.4. Thus, for atmospheric flow near the surface, in conditions of neutral equilibrium, equations (2) and (3) with k = 0.4 specify true values of the eddy viscosity, K_m, and we may now proceed to compare such values with those of the eddy diffusivity, K_v, deduced directly from the present measurements.

24. Making the reasonable assumption of negligible horizontal variation of absolute humidity, the eddy diffusivity, K_v, may be directly obtained from the appropriate integrated equation of transfer (neglecting the variations of air density with height), as follows

$$E + \left(K_v \frac{\partial \chi}{\partial z}\right)_z = \int_o^z \frac{\partial \chi}{\partial t} \partial z , \qquad \ldots\ldots\ldots(4)$$

employing the time mean values of $\frac{\partial \chi}{\partial z}$, $\frac{\partial \chi}{\partial t}$ and rate of evaporation E. Values of E, obtained directly from the water losses shown by the 4" soil evaporimeters, are included in Table 3. In practice, however, it was easily verified that for the shallow layer involved here the term on the right hand side of equation (4) could be neglected. Indeed the magnitude of

$$\frac{1}{E} \int_0^{z=200} \frac{\partial \chi}{\partial t} \partial z$$

for many of the observations was less than 0.005, while the maximum value, occurring with the low rates of evaporation of Observations 9, 23, 28, was only of the order of 0.02. Thus with sufficient accuracy

$$K_v = \frac{-E}{\frac{\partial \chi}{\partial z}} \qquad \ldots\ldots\ldots(5)$$

and in order to facilitate comparison with the form given for the eddy viscosity in equation (3), the above expression was reduced to the non-dimensional parameter

$$\frac{\frac{K_v}{z^2 \frac{\partial u}{\partial z}}}{} = \frac{\frac{-E}{}}{z^2 \frac{\partial u}{\partial z} \frac{\partial \chi}{\partial z}} , \qquad \ldots\ldots\ldots(6)$$

and is tabulated in this form in Table 3.

25. If eddy diffusivity and eddy viscosity are identical then from equations (3) and (6), we should have in neutral conditions,

$$\sqrt{z^2 \frac{-E}{\frac{\partial u}{\partial z}\frac{\partial \chi}{\partial z}}} = k = 0.4 .$$

Values of the above parameter, corresponding to cases with Richardson Number at 75 cm. (Ri_{75}) numerically less than 0.005, have been obtained from table 3 and are reproduced below. A considerable range of wind speed is covered by these cases.

Observation No.	Ri_{75}	U_{200} (cm/sec)	Height (cm)	$\sqrt{\left(z^2 \frac{-E}{\frac{\partial u}{\partial z}\frac{\partial \chi}{\partial z}}\right)}$
8	0.000	512	75	0.39
13	0.003	244	75	0.51
18	-0.001	422	37½	0.41
			75	0.44
			150	0.44
22	-0.004	490	37½	0.36
			75	0.37
			150	0.39
27	0.004	376	75	0.38
			150	0.37
30	-0.004	654	75	0.46
		Mean		0.41

It is seen that the above figures, though somewhat variable, as might well be expected from the nature of the measurements made, have a mean value very close to 0.40. As far as is known this provides for the first time a direct verification of the identity of the diffusivity for water vapour with that for momentum, and a justification of the use of equations (1) and (2) and (5) for the derivation of the natural rate of evaporation from the ground surface in atmospheric conditions close to those of neutral equilibrium.

26. For the derivation of rates of evaporation in practice it may be considered more convenient to use an integrated form of equation (5) which (see for example, Thornthwaite and Holzman, 1942) may be reduced to

$$E = \frac{k^2 (\chi_1 - \chi_2)(\bar{u}_2 - \bar{u}_1)}{\left(\log_e \frac{z_2}{z_1}\right)^2} \qquad \ldots\ldots\ldots(7)$$

The evaluation of equation (7) thus requires only measurements of wind speed and absolute humidity at two heights, provided these heights are large compared with the height of vegetation, as in the present measurements. Otherwise the so-called "zero displacement" correction must be applied and the equation becomes

$$E = \frac{k^2 (\chi_1 - \chi_2)(\bar{u}_2 - \bar{u}_1)}{\left[\log_e \left(\frac{z_2 - d}{z_1 - d}\right)\right]^2} \qquad \ldots\ldots\ldots(8)$$

Measurement of wind speed at three heights would be required to specify "d" from the following equation, which is deduced from the appropriate form of equation (1),

$$\frac{\bar{u}_2 - \bar{u}_1}{\bar{u}_3 - \bar{u}_1} = \frac{\log (z_2 - d) - \log (z_1 - d)}{\log (z_3 - d) - \log (z_1 - d)} , \qquad \ldots\ldots\ldots (9)$$

and which may be solved for "d" graphically.

27. The scatter in the values of the parameter given in paragraph 25 are presumably partly due to experimental error in the measurement of rate of evaporation, on account of the natural point to point variations in this quantity. It is most likely, therefore, that the use of the above method with k = 0.40 would give the short term rate of evaporation with an error less than ± 20%, while on the average the error would be 5% or less.

THE INFLUENCE OF ATMOSPHERIC STABILITY

28. We may now turn to a more general consideration of the data assembled in Table 3, where a substantial variation in the stability of the atmosphere is indicated by the wide range of the Richardson Number, which at a height of 75 cm. amounts to - 0.125 to + 0.123. This range may be appreciated in more practical terms from the following extreme cases of large temperature gradient and low wind speed.

Observation No.	U_{200} (cm./sec.)	Temp. Diff. ^{o}F (200 - 25 cm.)
28	197	3.5
16	258	- 1.7

29. The foregoing considerations have demonstrated that in conditions approaching neutral equilibrium both wind speed and absolute humidity vary linearly with the logarithm of the height in the surface layer and that the parameter

$$\frac{E}{z^2 \frac{\partial u}{\partial z} \frac{\partial \chi}{\partial z}} \quad \text{or} \quad \frac{K_v}{z^2 \frac{\partial u}{\partial z}}$$

is constant and equal to k^2, where k is von Kàrmàn's constant. In other atmospheric conditions we have already seen that the vertical profiles of wind speed and absolute humidity depart from the simple logarithmic relation in a similar and systematic fashion. It is now evident from Table 3 that in such conditions the above parameter is no longer constant but varies, between very wide limits, over the range of atmospheric stability involved. An obvious step was to investigate the possibility of a connection between this parameter and the Richardson Number, and the result is shown graphically in Figure 4, where all values are plotted. A consistent variation of the parameter with the Richardson Number is exhibited and there is no suggestion of a systematic separation of the points according to the height above the boundary. A close approach to a unique relation between $K_v / z^2 \frac{\partial u}{\partial z}$ and Richardson Number is thus established for the shallow surface layer here considered. For purposes of further discussion a smooth curve has been drawn through the points and corresponding values tabulated below

Ri	-0.25	-0.20	-0.15	-0.10	-0.05	0.00	+0.05	+0.10
$\frac{K_v}{z^2 \frac{\partial u}{\partial z}}$	0.72	0.58	0.45	0.34	0.24	0.17	0.11	0.07

The high negative values of the Richardson Number are of course associated with measurements at the height of 150 cm., the corresponding range for 37½ cm. being only −0.03 to +0.04.

30. An increase of the diffusive action of the atmosphere in unstable conditions, and a reduction in stable conditions, both of which are demonstrated by the above data, are to be expected on any plausible theory of turbulent action. The real interest of the present results lies in the specification of the magnitudes of these effects and in the demonstration of the importance of the height above the boundary. The figures show a 10-fold variation of the parameter $K_v/z^2 \frac{\partial u}{\partial z}$, but it should be remembered that the maximum variation is associated with the measurements at 150 cm., where the range of Richardson Number is the greatest. However the latter number decreases rapidly as the ground is approached. As mentioned above the range at 37½ cm. is accordingly reduced and with it is associated only a 2-fold variation of the eddy diffusivity parameter. Taking as an example the moderate range of stability covered by Observations 16 and 23, smoothed values of $K_v/z^2 \frac{\partial u}{\partial z}$ corresponding to the Richardson Numbers at the various heights may be read off the curve in Figure 4. These values are given below, expressed as fractions of the constant value obtaining at zero Richardson Number.

Height	Smoothed values of $K_v/z^2 \frac{\partial u}{\partial z}$ relative to value at Ri = 0	
(cm.)	Obs 23 Ri(75) = + 0.076 Moderate Stability	Obs 16 Ri(75cm) = −0.125 Marked Instability
37½	0.7	1.2
75	0.5	2.3
150	0.4	4.2

Alternatively, the departure of the above figures from unity indicates the order of error which would be incurred if in such conditions the identity $K_m = K_v$ and equations (1), (2) and (3) were assumed to apply. Neglect of the influence of atmospheric stability, and the application of equations (1), (2) and (3) over a moderate range of atmospheric stability, are thus reasonably justifiable only as long as attention is confined to very low heights, while over height ranges customarily adopted (1m. upwards) the errors so introduced would be very large. It is therefore necessary to give further consideration to the present results with particular reference to the influence of atmospheric stability.

31. An attempt to prescribe theoretically the effects of atmospheric stability on the form of the wind profile in the boundary layer has been made by Rossby and Montgomery (1935). This treatment, the details of which it is not proposed to discuss in this paper, leads to an expression relating wind shear and horizontal shearing stress, from which the following equation may be derived

$$\frac{K_m}{z^2 \frac{\partial u}{\partial z}} = \frac{k^2}{(1 + \sigma\, Ri)} \quad , \qquad \text{.....(10)}$$

where k is von Kàrmàn's constant and σ is a proportionality factor introduced in the relation between eddying energy and the potential energy associated with thermal stratification. Deacon (loc. cit.) finds that the Rossby-Montgomery formula does not agree with wind gradient observations at large stability and instability, and demonstrates that a modified formula, proposed by Holzman (1943) entirely on empirical grounds and in the interests of mathematical tractability, provides a better representation. Holzman's formula may be reduced to the form

$$\frac{K_m}{z^2 \frac{\partial u}{\partial z}} = k^2 (1 - \sigma\, Ri) \quad , \qquad \text{.....(11)}$$

which it will be seen is identical with equation (10) when the Richardson Number is small.

32. Equations (10) and (11) are readily compared with the experimental results summarised in paragraph 29 and it is found that neither equation can satisfactorily describe the whole range of results. It is noteworthy, however, that reasonable agreement with the results in stable conditions is provided by the Rossby-Montgomery equation with O equal to about 12, while in unstable conditions the Holzman formula gives corresponding agreement using the same value of O. No physical explanation is immediately evident for this feature, nor for the magnitude attached to O. Thus, in both functional and explicit form the equations put forward by Rossby and Montgomery and Holzman fail to provide a satisfactory basis for expressing the observed relation between eddy diffusivity and Richardson Number. To what extent this is due to deficiencies in the derivation of these equations or to a genuine difference between eddy diffusivity and eddy viscosity in non-neutral conditions is somewhat problematical.

33. An alternative basis for examining the present results is to be found in the generalised wind profile law recently put forward by Deacon (loc. cit.), who found that an extensive series of wind profiles, exhibiting characteristics similar to those shown in Figure 3(b), was well represented by the equation

$$\frac{\partial u}{\partial z} = az^{-\beta} , \qquad \text{.....(12)}$$

in which the exponent β is >1, 1 and <1 in unstable, neutral and stable conditions respectively. Integrating this empirical law and making use of the observed decrease in stability influence on the wind profile as the boundary is approached, Deacon obtained

$$\bar{u}_z = \frac{1}{k\,(1-\beta)} \sqrt{\frac{\tau}{\rho}} \left[\left(\frac{z}{z_o} \right)^{1-\beta} - 1 \right] \qquad \text{.....(13)}$$

which reduces to equation (1) for small values of z and values of β not differing greatly from unity. Actually it was found that β was not strictly constant with height, which led to unacceptable values of z_o when calculated from equation (13). To overcome this difficulty Deacon suggested the assumption of z_o independent of stability, though admitting that this may be unreasonable in marked stability, its value being determined by measurement of wind speed at two heights in neutral conditions and the application of equation (1). Using this value of z_o, equation (13) may then be used to calculate a mean value of β from measurements over the same surface in other conditions of stability.

34. From equation (13) it is easily shown that the eddy viscosity K_m is given by

$$K_m = k\sqrt{\frac{\tau}{\rho}}\, z_o^{1-\beta} z^{\beta} = k^2 z^2 \frac{\partial u}{\partial z} \left(\frac{z}{z_o} \right)^{2\beta-2} , \qquad \text{.....(14)}$$

$$\text{or} \quad \frac{K_m}{z^2 \frac{\partial u}{\partial z}} = k^2 \left(\frac{z}{z_o} \right)^{2\beta-2} .$$

a form which is easily comparable with our results for K_v and requires only the derivation of z_o, and β; for which the following process was adopted. For Observations Nos. 15 - 30, values of the wind ratio $U_{150}/U_{37\frac{1}{2}}$ (which involves the largest height interval to which instrument interchanging was applied) were plotted against the Richardson Number at 75 cm. and found to lie on a fairly smooth curve. The interpolated values of this ratio for Ri = 0 was 1.275, which with equation (1) gave z_o = 0.25 cm. From equation (13)

$$\frac{\bar{u}_{150}}{\bar{u}_{37.5}} = \frac{150^{1-\beta} - z_o^{1-\beta}}{37.5^{1-\beta} - z_o^{1-\beta}} \qquad \text{.....(15)}$$

and using the above value of z_o expression (15) was evaluated for a range of values of β. A graphical relation was thus obtained between β and the Richardson Number and hence between the latter and the parameter $k^2\left(\frac{z}{z_o}\right)^{2\beta-2}$, with k = 0.4 and z = 75 cm.

35. Interpolated values of the various parameters, together with the observed magnitudes of $K_v/z^2 \frac{\partial u}{\partial z}$ are tabulated below.

Richardson No. at 75 cm.	-0.125	-0.10	-0.05	+0.05	+0.10	+0.125
$\bar{u}_{150}/\bar{u}_{37.5}$	1.209	1.216	1.237	1.370	1.525	1.640
β	1.085	1.075	1.045	0.915	0.790	0.715
$k^2\left(\frac{z}{z_0}\right)^{2\beta-2} = \frac{K_m}{z^2 \frac{\partial u}{\partial z}}$	0.42	0.37	0.26	0.06	0.015	0.005
$-\frac{E}{z^2 \frac{\partial \chi}{\partial z} \frac{\partial u}{\partial z}} = \frac{K_v}{z^2 \frac{\partial u}{\partial z}}$	0.39	0.34	0.24	0.11	0.07	0.06

(z = 75 cm.)

A most striking agreement between the computed $K_m/z^2 \frac{\partial u}{\partial z}$ and the observed $K_v/z^2 \frac{\partial u}{\partial z}$ occurs in unstable conditions, the difference between the two values being less than 10%, which is almost as good as the agreement obtained in the corresponding comparison in neutral conditions (paragraph 25). With the increase of atmospheric stability, a considerable disparity appears in the two sets of values, which is consistent with the previous suggestion (see paragraph 33) that equation (13), with z_o independent of stability, is invalid at marked stability. It is important to realise here that equation (13), and hence equation (14), though fairly well established in functional form, has not been verified explicitly. Thus, the present considerations differ from those previously applied to neutral conditions in that a direct test of the identity of eddy diffusivity and eddy viscosity cannot be made. However these considerations do show that for the range of unstable conditions involved

$$K_v \text{ (observed)} = K_m \text{ (computed from equations (13) \& (14))}$$

provided, of course, the roughness parameter, z_o, is determined from equation (1) and wind profile measurements in neutral conditions over the same surface. Thus in practice the assumption $K_v = K_m$ (computed) and equations (5), (13) and (14) may be applied in unstable conditions to evaluate short term rate of evaporation from measurements of wind velocity gradient and humidity gradient, the accuracy obtainable being comparable with that obtained in neutral conditions.

36. As in the neutral case a composite expression incorporating wind speed and absolute humidity at two heights would be more convenient for practical application in unstable conditions. Integration of equation (5) with K_v assumed equal to K_m, and of the form given in equation (14), leads to an expression

$$E = \frac{(1-\beta)^2 k^2 z_o^{2(1-\beta)} (\chi_1 - \chi_2)(\bar{u}_2 - \bar{u}_1)}{\left(z_2^{(1-\beta)} - z_1^{(1-\beta)}\right)^2} \qquad \ldots\ldots\ldots(16)$$

For evaluation of this expression z_o is derived from equation (1) using measurements of wind speed at z_2 and z_1 in neutral conditions. In conditions other than neutral β may be evaluated graphically, as previously described, from measurements of wind speeds at the same heights. It should also be remembered that over tall vegetation it may be necessary to apply a "zero displacement" correction to the values of z_2 and z_1, this being obtainable from the measurement of wind speed at <u>three</u> heights in neutral conditions and the application of equation (9).

37. The foregoing analysis establishes that an acceptable approach to the problem of computing eddy diffusivity and natural rate of evaporation may be made in terms of appropriate wind profile laws for <u>aerodynamically rough flow in neutral</u>

and unstable conditions. In stable conditions the position remains obscure though a reasonable approximation is provided by the result discussed in paragraph 30, namely that stability influence on the eddy diffusivity decreases as the boundary is approached. Thus, if profile measurements are made in the first half metre above the surface equations (1) and (7) may be applied in moderate stability without introducing gross error. This approximation may be somewhat improved in the following way. An inspection of the relation between $K_v/\bar{u}z$ and Richardson Number indicates that although there is a less convincing approach to a unique relation (independent of height) than that shown by $K_v/z^2 \frac{\partial u}{\partial z}$, there is nevertheless a fairly systematic relation for each individual height. Furthermore it appears that $K_v/\bar{u}z$ decreases less rapidly with increase of stability than does $K_v/z^2 \frac{\partial u}{\partial z}$. For example, interpolating smoothed values of $K_v/\bar{u}$ for the moderately stable conditions represented by Observation No. 23, from $K_v/\bar{u}$, Ri graphs for 37½, 75 and 150 cm., the following relative values of K_v (constant wind speed) are obtained. The corresponding values of K_v (constant wind shear) are reproduced from paragraph 30 for comparison.

Height (cm.)	Smoothed values of K_v in moderately stable conditions relative to value in neutral conditions	
	Constant wind speed	Constant wind shear
37½	0.9	0.7
75	0.7	0.5
150	0.6	0.4

The above figures suggest that if in neutral conditions K_v (= K_m) is evaluated from equations (1) and (2) and in moderately stable conditions over the same surface is simply assumed directly proportional to $\bar{u}$ then the value obtained in the latter conditions (and hence the rate of evaporation obtained from equation (5)) will only be approximately 10% high provided the mean wind speed and humidity gradient are observed in the first half metre above the boundary.

HEAT BALANCE ANALYSIS AND DERIVATION OF EDDY CONDUCTIVITY

38. The balance of heat exchange processes near the earth's surface may be expressed as follows. If

R_s is the incoming solar radiation (direct and diffuse),

R_r reflected component of solar radiation,

R_z upward flux of long wave radiation (i.e. the net effect of radiation from ground and atmosphere), at height z,

S heat absorbed in the soil,

L heat associated with evaporation from the ground,

Q_z the upward turbulent flux of heat at height z,

then, in the absence of advection effects,

$$R_s = R_r + S + L + R_z + Q_z + \int_0^z c_p \rho \frac{\partial T}{\partial t} \partial z \quad \text{.....(17)}$$

T, ρ, and c_p being the absolute temperature, density and specific heat at constant pressure of the atmosphere. From the present measurements all terms with the exception of Q_z may be evaluated. Equation (17) may thus be solved for Q_z and finally, since by definition

$$Q_z = -K_H \left(\frac{\partial T}{\partial z} + \Gamma \right) \quad \text{.....(18)}$$

the magnitude of the eddy conductivity K_H may be derived. Evaluation of the terms R_s, R_r and L follows without further explanation and it will be seen later that in all present cases the term

$$\int_0^z c_p \rho \frac{\partial T}{\partial t} \partial z \quad ,$$

which represents the rate at which sensible heat is absorbed by the air layer up to height z, is completely negligible. The terms S and R_z remain to be discussed.

Evaluation of Heat Absorbed in Soil

39. The rate at which heat is absorbed in the soil per unit area of ground surface is given by

$$S = \int_0^\infty \rho' c \frac{\partial \theta}{\partial t} \partial z \quad ,$$

or

$$\int_0^z \rho' c \frac{\partial \theta}{\partial t} \partial z - \left(k' \frac{\partial \theta}{\partial z} \right)_z \quad , \qquad \text{.....(19)}$$

where z is the depth below the soil surface, θ, ρ', c, k' are the temperature, density, apparent specific heat and thermal conductivity of the soil medium. If, as frequently occurred in the present measurements, the soil temperature θ passes through a maximum or minimum value at depth z' then

$$S = \int_0^{z'} \rho' c \frac{\partial \theta}{\partial t} \partial z \quad , \qquad \text{.....(20)}$$

which may be readily evaluated graphically from the soil temperature measurements, using time mean values of $\frac{\partial \theta}{\partial t}$ and determined values for $\rho' c$ at various depths. The apparent density of the soil, ρ', is obtainable directly from the soil sampling data (see paragraph 8). The latter also gives the proportion of soil material and water, and from this and the specific heat of the soil material the apparent specific heat of the soil medium may be derived by simple proportion, the air component being neglected. Determination of the specific heat of the soil material had already been carried out by the School of Agriculture, on oven dried soil samples taken in the previous year from the present site, from depths 0-1, 2-3, 4-5 and 6-7 inches below the surface. The method of mixtures was employed, errors due to heat of wetting of the material being avoided by mixing it with a known quantity of water and performing the determination on the mixture. No systematic variation with the depth was apparent in the values of specific heat so obtained and the mean result was 0.20 ± 0.02. The value 0.20 was employed in all subsequent calculations.

40. The soil temperature data are summarised in Table 4 in the form of mean soil temperatures at the various depths for each of the observations which included radiation measurements. With these are given the corresponding changes of temperature during the observation period, from which mean values of $\frac{\partial \theta}{\partial t}$ may be derived by dividing by the effective observation period (57 minutes). Since the change of soil temperature occurs slowly and smoothly little error is likely to result from this simplified procedure. At the bottom of Table 4 are given the data relevant to the three sets of soil samples, the values of $\rho' c$ finally obtained being used in conjunction with the corresponding temperature observations. It may be noted that the water content of the 0-1 inch layer includes that associated with the vegetation cover and in the evaluation of $\rho' c$ for this layer it is implicitly assumed that the specific heat of the solid material of the vegetation is the same as that of the soil solids. Since the vegetation cover

could only represent a minute fraction of the mass of the 0-1 inch layer it is obvious that the error so involved is completely negligible. From graphs of $\rho' c$ against the mean depth of layer, smooth values were read off for the levels of temperature measurement (down to 8 inches). For observations in which z' was less than 8 inches (Nos. 9, 10, 15, 16, 19, 20, 24, 25 and 28) values of $\rho' c \frac{\partial \theta}{\partial t}$ were then plotted against depth z and expression (20) evaluated by measuring the area between the z axis and the curve so obtained, with a planimeter, over the limits $z = o$, $z = z'$.

41. In the cases when the soil temperature did not pass through a maximum or minimum above the 8 inch level it was necessary to use equation (19), with z = 8 inches, and to evaluate k' at this level. This was readily achieved from the previous cases (z' < 8 inches) since we may write

$$\int_{z'}^{z = 8"} \rho' c \frac{\partial \theta}{\partial t} \partial z = \left(k' \frac{\partial \theta}{\partial z} \right)_{z = 8"} , \qquad \text{.....(21)}$$

the left hand side of the equation being evaluated as above and $\frac{\partial \theta}{\partial z}$ being obtained by drawing tangents to the soil temperature profiles. Inspection of the soil temperature data summarised in Table 4 indicates the small value of $\frac{\partial \theta}{\partial z}$ normally occurring in the region of 8 inches depth, so that in general the term $k' \frac{\partial \theta}{\partial z}$ is of the nature of a small correction. The magnitudes of k' obtained as described from four observations (Nos. 9, 10, 19, and 20) in which the magnitudes of $\frac{\partial \theta}{\partial t}$ and $\frac{\partial \theta}{\partial z}$ were highest, were 20, 58, 18 and 35 x 10^{-4} cals/^{o}C.cm.sec respectively and a mean value of 33 x 10^{-4} was used throughout for the few observations (Nos. 6, 7, 8, 17, 18) where it was necessary to apply equation (19).

Estimation of Long Wave Radiation Term

42. The long wave radiation component from ground and atmosphere was derived by the empirical method described by Robinson (1947), employing for convenience the radiation chart described on page 146 so as to obtain directly the net outward radiation at a given level. The "water-path" - temperature curve below 2 m. - was constructed from the present temperature and humidity profiles, and the "radiative temperature" of the ground measured as described in paragraph (9), using intervals 0-37½, 37½-50, 50-75, 75-100, 100-150, 150-200 cms. For the remainder of the curve the necessary values of upper air temperature and humidity (starting from the 1000 or 950 mb. levels according as the surface pressure was above or below 1025 mbs.) were interpolated from previous and succeeding Downham Market ascents. In all cases inspection of the synoptic situation suggested that no substantial differences in air mass properties were likely to exist over the distances separating the present site and Downham Market (30 miles). Furthermore the observations for which these computations were made were carried out in good visibility with in general, less than 3/10 diffuse high cloud (see cloud data in Table 3), the cloud being in most cases at a fairly low angular elevation. No attempt has been made to allow for the effects of these small cloud amounts. The worst case is observation No. 20 when the mean cloud amount was 3/10 Ci and 2/10 Ac. and this has been included deliberately in the present analysis so as to provide some appraisal of the errors likely to accrue from neglect of the cloud factor.

Discussion of Heat Exchange Factors and Derivation of Eddy Conductivity

43. The observational and computed data relevant to the heat balance analysis are summarised in Table 5. The net outward long wave radiation is given for the 75 cm. level, since succeeding considerations are largely confined to this level, but it is of interest at this stage to consider how far this value is representative of the long wave flux at other heights up to 2 m. For a selection of cases the radiation chart curves were constructed for the 37½ and 150 cm. levels, as well as the normal 75 cm. level and it was found that there was a measurable difference in the upward flux at the three levels. However, the difference between the flux at the two extreme heights (37½ and 150 cms.) was at the most 0.002 cals/cm^{2}min., whereas it will be seen from Table 5 that the magnitude of the flux at 75 cms. is invariably 0.1 cals/cm^{2}min. or more. Thus to a high degree of approximation the

net upward component of long wave radiation may be assumed invariant with height in the surface layer considered.

44. In view of the number of separate heat exchange factors it is clear that considerable error could arise in the resultant of these factors, and indeed this constitutes one of the greatest potential drawbacks in employing heat balance considerations in order to evaluate one or other of the factors. The heat balance components are specified in Table 5 to 0.001 cals/cm^{2}min. though it will be seen that this limit is considerably smaller than the accuracy which may be attributed to the values. For the estimation of long wave radiation, Robinson (loc. cit.) suggests that the net outward radiation with clear skies is given by the radiation chart with a probable error of $\pm$ 0.015 cals/cm^{2}min. The comparison of the solarimeter with a Gorczynski or Angström instrument suggested that the indications of the former may be in error up to $\pm$ 2%, i.e. $\pm$ 0.015 cals/cm^{2}min. in the maximum values of incoming solar radiation measured in the present investigation. As for the errors likely to result from heterogeneity of the thermal properties of the soil it may be noted that if point to point variations in soil moisture content are of the order of the differences shown by the separate measurements summarised at the bottom of Table 4, then these could lead to errors of up to $\pm$ 10% in the heat capacity per unit volume of soil and hence $\pm$ 0.015 cals/cm^{2}min. in the maximum value of the heat absorbed in the soil. These few examples suffice to indicate the very considerable error which might accumulate in the balance of the various factors and it would be somewhat problematical to assign a figure to this resultant error. Fortunately the present observations include two cases in which a direct assessment of the resultant error is possible.

45. For observations Nos. 8 and 18, which were carried out on widely separated occasions, the gradient of potential temperature was practically zero. In point of fact the values at 75 cm. were respectively 2 x 10^{-4} ^{o}C/cm. and -1 x 10^{-4} ^{o}C/cm. respectively. The turbulent transport of heat should therefore be very small and the remaining components, i.e. those measured and estimated, should almost balance. Actually it will be seen from Table 5 that the magnitudes of the upward components which cannot be accounted for except by a low order of turbulent transport and by error in estimation are

-0.004 cals/cm^{2}min. for No. 8 with $\left(\frac{\partial T}{\partial z} + \Gamma\right)$ + 2 x 10^{-4} ^{o}C/cm.

$+0.038$ cals/cm^{2}min. for No.18 with $\left(\frac{\partial T}{\partial z} + \Gamma\right)$ -1 x 10^{-4} ^{o}C/cm.

Only a small fraction of the greater of these components (No.18) could be ascribed to an eddy conductivity of the anticipated order of 10^3 and since the change of air temperature during any observation (effective period 57 minutes) was less than 5^{o}F the error involved in neglecting the term $\int_0^z \rho c_p \frac{\partial T}{\partial t} \partial z$ could not be more than 0.001 cals/cm^{2}min. It follows therefore that the various components derived for observations 8 and 18 balance to within about 0.04 cals/cm^{2}min. and it would seem reasonable to assign this order of casual error to the resultant upward components attributed to turbulent transport in the remaining observations.

46. Considering now the magnitudes of the upward heat transport which must be attributed to turbulent exchange we may apply equation (18) and derive apparent values of the eddy conductivity K_H. The appropriate magnitudes of $\left(\frac{\partial T}{\partial z} + \Gamma\right)$ at 75 cm. and the resulting values of K_H at this level are reproduced in Table 5 together with the corresponding values of K_v, the eddy diffusivity. It will be seen that eddy conductivity and diffusivity are of similar magnitudes, and range from 10^3 to 10^2 in order. A more critical comparison of the two diffusion coefficients may be made in terms of the non-dimensional parameters $K_H/z^2 \frac{\partial u}{\partial z}$ and $K_v/z^2 \frac{\partial u}{\partial z}$, which are tabulated in Table 5 and plotted against the Richardson Number at the same level in Figure 5.

47. In Figure 5 a quite remarkable identity is shown between the eddy conductivity and eddy diffusivity in stable conditions, while with the increase of instability the former coefficient is systematically the greater of the two.

These results provide the first direct evidence of the existence of a difference in the eddy diffusivities for water vapour and heat and it is therefore of the utmost importance that the accuracy of the results should be carefully appraised. So far discussion has been confined to casual error (paragraph 44), which would result merely in a "scatter" of the experimental results. With the exception of Observation 20 all the values of eddy conductivity conform to the line drawn through them to a degree which is consistent with errors in the turbulent transport component of well within $\pm$ 0.04 cals/cm^{2}min. The consistency is better than might be anticipated from the previous discussion (paragraph 45) on the magnitude of casual error to be assigned to the values derived for this component.

48. A feature which is of more critical importance in the present considerations is the possibility of systematic error in unstable conditions. In order to explain the observed differences in eddy conductivity and diffusivity the error would have to be in the form of an overestimation of the turbulent heat flux to an extent of about 100%. Considering the heat balance terms there is no reason to suspect any appropriately systematic error in the measured terms, solar radiation, soil heat and heat of evaporation, and the only other term is the computed long-wave radiation flux. In the computation of this factor the effects of small amounts of high cloud were neglected, but this would result in a slight overestimation of the upward flux of long-wave radiation and hence an underestimation of the turbulent heat flux. Furthermore, the computation assumes the radiative temperature of the ground to be given in all cases by a spirit thermometer laid on the ground. Actually Robinson's unpublished data suggest that with the sun's altitude above 30° the method overestimates the radiative temperature. As far as can be inferred from this somewhat limited data the error in the present computations would amount at most to an overestimation of the order of 0.01 cals/cm^{2}min. in the upward flux of long-wave radiation and hence a corresponding underestimation in the upward turbulent flux. Again, Robinson (1947 p.146) has observed that the computed downward flux of atmospheric radiation may be too high. This would lead to an overestimation of the turbulent heat flux. However the indications of Robinson's measurements are that this error could be of the order of 0.02 cals/cm^{2}min. These features are clearly insufficient to modify the higher values of the turbulent heat flux (ca 0.20 cals/cm^{2}min.) to the required extent.

49. The above considerations of the accuracy of estimation of the turbulent heat flux provide no grounds for suspecting that the observed differences in diffusivity and conductivity are other than genuine and of the correct order of magnitude. Although it is not proposed in this paper to deal in any detail with theories of the turbulent mixing it may now be noted that, in qualitative implication at any rate, the present results are not entirely unexpected. In a recent treatment of convection near the ground Sutton (1948) has assembled evidence for the existence of a difference in heat and momentum transport. Furthermore, Priestley and Swinbank (1947) have recently put forward a modification of the classical theory of turbulent transport of heat, taking into account buoyancy effects. The latter treatment possesses implications as regards the transfer of physical quantities other than that of heat and provides reasons for expecting different values of the eddy diffusion coefficient to apply to the transport of different properties. So far as is known the present results provide the first direct experimental confirmation of the reality of this feature in the cases of matter and heat.

50. Consideration has been restricted so far to the absolute magnitudes of the eddy conductivity at a single level, 75 cm, though we may readily examine the variation with height of this factor. From the previous discussion of the variation with height of the net outgoing long-wave radiation (paragraph 43) and the magnitude of the term $\int_0^z \rho c_p \frac{\partial T}{\partial t} \partial z$ (paragraph 45) it follows that in the present measurements the turbulent heat flux cannot vary with height by more than 0.003 cals/cm^{2}min. in the surface layer considered (0-2 metres). This is of a very small order compared with the magnitude of the turbulent heat flux derived for the 75 cm. level (which except in "near neutral" conditions is of the order of 0.1 cals/cm^{2}min.) and moreover is much less than the overall accuracy which could be assigned to the determinations. It is thus justifiable to neglect the variation with height of the turbulent heat flux and to adopt the value at 75 cm. as appropriate to the layer 0-2 metres; in which case from equation (18)

$$K_H \propto \left(\frac{\partial T}{\partial z} + \Gamma\right)^{-1} .$$

The variation of eddy conductivity with height may then be readily appraised by reference to the values of the vertical temperature gradient given in Table 3 for the heights 37½, 75 and 150 cm.

51. As an illustration of this point we may take the groups of observations considered in the earlier discussion on the form of the vertical profiles (paragraphs 17 and 18) and evaluate the exponent α in the expression

$$K_H \propto \left(\frac{\partial T}{\partial z} + \Gamma\right)^{-1} \propto z^{\alpha}$$

These are given below together with the corresponding smoothed values of β obtained from the wind profile equation (15)

	Mean Richardson No.	Obs. 15, 17, 20	Obs. 19, 23
	at 75 cm.	- 0.058	+ 0.067
α	from temp. gradient at 37½ and 75 cm.	1.23	0.76
α	from temp. gradient at 75 and 150 cm.	1.15	0.85
α	Mean of above	1.19	0.81
β	From $U_{150}/U_{37\frac{1}{2}}$	1.05	0.87

The above figures illustrate the more rapid increase of K_H with height in unstable conditions than in stable conditions and also suggest that in unstable conditions the eddy conductivity increases much more rapidly than the eddy viscosity since it follows from equation (14) that $K_m \propto z^{\beta}$.

52. Turning finally to the practical object of this study, i.e. the specification of natural rate of evaporation in terms of other factors, we may note that an application of the heat balance equation has been made by a number of workers (notably Angström (1920) and Cummings (1940)) in prescribing evaporation loss from lakes. In this application the assumption is made that the eddy conductivity and eddy diffusivity are identical, so that in our nomenclature the heat balance equation may be written

$$R_s = R_r + R_z + S - K_v \left[\rho c_p \left(\frac{\partial T}{\partial z} + \Gamma\right) + L' \frac{\partial \chi}{\partial z} \right], \qquad \ldots\ldots\ldots(22)$$

where L' is the latent heat of evaporation of water. From measurements or estimations of incoming and reflected solar radiation, net outgoing long-wave radiation, heat absorbed in the soil and the vertical gradients of temperature and absolute humidity in the air the above equation may be used to derive K_v, and hence $-K_v \frac{\partial \chi}{\partial z}$ i.e. the rate of evaporation. We may now assess the validity of applying this procedure to the derivation of evaporation from a land surface. From the agreement shown in Figure 5 between eddy conductivity and eddy diffusivity in stable conditions it follows that rate of evaporation would be adequately prescribed by the above method. In occasions of marked instability, however, for which the present results indicate an eddy conductivity some 100% higher than the eddy diffusivity the assumption of equality of the two coefficients would lead to an overestimation of the rate of evaporation. In the present cases when the heat absorbed in the evaporation process is of a similar magnitude to that transported by turbulent action this overestimation would thus amount to approximately 50% and in other

circumstances would be greater or less than this according as the heat associated with the evaporation process is less or greater than the turbulent transport of heat.

REFERENCES

Angström, A., 1920, Geog. Ann. 2, 237-252.

Calder, K. L., 1947, in course of publication.

Cummings, M. W., 1940, Amer. Geophys. Union Trans., 21, 512-522.

Deacon, E. L., 1948, Porton Technical Paper No. 39.

Holzman, B., 1943. Ann. New York Acad. Sci., XLIV, Art. 1, 13-17.

Priestley, C. H. B., and Swinbank, W. C., 1947, Proc. Roy. Soc., A, 189, 543-561.

Robinson, G. D., 1947, Q.J.R. Met. Soc., LXXIII, Nos. 315-316, 127-150, and unpublished results.

Rossby, C. G., and Montgomery, R. B., 1935, Papers Phys. Oceanogr. and Met., Mass. Inst. Tech., 3, No. 3.

Sheppard, P. A., 1947, Proc. Roy. Soc., A, 188, 208-222.

Sverdrup, H. U., 1936, Ann. der Hydrog. u. Marit. Met. 64, 41-47.

Thornthwaite, C. W., and Holzman, B., 1942, U.S. Dept. Agric. Tech. Bull. No. 817.

Table 1. Evaporation Data

Group and Date	Period of Evap. (mins.)	Obs. No.	GMT. (ϕ)	Relative loss from Evaporimeters							Actual loss in gm. from No.5
				2"	3"	No.1	No.2	No.3	No.4	No.5	
	75	6	10.33	1.62	1.21	1.38	2.19	1.40	1.33	1.00	0.82
A	75	7	12.30	1.69	1.14	1.17	1.62	1.24	1.25	1.00	1.00
	75	8	15.30	1.69	1.03	1.33	1.49	1.27	1.39	1.00	0.75
10.3.48	75	9	17.30	2.92	2.17	1.83	2.83	2.17	4.17	1.00	0.12
	75	10	19.30	1.82	1.00	1.29	1.41	1.29	1.82	1.00	0.17
	75	11	12.35	0.75	0.75	0.79	0.85	0.83	0.89	1.00	2.48
B	75	12	14.30	0.74	0.74	0.77	0.88	0.88	0.82	1.00	1.60
18.3.48	75	13	16.34	0.73	0.68	0.85	0.80	0.89	0.68	1.00	0.99
	75	14	19.32	0.53	0.61	0.66	0.84	0.76	0.68	1.00	0.38
				2"	No.4	No.5	No.6				No.5
	70	15	09.38	0.78	0.92	1.00	0.95				1.18
C	70	16	11.30	0.69	0.87	1.00	0.89				1.47
	70	17	14.30	0.68	0.84	1.00	0.82				1.45
24.3.48	70	18	16.30	0.77	0.87	1.00	1.06				0.88
	70	19	18.30	0.63	0.77	1.00	0.67				0.30
	70	20	10.30	0.71	0.91	1.00	0.84				1.14
C_1	70	21	12.35	0.76	0.92	1.00	0.99				1.43
25.3.48	70	22	15.30	0.80	0.84	1.00	0.87				1.28
	70	23	18.30	0.62	0.85	1.00	1.00				0.28
	70	24	10.30	0.81	0.89	1.00	0.91				1.52
D	70	25	12.33	0.76	0.83	1.00	0.86				1.62
	70	26	14.42	0.82	0.97	1.00	0.97				1.10
28.3.48	70	27	16.30	0.64	0.91	1.00	0.94				0.64
	70	28	19.00	0.11	0.67	1.00	1.00				0.09
D_1	70	29	11.39	0.94	1.08	1.00	1.00				1.35
29.3.48	70	30	14.03	0.79	0.98	1.00	1.00				0.85

N.B. Evaporimeters Nos. 1-6 are of depth 4 inches.

ϕ Refers to mid-time of observation period of approximately 60 min.

Table 2. Wind Speed, Air Temperature and Humidity Data

GROUP AND DATE		A 10.3.48					B 18.3.48				C 24.3.48					C^1 25.3.48				D 28.3.48					D^1 29.3.48	
OBS. NO.		6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
G.M.T. (ø)		1033	1230	1530	1730	1930	1235	1430	1634	1932	0938	1130	1430	1630	1830	1030	1235	1530	1830	1030	1233	1442	1630	1900	1139	1403
Wind speed in cm/sec. at hts. in cm.	200	443	379	512	282	357	386	280	244	241	253	258	323	422	277	327	479	490	280	522	570	525	376	197	781	654
	150	426	365	484	266	337	370	268	232	228	247	249	313	393	256	314	458	469	231	500	547	500	358	166	743	624
	100	403	351	468	247	319	354	257	220	205	233	238	298	382	238	298	438	435	204	472	506	464	333	142	707	589
	50	358	308	401	207	272	309	224	190	172	210	215	265	336	200	264	381	385	168	420	450	407	293	106	612	514
	37½	335	292	†	191	257	293	211	181	163	200	206	253	317	189	254	365	363	157	398	427	386	276	101	594	493
	25	299	257	335	163	223	268	191	161	143	178	184	231	292	170	225	325	330	136	360	389	349	258	85	529	426
Air Temp. in ^{o}F at hts. in cm.	200	51.7	54.2	56.2	53.1	48.8	48.9	49.9	50.6	44.4	47.9	52.2	55.1	53.0	45.0	54.5	58.3	58.8	51.0	54.1	56.9	57.6	56.4	47.8	56.7	56.4
	150	51.88	54.39	56.20	52.80	48.60	49.16	50.01	50.47	44.16	48.11	52.4	55.29	53.01	44.71	54.80	58.47	58.89	50.52	54.50	57.13	57.71	56.39	47.18	56.95	56.52
	100	52.15	54.69	56.20	52.34	48.32	49.44	50.18	50.34	43.79	48.21	52.70	55.55	53.00	44.30	54.93	58.66	58.94	49.85	54.74	57.54	57.86	56.35	46.39	57.03	56.54
	50	52.72	55.35	56.18	51.41	47.80	50.02	50.49	50.33	43.19	48.68	53.38	55.96	53.01	43.66	55.45	59.04	59.07	48.82	55.27	58.15	58.11	56.18	45.19	57.40	56.68
	37½	53.00	55.66	56.19	51.05	47.61	50.32	50.66	50.32	42.97	48.81	53.58	56.22	53.03	43.43	55.74	59.22	59.12	48.44	55.53	58.41	58.23	56.26	44.85	57.50	56.71
	25	53.39	56.06	56.16	50.55	47.34	50.68	50.86	50.25	42.65	49.09	53.92	56.45	53.00	43.14	56.07	59.43	59.16	47.91	55.77	58.76	58.37	56.29	44.31	57.65	56.74
Abs. Humidity in gm/cu.m at hts. in cm.	200	6.07	5.97	6.04	5.84	7.68	5.34	4.99	4.78	5.21	4.67	5.36	5.13	5.01	4.47	4.95	*	3.26	3.70	4.94	*	5.36	5.60	5.39	8.11	7.63
	150	6.13	6.07	6.11	5.90	7.69	5.48	5.07	4.84	5.23	4.74	5.46	5.19	5.06	4.48	4.99	4.34	3.30	3.74	4.90	5.18	5.42	5.65	5.40	8.12	7.61
	100	6.24	6.17	6.17	5.93	7.75	5.58	5.19	4.97	5.27	4.86	5.51	5.31	5.17	4.56	5.13	4.46	3.46	3.86	5.02	5.31	5.53	5.76	5.45	8.21	7.68
	50	6.41	6.45	6.39	6.09	7.79	5.89	5.46	5.13	5.35	5.13	5.85	5.63	5.37	4.63	5.45	4.75	3.74	4.01	5.15	5.39	5.74	5.98	5.55	8.38	7.75
	37½	6.49	6.50	6.40	6.13	7.84	6.04	5.54	5.19	5.37	5.21	5.95	5.70	5.40	4.68	5.49	4.82	3.78	4.05	5.25	5.60	5.81	5.95	5.53	8.41	7.81
	25	6.61	6.68	6.49	6.18	7.86	6.17	5.67	5.29	5.42	5.39	6.18	5.95	5.55	4.73	5.69	5.03	3.97	4.13	5.39	5.70	5.93	6.13	5.58	8.51	7.89

† Anemometer Reading obviously spurious.

* Wet bulb dry at tip.

ø Refers to mid-time of observation period of approximately 60 minutes.

GROUP AND DATE		A		1	D 28.3.48					D[1] 29.3.48	
OBS. NO.		6	7		24	25	26	27	28	29	30
G.M.T. (ϕ)		1033	1230	1	1030	1233	1442	1630	1900	1139	1403
U at 2 m. cm/sec.		443	379		522	570	525	376	197	781	664
Wind Dir.		WNW	NW		ESE	ESE	ESE	SSE	SE	S	SSW
Cloud		3/10 Cl	3/10 Cl	3/1	Tr.Ac	Tr.Ac	1/10 Tr.Cu 10 Ac	2/10 Ac	1/10 Ac	3/8 10Cusc 10 Ac	8/8 10Cusc 10 Ac
$10^8 \times E$ gm/cm^{2}sec.		340	355		430	440	330	185	25	420	255
$-10^{10} \times \frac{\partial \chi}{\partial z}$ gm/cm^4	37½ cm.				85	108	75			50	49
	75 cm.	33	42		31	37	37	57	13	28	17
	150 cm.						16	29		10	
$10^2 \times \frac{\partial u}{\partial z}$ sec^{-1}	37½ cm.				224	234	223	137		313	317
	75 cm.	85	67		97	116	110	79	60	143	125
	150 cm.				48	62	59	41		71	63
$10^4 \times \frac{\partial T}{\partial z}$ ^{0}A/cm	37½ cm.				-107	-131	-55			-54	
	75 cm.	-58	-66		-56	-68	-28	+7	+125	-30	-19
	150 cm.						-14	+3		-18	
$-\frac{10^5 E}{\frac{\partial \chi}{\partial z}\frac{\partial u}{\partial z} z^2}$	37½ cm.				160	125	140			190	
	75 cm.	215	225		255	180	145	145	60	190	210
	150 cm.						150	135		265	
$\frac{10^8 g\left(\frac{\partial T}{\partial z} + \Gamma\right)}{T\left(\frac{\partial u}{\partial z}\right)^2}$	37½ cm.				-7	-9	-4			-2	
	75 cm.	-27	-50		-19	-17	-8	+4	+123	-5	-4
	150 cm.						-12	+8		-11	

ϕ Refers to mid-

Table 4. Soil Temperature Data and Heat Capacities

GROUP AND DATE		A 10.3.48					C 24.3.48					C[1] 25.3.48	D 28.3.48		
OBS. NO.		6	7	8	9	10	15	16	17	18	19	20	24	25	28
G.M.T. (ϕ)		1033	1230	1530	1730	1930	0938	1130	1430	1630	1830	1030	1030	1233	1900
Soil Temperature in °F at depths in inches	0	52.25	57.05	56.50	49.55	46.25	52.90	61.85	61.30	53.35	43.25	57.80	55.35	61.95	44.40
	½	49.65	53.35	54.70	50.30	47.50	47.25	53.60	56.30	52.50	46.35	49.45	49.35	55.20	47.05
	1	48.45	51.75	52.80	50.60	48.10	44.35	49.40	53.70	52.25	48.25	45.75	46.30	51.70	48.65
	2	47.40	50.00	51.90	50.70	48.75	42.85	46.65	51.60	51.85	49.40	43.20	44.25	48.80	49.45
	4	46.55	47.95	50.05	50.00	49.15	42.55	44.25	48.55	50.00	49.60	41.80	42.85	45.60	49.30
	6	46.30	46.80	48.35	48.85	48.70	43.35	43.80	46.60	48.00	48.50	42.30	43.05	44.30	48.15
	8	46.20	46.20	47.05	47.55	47.85	44.35	44.10	45.50	46.50	47.30	43.20	43.80	44.05	46.95
	12	45.95	45.80	45.85	46.25	46.55	45.75	45.35	45.45	45.70	46.15	44.95	45.25	44.90	45.60
	16	45.25	45.20	45.20	45.35	45.60	46.15	45.85	45.85	45.80	45.85	45.65	45.75	45.45	45.45
Final Temp. - Initial Temp. in °F (effective period 57 mins.) at depths in inches	0	+2.80	+1.70	-1.45	-4.30	-1.90	+6.05	+2.65	-2.15	-6.40	-4.15	+2.00	+4.95	+0.95	-1.10
	½	+1.80	+1.60	-1.05	-2.40	-1.25	+3.75	+2.40	-0.60	-3.65	-2.50	+2.60	+3.45	+1.80	-1.65
	1	+1.55	+1.50	-0.65	-1.60	-1.25	+2.50	+2.25	+0.25	-1.90	-1.75	+2.60	+2.80	+2.10	-1.65
	2	+0.90	+1.20	-0.30	-0.85	-1.10	+1.50	+2.05	+0.80	-0.70	-1.35	+2.35	+2.10	+2.00	-1.25
	4	+0.25	+0.80	+0.20	-0.20	-0.75	+0.40	+1.30	+1.00	+0.20	-0.55	+1.10	+0.90	+1.50	-0.60
	6	-0.15	+0.35	+0.30	+0.10	-0.55	-0.15	+0.45	+0.80	+0.40	0.00	+0.40	+0.20	+0.80	+0.15
	8	-0.35	+0.10	+0.20	+0.25	-0.30	-0.30	+0.05	+0.50	+0.45	+0.35	-0.15	-0.10	+0.35	+0.30
	12	-0.30	-0.10	+0.10	+0.10	-0.25	-0.25	-0.30	0.00	+0.05	+0.35	-0.30	-0.25	-0.05	+0.35
	16	-0.15	-0.10	-0.00	+0.05	-0.25	-0.05	-0.30	-0.10	-0.05	+0.15	-0.10	-0.15	-0.05	+0.20

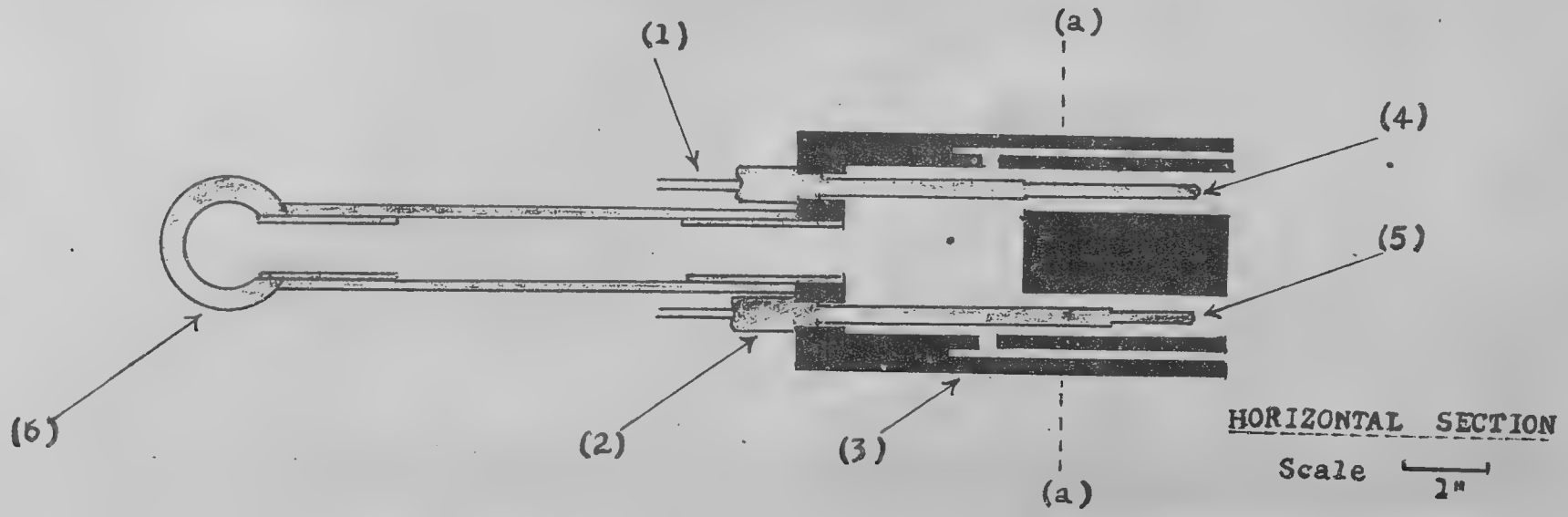
Date of soil sampling	10.3.48					24.3.48					29.3.48				
Layer (inches)	0-1	1-2	2-4	4-6	6-8	0-1	1-2	2-4	4-6	6-8	0-1	1-2	2-4	4-6	6-8
Apparent Sp. Gr.	1.38	1.69	1.71	1.76	1.79	1.80	1.85	1.82	1.61	1.77	1.31	1.55	1.66	1.68	1.68
% by wt. of water	23.3	19.7	20.6	21.3	20.5	19.8	17.8	18.6	19.5	19.9	24.3	17.0	18.2	19.4	20.1
Apparent Sp. Ht.	0.39	0.36	0.37	0.37	0.36	0.36	0.34	0.35	0.36	0.36	0.39	0.34	0.34	0.35	0.36
Heat Capacity per unit volume, cals/°C cm.^{3}	0.53	0.61	0.62	0.65	0.65	0.64	0.63	0.63	0.57	0.64	0.52	0.52	0.57	0.60	0.61

ϕ Refers to mid-time of observation period of approximately 60 minutes.

DS 79487/1/7149 75 11/48 TL

Table 5. Heat Balance Analysis

GROUP AND DATE	A 10.3.48					C 24.3.48					C^1 25.3.48	D 28.3.48		
OBS. NO.	6	7	8	9	10	15	16	17	18	19	20	24	25	28
G.M.T.	1033	1230	1630	1730	1930	0938	1130	1430	1630	1830	1030	1030	1233	1900
Temperature indicated by spirit thermometer on ground ^{o}C	16.3	17.9	13.4	7.3	5.7	14.4	18.0	16.3	11.2	3.8	17.6	15.0	18.3	3.4
$10^8 \times E$ ($gm/cm^2 sec.$)	340	355	275	80	65	345	410	390	260	75	320	430	440	25
Heat Components in $cals/cm^2 min.$														
Incoming Solar Radiation	.715	.770	.336	.031	nil	.637	.835	.692	.314	.006	.764	.803	.903	nil
Reflected Solar Radiation	.123	.128	.073	.008	nil	.129	.157	.137	.071	.003	.137	.156	.168	nil
Net outward long wave Radiation (at 75 cm.)	.191	.196	.153	.116	.109	.186	.200	.185	.155	.119	.195	.180	.192	.099
Heat Absorbed in Soil	.057	.105	.017	-.048	-.063	.117	.154	.089	-.042	-.088	.135	.129	.148	-.053
Heat Used in Evaporation	.120	.126	.097	.029	.023	.121	.144	.138	.092	.026	.112	.152	.155	.009
Upward Component to be accounted for by turbulent transport	.224	.215	-.004	-.074	-.069	.084	.180	.143	.038	-.054	.185	.186	.240	-.055
$10^4 \times \left(\frac{\partial T}{\partial z} + \Gamma\right)$, $^{o}A/cm.$, at 75 cm.	-57	-65	+2	+94	+54	-36	-59	-48	-1	+70	-48	-55	-67	+126
Eddy Conductivity, K_H, cm^2/sec, at 75 cm.	2,150	1,820		430	690	1,270	1,680	1,650		420	2,140	1,890	2,020	240
Eddy Diffusivity, K_V, " "	1,030	840	950	370	570	780	930	830	810	390	690	1,400	1,180	190
$10^8 \times \frac{K_H}{\partial u/\partial z\ z^2}$ at 75 cm.	450	485		105	165	520	740	520		115	670	345	310	70
$10^3 \times \frac{K_V}{\partial u/\partial z\ z^2}$ " "	215	225	155	90	135	320	410	260	195	105	215	255	180	60
$10^3 \times$ Richardson Number at 75 cm.	-27	-50	0	+64	+33	-66	-125	-52	-1	+58	-51	-19	-17	+123



HORIZONTAL SECTION

Scale 1"

KEY

(1) Tubes carrying thermocouple wires.
(2) Stem of thermo-elements.
(3) Double-walled Tufnol housing.
(4) Wet bulb.
(5) Dry bulb.
(6) Common aspiration mast.
(7) Water chamber.
(8) Cotton feed to wet bulb.

VERTICAL SECTION AT a----a

Scale 1"

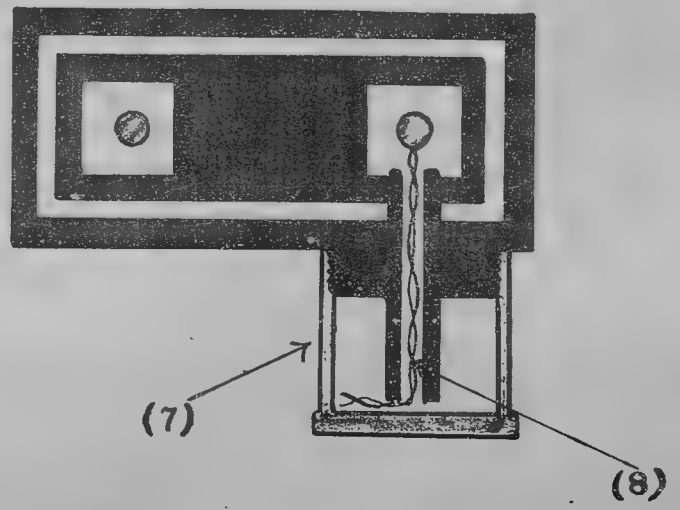


FIGURE 1. SECTIONAL REPRESENTATION OF MODIFIED THERMOCOUPLE PSYCHROMETER.

DS 7948

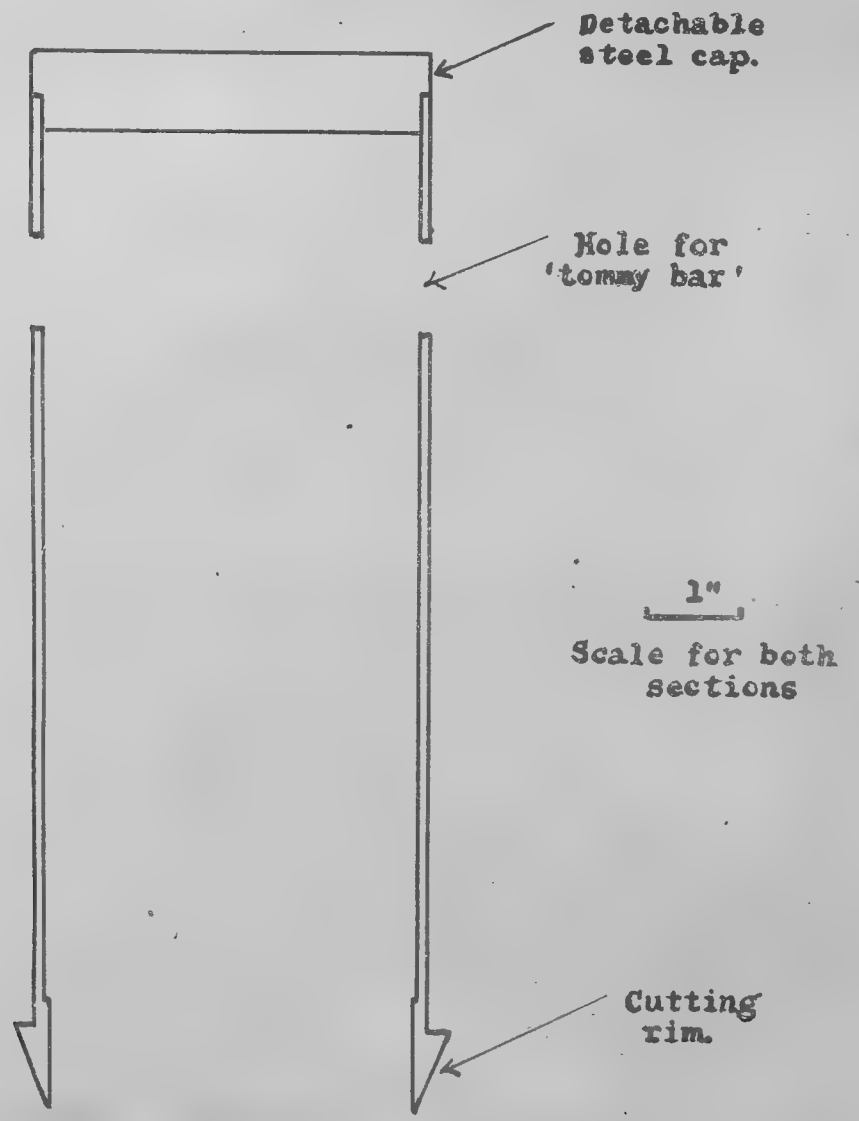


FIGURE 2(a). VERTICAL SECTION OF 'UNDISTURBED' CORE' SOIL SAMPLER.

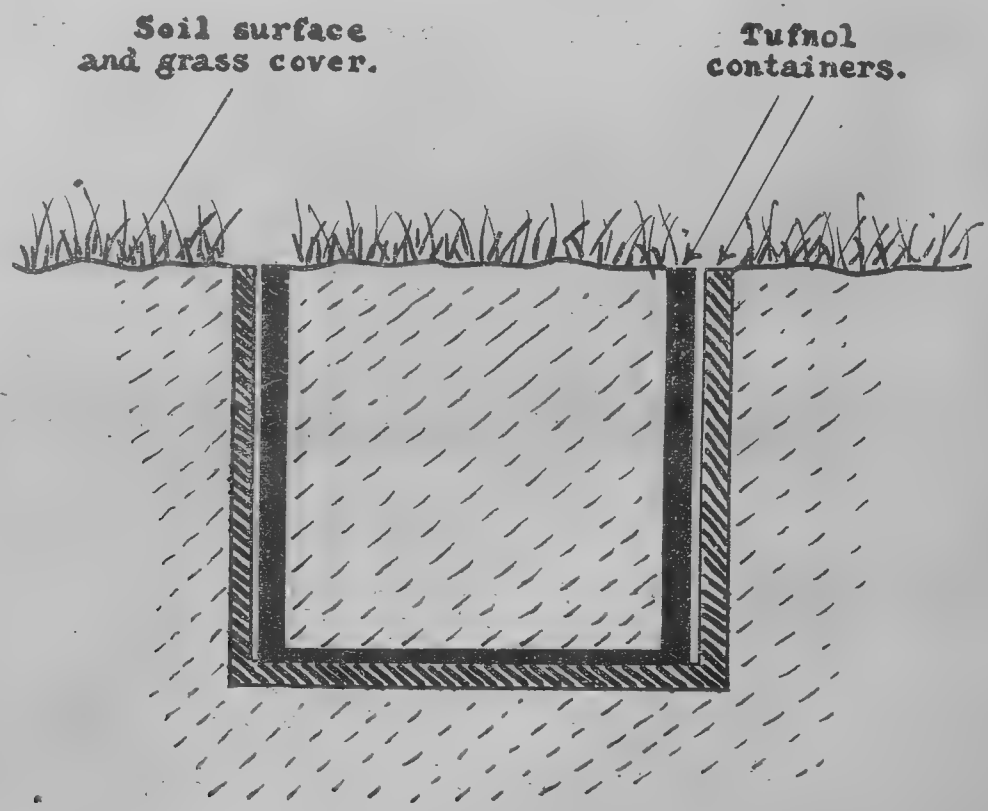


FIGURE 2(b). VERTICAL SECTION OF SOIL EVAPORIMETER

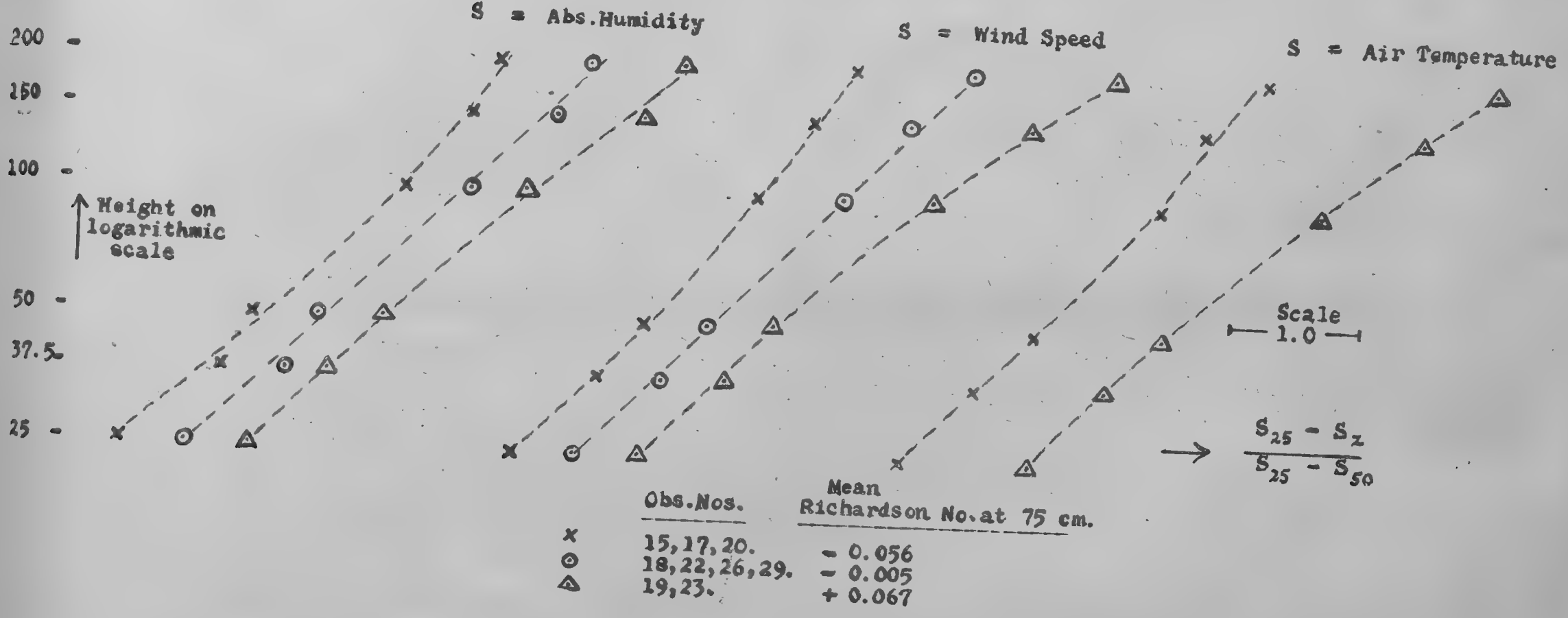


FIGURE 3(a) VERTICAL PROFILES OF ABSOLUTE HUMIDITY, WIND SPEED AND AIR TEMPERATURE ABOVE SHORT GRASS.

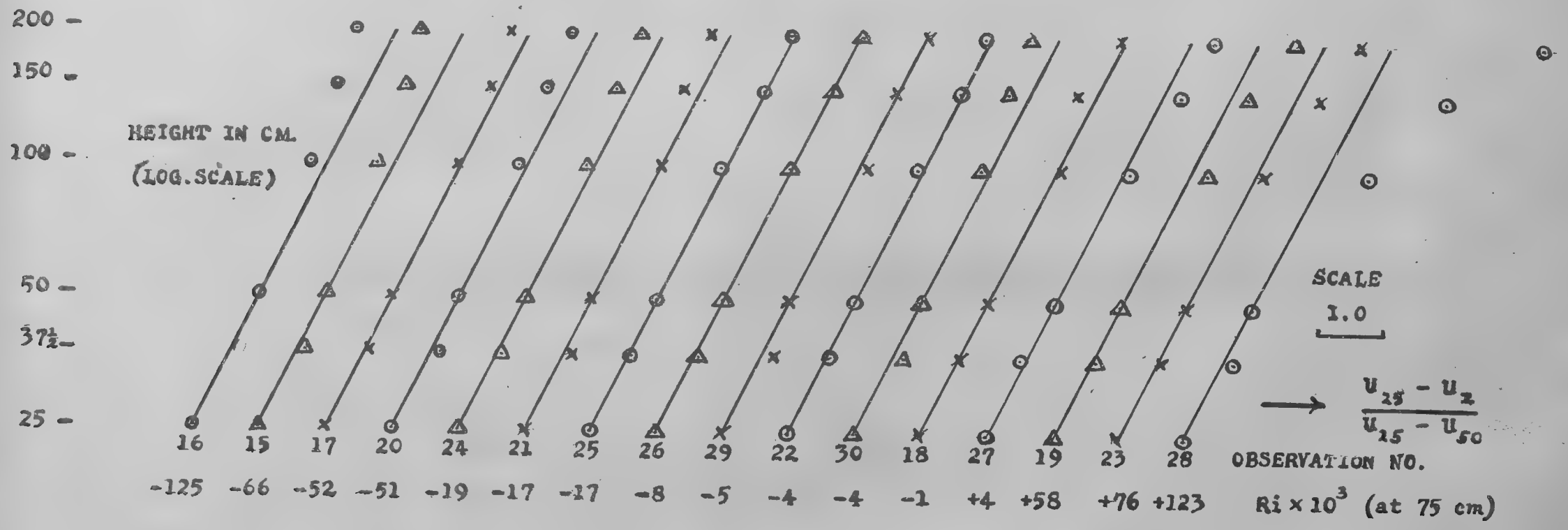


FIGURE 3(b). VERTICAL PROFILES OF WIND VELOCITY OVER A SHORT GRASS SURFACE IN RELATION TO RICHARDSON NUMBER (Ri).

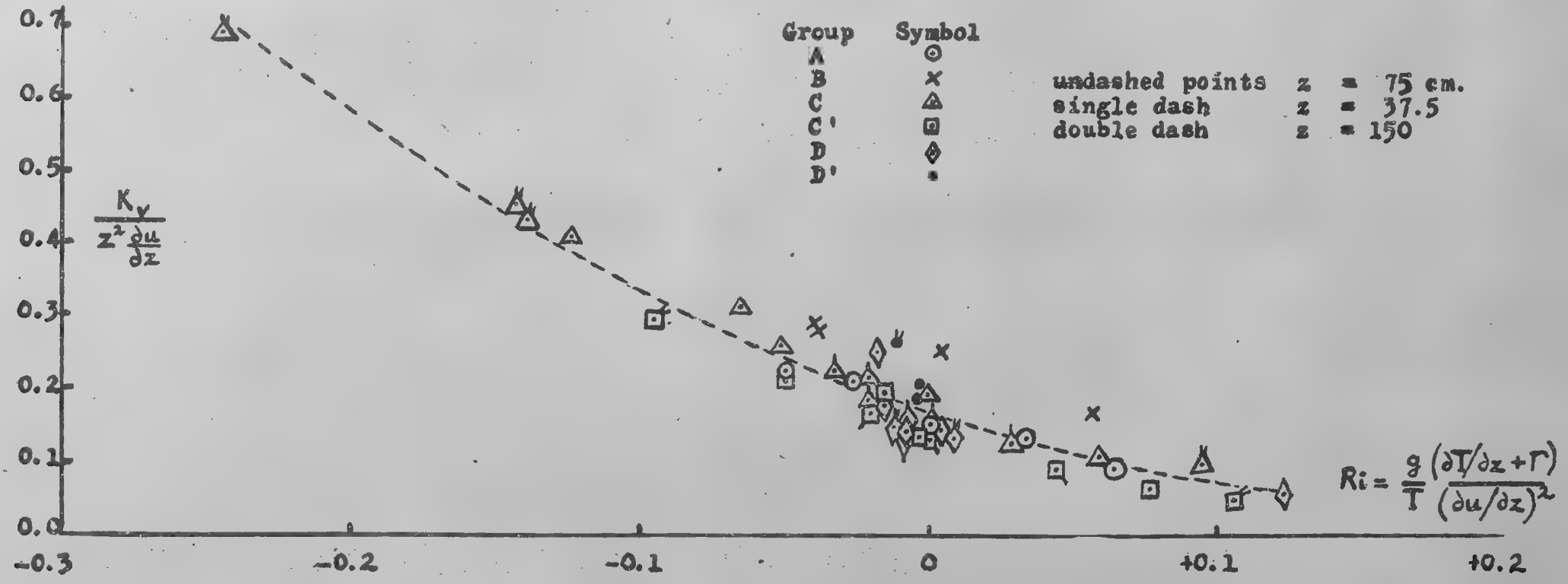


FIGURE 4. RELATION BETWEEN EDDY DIFFUSIVITY (K_v), VERTICAL WIND GRADIENT ($\partial u/\partial z$) AND RICHARDSON NUMBER (Ri) AT VARIOUS HEIGHTS ABOVE A SHORT GRASS SURFACE.

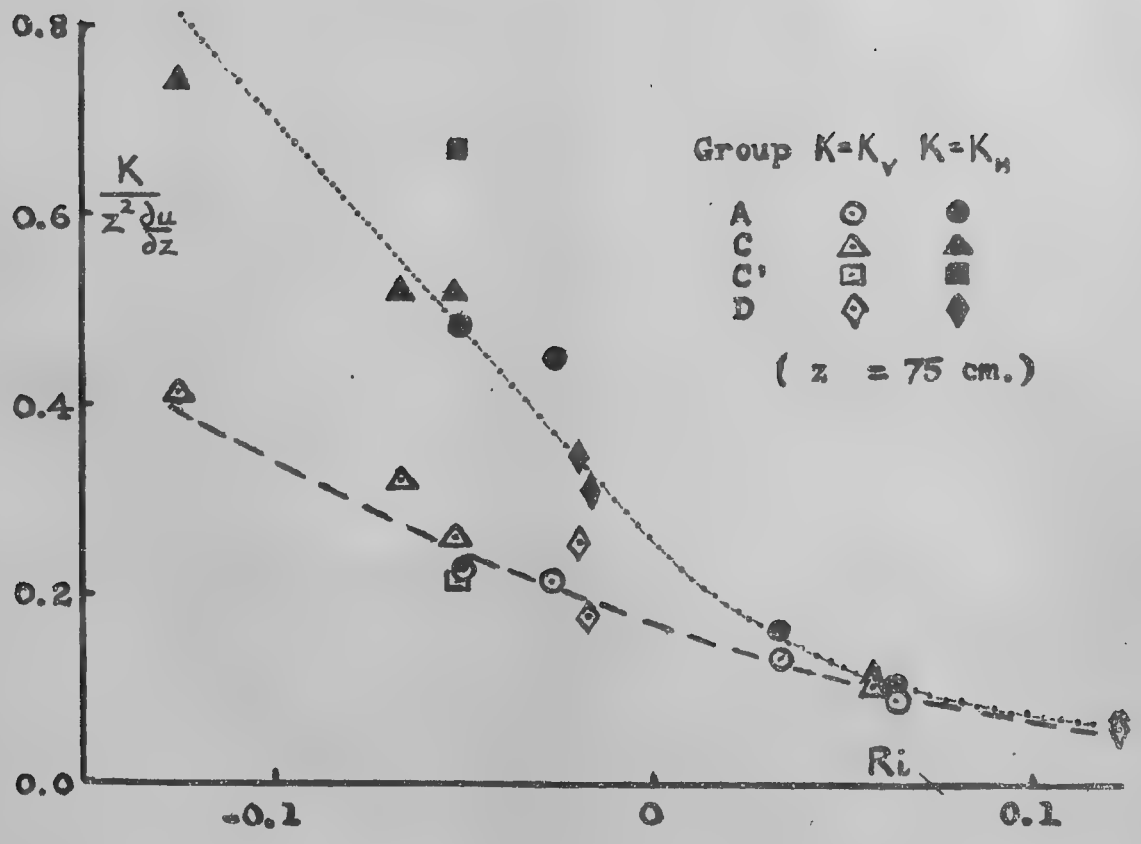


FIGURE 5. OBSERVED VALUES OF EDDY CONDUCTIVITY (K_H) AND EDDY DIFFUSIVITY (K_v) IN RELATION TO WIND VELOCITY GRADIENT ($\partial u/\partial z$) AND RICHARDSON NUMBER (Ri) AT 75 cm. OVER A SHORT GRASS SURFACE.

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

NOV 17 1952

M.R.P. 428
S.C.II/15
16th September, 1948

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

Measures of Atmospheric Circulation

by A.H.R. Goldie.

AFM-172157

1. Introduction and Summary

It is generally agreed that long-range weather forecasting is unlikely to have any substantial success until a better understanding of the general atmospheric circulation is obtained.

An atmospheric circulation index has distinct claims to study in this connection. It can be shown that the magnitude of the index is associated with certain characteristics of the weather situations, whether the day, week, month, season or even the year is taken as the unit of time. The index can conveniently be taken as the barometric gradient e.g. that for West-East flow of air can be pressure difference between two fixed points in different latitudes but in approximately the same longitude. The index for total resultant flow may be got by compounding gradients for W-E and S-N flow of air.

The second section of this note deals with the variation of the index from year to year and the relation to it of British weather and mentions in particular the inverse relationship between the W-E annual index and the latitude exchange of air as deduced from components of hourly values of wind.

The third section outlines physical explanations given by J. Bjerknes and Rossby.

The fifth section deals with the relation of the index to exceptional conditions of weather affecting long periods as for example the last ten years.

2. Circulation Index and Weather in the British Isles

Here are some deductions given in a paper written in 1935 on the Mean Annual Circulation over the British Isles (1), taking pressure difference between two stations approximately on a North-South line (Lerwick and Eskdalemuir) as an index of mean annual West-East circulation for the British Isles. I found (1) that in the previous 32 years about every 4th year had had a relatively high index, and that the index for the Western American seaboard ran through similar changes in virtually the same phase; (2) that in the years of high index the British Isles had on the whole higher mean pressure, higher temperature, substantially higher summer sunshine and lower annual rainfall; the intermediate years had more rain and most noticeably more gales. (3) From a cross section of pressure from Upernivik in Greenland to Cairo it was seen further that the years of high index showed an accentuation of low pressures in the Angmagsalik latitude and a diminished frequency of low pressure in the latitude from about Aberdeen to Greenwich and a tendency of the Azores high to extend as far as the British Isles; and that in the years of low index these positions were reversed. And further, (4) a study of the components of hourly values of wind over the British Isles over the shorter period 1911 - 21, showed that the longitude exchange of air (i.e. W + E components) reached a maximum in the years of maximum index and that the latitude exchange of air (S + N) reached a maximum in the years of low index. To these I should now add further (5) a very low seasonal or annual index brings greater liability to temporary extremes of heat or cold or gales or precipitation.

/From

From this it seems that the atmospheric circulation of our zone tends to oscillate between two conditions, the one a good steady westerly circulation with the principal "lows" far north, the other a tendency for this circulation to be broken up by latitude exchange of air with "lows" relatively far south. Though there is probably no real periodicity, the fact that the Western American seaboard ran through similar changes of index in virtually the same phase shows that it is no mere local phenomenon.

Curves of pressure differences, brought up to 1947, are shown in Fig. 1 (a) and 1(b). The upper curve, full-line, shows the difference Eskdalemuir-Lerwick. The lower curve, full-line, is the resultant barometric gradient compounded from the gradients in two directions approximately at right angles to one another, namely Eskdalemuir-Lerwick and Aberdeen-Stornoway. The resultant gradient smoothed by running four-year means is also shown on the lower diagram as a broken line. On the upper diagram the broken line shows the sunspot figures.

The extension of the curves beyond 1934 shows a change in the behaviour of the index. The period of the swings has increased from rather under four years to about five or six years and the range has increased in about the same proportion. In particular, the index for W-E flow was much lower in 1937 than at any time in the 22-year period since 1915 and again in 1939-42 and 1947 it fell to conspicuously low values. This feature of recent years will be referred to again in paragraph 5.

3. Rossby's Circulation Patterns

The physical explanation of the accentuation of low pressures in relatively low latitudes, in the years when latitude exchange of air becomes great, is rather nicely connected with a result given by Rossby in 1939, (2) following upon a paper by J. Bjerknes in 1937. (3) If f represents the Coriolis parameter ($2\Omega \sin \phi$) and ζ the vertical component of the vorticity of a mass of air relative to the earth's surface then each column of air must satisfy the condition -

$$f + \zeta = \text{constant}$$

Thus an air mass which is displaced to more southerly latitudes where the earth's rotation is weaker will experience an increasing cyclonic rotation and a mass which is displaced northward will experience a decreasing cyclonic rotation. Thus maximum latitude exchange of air should be connected with more "lows" in relatively far south positions (and corresponding weather over the British Isles) and fewer lows on the northerly tracks and vice versa.

From the vorticity equation given above it is seen that

$$\frac{d\zeta}{dt} = -\frac{df}{dt} = -v\frac{\partial f}{\partial y} \quad \left(\begin{array}{l} \text{if the y axis} \\ \text{points northward} \end{array} \right)$$

$$= -\beta v$$

where $\beta = \dfrac{2\Omega \cos \phi}{R}$ R being the mean radius of the earth.

/Rossby

Rossby applies this equation to the case of a zonal current of uniform velocity U upon which is superposed a perturbation with components u' and v' . He finds that for a sinusoidal disturbance of wave length L the rate of propagation should be

$$c = U - \frac{\beta L^2}{4\pi^2}$$

For stationary waves (i.e. $c = 0$) the length L_s is given by

$$U = \frac{\beta L_s^2}{4\pi^2} \quad \text{or} \quad L_s = 2\pi\sqrt{U/\beta}$$

Waves of greater length than L_s travel westward (c negative), shorter waves travel eastward.

Rossby computes a table of the stationary wavelengths for different values of the zonal current U and different latitudes. Obviously the stationary wavelength increases with the zonal current and with latitude. Corresponding to some values of maximum zonal velocity for February as given by J. Bjerknes there would be only 2 or 3 perturbations in latitude 60° and slightly above 4 in latitude 30°. This result is in fair agreement with the maximum observed dimensions both of the permanent lows and of the subtropical high pressure cells.

Rossby thus puts forward the view that in the Pacific and Atlantic Oceans with high index there should exist single high pressure cells of large area (with highs also over the continents) and that the Aleutian and Icelandic Lows should exist as single centres, probably to the east of their normal positions.

By contrast, with a low index the Pacific and Atlantic Highs should consist of small or indistinct systems with several centres, and the Aleutian and Icelandic Lows should move west of their normal positions with possibly a second centre in the normal position. A paper by Allen (4) gives some excellent examples of the two types. On six weeks with maximum index an average of 2.5 Low centres per chart appeared; on six weeks with minimum index an average of 4.2 per chart.

Rossby takes the circulation index as the mean around the globe of the pressure difference between latitude 35° and latitude 55° and takes the theory as applying to the weekly or 5-day average charts rather than to the daily synoptic situation. The correlation between the index in one longitude and the mean around the globe is found to be high.

4. Circulation Index as a measure of large scale turbulence

The whole picture fits in very consistently with the idea of the circulation index as a pointer to the large-scale atmospheric turbulence, the high index corresponding to efficient non-turbulent west-east circulation with few and large cells, the low index to a maximum of turbulence in this zonal circulation brought about by numerous smaller cells.

It is obvious that any long continued sequence of high W-E index, i.e. comparative absence of latitude exchange of air, would permit a strong temperature gradient between polar and sub-tropical latitudes to be built up, thus promoting the conditions compelling latitude exchange of air; and similarly that after a thorough mixing in latitude, the conditions conducive to

laminar West-East flow would be improved. Fluctuations of a roughly periodic character in the index would thus be not unlikely and the longer the index remained low the greater might the subsequent swing to high index be expected to be. The tendency to recurrences of maxima in the annual means at intervals of about four years up to about 1934 is shown in Figs. 1(a) and (b). The greater period and greater range since 1934 have already been mentioned. The mean range of the fluctuations is very considerable - about 68% of the mean value of index in Fig. 1(a), which means in the years of high index the W-E flow is on the average twice as great as in the years of low index; whilst in a year of very high index, like 1938, the W-E flow was about four times as great as in a very low year like 1937 or 1947. Such differences in an annual value should have some strong significance.

5. Exceptional Conditions in particular epochs

Daily values of a circulation index for the N. Atlantic area are discussed in M.R.P.80. The values of this index fell away so notably from the summer of 1939, as to suggest at first that from the outbreak of war the observations of pressure over the Atlantic had become inadequate for the construction of a reasonably complete synoptic chart, and on this account shortly afterwards the tabulation of the index was stopped. Figs. 1(a) and 1(b) however, showing the mean annual values of an index deduced from British observations only, - which remained complete in the war years - confirm that the four consecutive years, 1939, 1940, 1941 and 1942 (and also 1936, 1937) had lower indices than had occurred at any time previously in the period dealt with, unless in 1915.

The years 1938 to 1941 inclusive were also exceptional in that the resultant drift for these years was approximately from W. to E. in each year (actually 264° to 273°). The direction of drift has not veered so far since 1917. The average direction of the annual mean drift over 40 years is 250°. (The tabulations from which latitude exchange of air, S + N, could be calculated are not available for recent years). It now seems that the remarkable lowering of the W-E circulation in these years accounts for the exceptionally "continental" nature of the winters 1939 - 40, 1940 - 41, and 1941 - 42. In 1947 the W-E index became lower than ever and the mean direction backed to the exceptional value of 217°, not hitherto attained in the series (i.e. since 1903). The exceptional cold of the winter and dryness of the early summer of that year are fresh in our memory. The record breaking tendency of the last decade is thus coupled with subnormality in the general circulation.

Specially cold spells occurred in the winter months of 1917 and 1919, i.e. following upon the previous period of notably low index (1915) when also the direction of the mean drift went through a similar series of changes but did not veer quite so far as in 1938-41.

Clearly it would be interesting to carry the examination of the mean drift of air back to earlier years. This is not so easy as might at first sight appear, owing to the scarcity of long period observations, averaged and reduced in exactly the same way, for northerly stations. Exact computation would require a good deal of work. The most notable depression in the index, prior to that around 1915, was probably, however, from 1876 to 1878, the index being rather low also in 1875 and 1879. The winter months 1878 - 79, 1879 - 80 and 1880 - 81 were exceptionally severe.

It will be noticed that a rough approximation to the Bruckner period separates these three epochs of subnormal circulation and exceptional weather conditions.

A curve of sunspot numbers added to Fig. 1(a) shows rather outstanding sunspot numbers around two of the epochs referred to above, with an exceptionally high sunspot number in 1947, the record breaking year of weather.

/Going

Going back to early years, however, the sunspot numbers were not high around 1877, but had been unusually high in the years 1870 - 72, higher than at any time for at least 60 years before 1917.

6. The choice of an Index

Four kinds of index have been mentioned above, (1) the annual or monthly mean pressure difference between two fixed points (Esk., Ler.) only some 7° apart in latitude, (2) the resultant gradient across a fixed area of similar dimensions, (3) the Rossby index - means round the globe for 5 day periods for a wide but fixed belt, namely 35° to 55°N, (4) a daily index as used in MRP/80 of total circulation in the N. Atlantic, in the form of the difference between the highest pressure within the area 10°W - 70°W and southward of 50°N and the lowest pressure within the area 20°E - 60°W and northward of 50°N. (Mostly this means the highest pressure in the Azores High and the lowest in the Icelandic Low).

When the object is correlation with European or British weather, the last-named index has claims to attention, though it would be desirable to take into account also the mean latitude of the major air stream whose flow it measures. Shifts in latitude presumably signify equally with changes in speed.

The inertia of this Atlantic air stream is considerable. The mean day-to-day fluctuation of this index is found to be about 16% and it is seldom that twice this rate of change is exceeded. When 5-day means are used, the mean rate of change per 24 hours ranges from some 2% in quiet periods to about 5% in very disturbed periods. The tendency to recurrences of high values of this index is discussed to some extent in MRP.80. It is shown that these recurrences are consistent with common winter and summer wave-lengths of the order of 3,600 and 2,900 miles respectively. This is in agreement with Rossby's theoretical conclusion that the predominant wave lengths in middle latitudes should be of the order of 3,000 miles, more in winter, less in summer.

The westerly circulation in high southern latitudes where the track is almost entirely a sea track uninterrupted by continents with their seasonal variations of temperature is more intense and appears to be much more uniform in summer and winter than in northern latitudes; it would therefore be of special interest to know in greater detail how it does in fact vary from season to season or year to year. It may be recalled that Sir Gilbert Walker found a periodicity of just under 4 years - which he regarded as real - in the pressure of Darwin, Australia.

References

1. Some Characteristics of the mean Annual Circulation over the British Isles. Q.J.R. Met. Soc. LXII No. 263 Jan. 1936.

2. Journal of Marine Research Vol. II No. 1 (1939)

3. Die Theorie der aussertropischen Zyklonenbildung Met. Zeit. 54 (1937)

4. Statistical Studies of certain characteristics of the general circulation of the northern hemisphere. Q.J.R. Met. Soc. 66 Suppt. (1940).

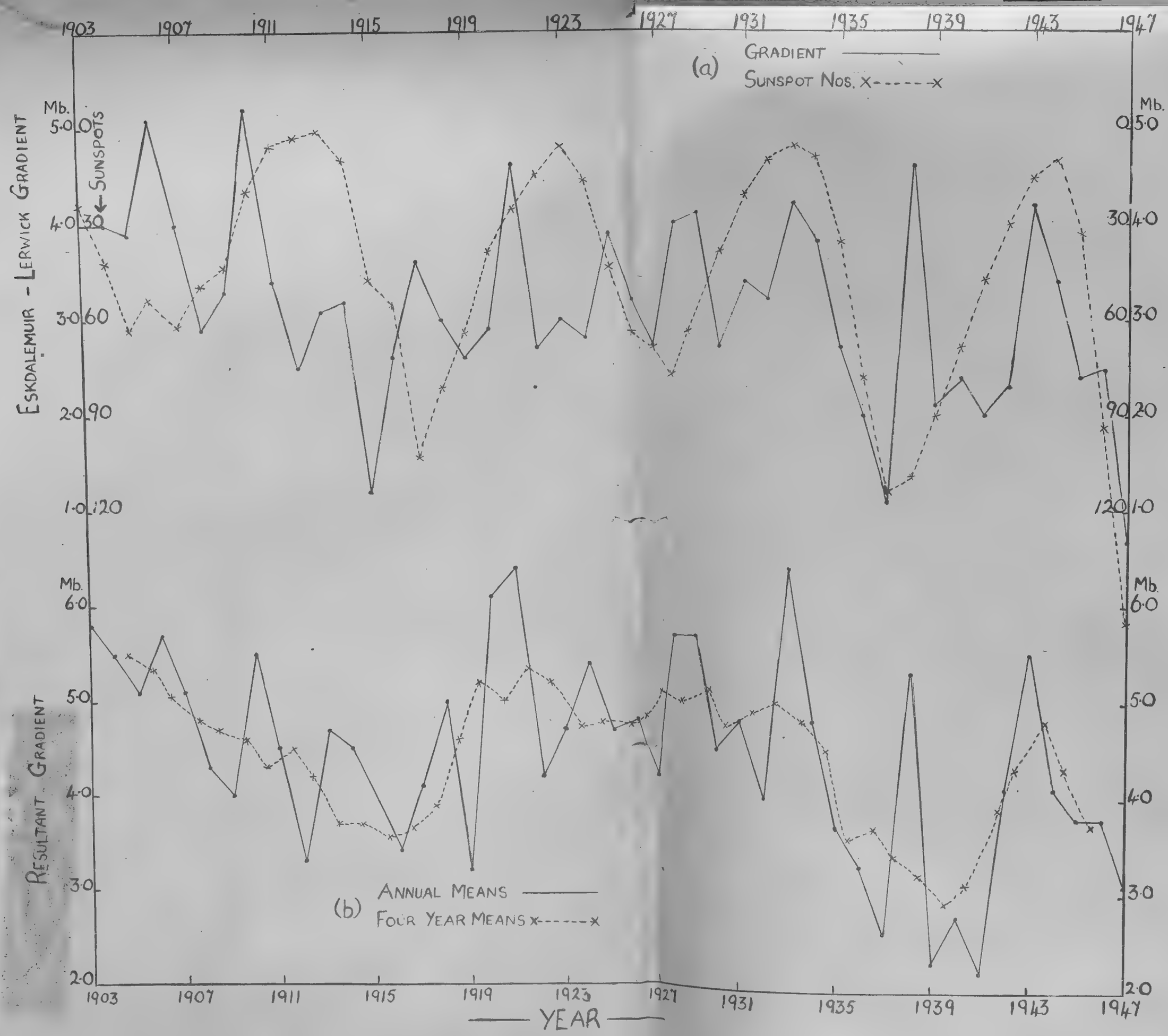


FIG 1 VARIATION OF ATMOSPHERIC CIRCULATION FROM 1903-1947.

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

M.R.P. 436
S.C.III/12
1st October, 1948.

AIL-9453

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

Meteorological Factors in the Occurrence of Turbulence at High Altitudes : Interim Report.

by

J.K. Bannon

NOV 17 1952

Summary

The data given in R.A.E. Report No. AERO 2235 have been studied further and observations of turbulence by the Meteorological Research Flight and by the BEA Gust Research Flight have also been analysed.

It appears that bumpiness at high altitudes is often associated with strong wind shear in the horizontal and also with strong winds; the latter is probably a consequence of the former. Bumpiness is frequently experienced at the tropopause or other discontinuity in the temperature lapse rate with height; this is not inconsistent with horizontal wind-shear being the immediate cause of this turbulence.

Regions are indicated on the weather map where strong wind shear in the horizontal is usually found, at heights above 20,000 ft.

1. Introduction

In previous work on this subject (references (1) and (2)), attempts were made to correlate the occurrence of bumpiness between the heights of 20,000 and 40,000 ft. with the temperature lapse rate with height, the rate of change of wind with height, the 'Richardson' criterion, the tropopause and the height. Further data have become available since these reports were prepared and these have been analysed from the meteorological aspect; the data presented and discussed in (2) have also been studied further.

Some of the meteorological factors are difficult to measure accurately and in many cases they have been interpolated either in space or in time (or both) from observations made at the radio-sonde stations in the British Isles. Individual instances, therefore, may give misleading results since the interpolation was necessarily linear and this may not represent the facts. However, when a number of observations are considered, it is thought that a statistical result from them will be significant.

A noteworthy aspect of high altitude turbulence, is that it is often experienced through a shallow layer only. Present wind-finding techniques and reporting codes are not sufficiently accurate to measure sharp discontinuities in wind, (wind observations used and quoted in this report were measured as means through layers about 3,000 ft. thick, centred on the particular height in question), and it may be that, for example, a sharp change in wind at a particular height which cannot be identified as a sharp change at present, can give rise to a shallow layer of bumpiness. However on 7 occasions (the first 7 of Table II) the original observations were used to evaluate winds over layers between 1,000 and 1,500 ft. in thickness; (the results have not been utilised in Table II). Greater wind shears with height were found than were apparent from the ordinary observations meaned through layers of about 3,000 ft. (as given in Table II) but these greater shears were not, except in one instance, at the bumpy level, as far as could be judged.

2. R.A.E. Spitfire Data

The observations given in Table I of (2) together with 9 other observations made by the same aircraft, have been re-analysed from the meteorological point of view; the results are given in Table I. Owing to the small amount of

/information

information available on the subject of bumpiness in the upper air it has been considered desirable in this Table and in Tables II and III to give details of the accompanying meteorological structure very fully.

The bump frequency given in this Table I is only a very rough indication of the degree of turbulence encountered for it takes no account of severity, being merely the frequency of occurrence of any sized gust; for full details of the gusts measured, reference must be made to the original paper.

$\partial v/\partial z$, the rate of change of wind speed with height, was in most cases taken from the wind observation at Larkhill nearest in time to the flight over Farnborough; in some cases observations made at Downham Market or Shoeburyness were utilised. As emphasised in Section 1 above, these wind observations will not show a sharp change in wind over a small height.

$-\partial T/\partial z$, the temperature lapse rate with height, was evaluated from the observations of temperature made by the investigating aircraft or from the observations of upper air temperatures made at Larkhill or Downham Market by radio-sonde when the former appeared unreliable. (On 18 occasions very large lapse rates of temperature, exceeding the dry adiabatic lapse rate, were recorded by the aircraft and the temperatures did not agree, even approximately, with those observed at adjacent radio-sonde stations). Since the aircraft temperatures were given to the nearest ^{o}F, the radio-sonde observations were probably preferable for computing $\partial T/\partial z$ in any case.

The horizontal wind-shear was calculated using the wind observations nearest in time for the 3 stations Larkhill, Downham Market and Fazakerly (Liverpool); this, strictly speaking, should give the horizontal shear over the Midlands but has been applied to Farnborough as it is the best observation possible from the data available. Occasionally a wind determination at Shoeburyness was of use. The convention as to sign was as follows: if v is the horizontal wind and n is the direction at right angles to v and to the right (i.e. away from low pressure), and if r is the radius of curvature (positive for cyclonic motion) of the streamline through the point in question, then the horizontal wind-shear

$$= \frac{\partial v}{\partial n} - \frac{v}{r}$$

From the three observations of wind the calculation of $\partial v/\partial n$ was not difficult; the estimation of r, however, was often difficult and the values of the horizontal wind-shear deduced may sometimes be considerably in error. On many occasions one or more of the wind observations was missing, or it was impossible to estimate r, and in these cases it was not possible to evaluate the horizontal shear.

The wind speed and direction were taken from the observation, nearest in time, at Larkhill, or interpolated from other observations.

It will be noted that the height of the tropopause as given in Table I does not always agree with that given in the original report: this is because in the latter, if a double tropopause was observed, the height of the upper tropopause was always given.

3. Meteorological Research Flight Data. (Mosquito aircraft)

41 successful high altitude flights were made by the M.R.F. in the period 8th December, 1947 to 18th August, 1948, inclusive; the greatest height attained on each flight was usually between 38,000 and 40,000 ft. On 23 of these ascents bumpiness, often only slight, but occasionally severe, was experienced. Table II gives details of these flights and the weather at the time.

As in Table I the wind observations, including shear in the vertical and shear in the horizontal, were interpolated from the observations at the radio-sonde stations Larkhill, Fazakerly and Downham Market, with the aid of occasional observations from Shoeburyness.

/The

The lapse rate of temperature and the position of the tropopause were determined from the careful observations of temperature made from the aircraft; these observations, sometimes made every 500 ft. but usually every 1,000 ft., are thought to be accurate to about ± 0.3°F.

The slope of the tropopause surface was estimated from the aircraft observations and from the observations at the radio-sonde stations; (Larkhill, Fazakerly, Downham Market, Penzance and occasionally those on the Continent). In some instances the tropopause surface may have been discontinuous so that the slope, as given, will have no real meaning.

It is of interest to note that bumpiness was reported at Lyneham on 18th July, 1948, at similar heights as bumpiness was found above Farnborough about an hour later.

4. Data from B.E.A. Research Mosquito Aircraft and other sources

The BEA Mosquito Flight have made 37 flights to investigate bumpiness above 20,000 ft., between 3rd March and 5th August, 1948. The majority of the flights were over the British Isles but several continental flights were also made. On most of these flights at least a few small bumps were encountered; on 10 of them the bumpiness was appreciable or worthy of study and details of these flights are given in Table III. Two cases of bumpiness reported by R.A.F. aircraft over N.W. Germany are also listed in the same Table.

As in the other Tables wind data have been interpolated from the radio-sonde observations available from the British Isles or N.W. German network of stations.

The height of the tropopause and its slope were determined by the radio-sonde observations in the neighbourhood and the temperatures recorded by the aircraft (which were in reasonable agreement). The lapse rate of temperature was also interpolated from the radio-sonde observations except when special temperature readings, every 1,000 ft., were made from the aircraft. (6th April and 6th July).

Outside the British Isles, except over or near N.W. Germany, upper air observations, especially of wind, were very sparse and many details have had to be omitted.

5. Bumpiness and Horizontal Shear of Wind

It was noticed that on several occasions of bumpiness there was a large change of wind in the horizontal. Accordingly the occurrence of bumps has been analysed in relation to the horizontal wind-shear. (See Section 2 for definition).

(a) Normal Frequency of Occurrence of Various Horizontal Wind-Shears

The average or 'normal' frequency of occurrence of various horizontal wind-shears was determined by evaluating the horizontal shear over the Midlands using the three wind-stations Larkhill, Downham Market and Fazakerly, for each of the heights 450 mb. (20,000 ft. approximately), 300 mb. (30,000 ft. approximately) and 200 mb. (38-40,000 ft.), for every occasion during the months October 1946, January 1947, April 1947 and June 1947, for which there were observations. The frequencies did not vary greatly from season to season. Table IV shows the percentage frequencies of occurrence of various wind shears in the horizontal for the three levels, for all the seasons together.

/Table IV.

Table IV

Percentage Frequency of Occurrence of Various Horizontal Wind Shears

(All Seasons)

Horizontal Wind-Shear (hours^{-1})

Height in ft.	0 - 0.04	0.05- 0.09	0.1 - 0.14	0.15- 0.19	0.2 - 0.24	0.25- 0.29	0.3 - 0.34	0.35- 0.39	0.4 up.	No.of obs.	Mean shear
20,000	43	30	14	6	2.6	1.5	0.9	1.2	0.6	341	0.074
30,000	36	33	17	7.7	2.3	1.1	0.8	0.4	1.9	261	0.083
40,000	52.5	24	9	6	1.2	3.6	2.4	1.2	0	166	0.072
Mean of above	42	30	14	7	2.2	1.8	1.2	0.9	0.9	768	0.077

It will be noted that horizontal shear is slightly larger at 30,000 ft. than at 20,000 ft. and 40,000 ft. Wind observations are fewer in number at 30,000 ft. than at 20,000 ft. and still less at 40,000 ft; the reason is that often the balloon and radar target are carried out of range at great heights when the wind is strong. Thus the observations may not be representative of all conditions since strong wind occasions will not appear. It is likely that occasions of strong wind will be more frequently associated with large shear than light winds and so Table IV may underestimate the frequency of occurrence of the larger shears. However, since these 'normal' frequencies will be compared with frequencies for bumpy occasions which were also liable to be non representative of high wind, the discrepancy is probably unimportant for our purpose.

(b) R.A.E. Spitfire Data

Bumpy layers of 4,000 ft. or more in depth were noted (see Table I) on 16 occasions for which horizontal shear data were available. The percentage frequency of occurrence of various horizontal wind shears in these bumpy layers is given in Table V. The mean horizontal shear in these 16 cases was 0.31 hours^{-1}

Table V

Percentage Frequency of Occurrence of Various Horizontal Wind-Shears in Bumpy Layers 4,000 ft. or more in depth.

Horizontal Wind-Shear (Hours^{-1})

	0 - 0.09	0.1 - 0.19	0.2 - 0.29	0.3 - 0.39	0.4 and over	Total no. of occasions
Percent. Frequency	12	38	12	19	19	16

The total number of bumpy layers observed, regardless of depth, for which there were horizontal shear observations, was 49. Table VI gives the percentage frequency of occurrence of various horizontal wind-shears in these bumpy layers. The mean horizontal wind-shear in these 49 cases was 0.17 hours^{-1}

/Table VI

Table VI

Percentage Frequency of Occurrence of Various Horizontal Wind-Shears in Bumpy Layers of all Depths

	Horizontal Wind-Shear (hours $^{-1}$)					
	0 - 0.09	0.1 - 0.19	0.2 - 0.29	0.3 - 0.39	0.4 and over	Total no. of observations
Percent. Frequency	39	33	12	6	10	49

If there is no relation between horizontal wind shear and the occurrence of bumpiness then the ratio of the frequency of occurrence of various horizontal wind shears in the turbulent and 'normal' cases should be unity. That this is far from the case is easily seen from Tables IV, V and VI.

Figures 1 and 2 show these ratios for the cases of deep bumpy layers (4,000 ft. or more), and all bumpy layers, respectively, the 'normal' frequencies being taken as the means over the three heights, as given in Table IV.

It is seen from these that horizontal wind-shear, or something closely connected with it, is an important factor in the occurrence of turbulence at high altitudes. The significance of these figures lies not only in the ratio F_{BD}/F_S being greater than unity but also in the way the ratio increases rapidly (particularly in Fig. 1) as the shear increases.

(c) M.R.F. Data

Wind shear in the horizontal was calculated for 18 out of the total of 39 bumpy layers observed; the mean horizontal shear for these 18 cases was 0.15 hours $^{-1}$. It was not possible to estimate the shear in the other cases.

Table VII shows the percentage frequency of occurrence of various horizontal wind-shears in the bumpy layers for all these 18 cases, also for those cases of bumpiness not associated with the tropopause, and also those cases not at the tropopause nor at a check in the lapse rate of temperature with height. (See Section 6 below). (As only 7 cases of bumpiness associated with the tropopause were noted by the R.A.E. Spitfire and as the detailed temperature structure of the air in the vertical was not known, this sub-division was not attempted in para. (b) above).

Table VII

Percentage Frequency of Occurrence of Horizontal Wind-Shears

(i) all occasions.

(ii) all occasions not associated with the tropopause.

(iii) " " " " " " " and not at a check in the lapse rate of temperature with height, and

(iv) for comparison, the 'normal' frequency from Table IV.

	Horizontal Wind-Shear (hours^{-1})						
	0 - 0.09	0.1 - 0.19	0.2 - 0.29	0.3 - 0.39	0.4 and over	Mean Shear	Total no. of obs.
(i)	43	28	17	6	6	0.15	18
(ii)	34	34	16	8	8	0.18	12
(iii)	33	45	22			0.13	9
(iv) 'normal'	72	21	4	2	1	0.077	-

(d) B.E.A. and other Data

Horizontal wind-shear observations were available for 7 out of the 14 bumpy layers observed. (see Table III), the mean shear being 0.12 hours^{-1}; omitting one case in which the bumpiness appeared to have been caused by convectional currents, (14th June, 1948), the mean was 0.13 hours^{-1}. In two of the other cases at least, (15th, 16th July, 1948), for which horizontal shear data was not available, the horizontal shear may well have been above average.

The number of observations was too small to consider frequencies.

In all the data, R.A.E., M.R.F. and B.E.A., no relation could be found between horizontal wind-shear and either the frequency of bumps or the magnitude of the greatest bumps.

6. Bumpiness and the Tropopause or other Check in the temperature lapse rate with height

Reference (1) indicated that bumpiness was frequently associated with the tropopause or other temperature lapse rate discontinuity, 13 occasions occurring within 1,000 ft. of the tropopause and 5 at a marked slowing up of the temperature lapse rate, out of a total of 37 bumpy layers.

As stated in reference (2), this was not confirmed by the R.A.E. observations, a possible explanation of the difference between the two results being that usually only accelerations of ± 0.1g or more were considered in the R.A.E. report while those observations made at Aldergrove were qualitative and may well have included many instances of bumps which would have been too low to be recorded by the R.A.E. accelerometers.

The M.R.F. and B.E.A. results confirm the first report (1). Of the 39 bumpy layers observed by the M.R.F., 17 were at or just above, (within 1,000 ft. of), the tropopause or between a double tropopause, and a further 5 were at a temperature lapse rate check. As emphasised in Section 3, the temperature observations on these flights were made frequently and carefully and it is seen that the bumpy layers were not below the inversion or slowing up of the temperature lapse rate but at it or in the smaller lapse rate. The mean lapse rate for these 22 cases, (tropopause or check in lapse rate), was - 0.9°F/1,000 ft. (i.e. an inversion); of the 17 other observations, the mean lapse rate in the troposphere was + 2.8°F/1,000 ft., (13 cases), that in the stratosphere being - 1.7°F/1,000 ft., (4 cases).

Of the 14 B.E.A. and other observations, 7 were at the tropopause, (including 2 deep layers which overlapped the tropopause), and one was in a marked temperature inversion; the mean lapse rate of temperature at these layers was - 3.9°F/1,000 ft. as compared with a mean lapse of 4°F/1,000 ft. for the other (4) cases in the troposphere and - 0.7°F/1,000 ft. in the stratosphere, (3 cases).

It must be remembered that the M.R.F. observations of turbulence were qualitative only, as was the case of the observations quoted in (1), but on the two most bumpy occasions, (6th and 26th February, 1948), the turbulence was associated with the tropopause. The B.E.A. observations, however, were made with recording accelerometers; of the cases associated with the tropopause or check in the lapse rate of temperature, 5 out of the 8 occasions showed accelerations of more than 0.1g, the other 3 occasions having bumps of 0.1g.

It would appear therefore that bumpiness is often associated with the tropopause or other check in the temperature lapse rate with height, and that though the bumpiness may well be small on many of these occasions, (most observations being qualitative only), considerable turbulence is sometimes found there, - perhaps more often than the report (2) would indicate.

/7.

7. Sloping Surfaces of discontinuity (check) in temperature lapse rate with height

It is easily seen that though there may be little horizontal wind-shear below a surface marking a temperature inversion, if the surface is sloping there may be appreciable horizontal wind-shear above the surface, even though the temperature structure of the air above the surface is quite regular.

This may be illustrated quite simply by the following model. Suppose that the general wind direction at all heights is along the x axis. The z axis is taken vertical and the y axis is such that the three axes form an orthogonal right-hand system. Suppose also that the lapse rate of temperature with height is constant in the troposphere.

$$\text{i.e.} \quad -\partial T_1/\partial z = L \qquad (\text{suffix}_1 \text{ refers to troposphere})$$

Also that temperature decreases at a constant rate in direction y.

$$\text{i.e.} \quad -\partial T_1/\partial y = N$$

and L is independent of z. This implies an increasing wind with height and a situation approximating to this is sometimes experienced in the troposphere. (A somewhat similar model is considered by Charney, (3)).

It is well known that, to a first approximation, the wind blows along the contours of the tropopause surface with low tropopause over the cold troposphere. (A casual glance at a series of charts of tropopause contours will be sufficiently convincing). We may therefore assume that the tropopause above our model troposphere slopes downwards in direction y, at an angle α say.

Assume now that there is a constant increase of temperature with height in the stratosphere,

$$\text{i.e.} \quad \partial T_2/\partial z = I \qquad (\text{suffix}_2 \text{ refers to stratosphere})$$

Then it can easily be shown that continuity in T across the tropopause requires:-

$$\frac{\partial T_2}{\partial y} = (I + L) \tan\alpha - N$$

In this model the horizontal shear of the geostrophic wind, u_g, (assumed constant at $z = 0$), is everywhere zero in the troposphere. It can be shown, (the algebra is simple if rather long), that the horizontal shear of the geostrophic wind above the tropopause is not zero but is in the form of a quadratic in $\tan\alpha$. If, for example, we take:-

$L = 4^\circ F/1{,}000$ ft.

$N = 2.9^\circ F/100$ miles (corresponds roughly to a 'thermal wind' of 2mph/1,000 ft.)

$I = 1^\circ F/1{,}000$ ft,

/then

then if $\tan \alpha$ = 4,000 ft. in 400 miles

i.e. = 1/500 approximately,

then $\partial u_g/\partial y$ = -0.04 hours $^{-1}$ in the stratosphere.

If the slope of the tropopause is twice as great, then

$\partial u_g/\partial y$ = - 0.16 hours $^{-1}$.

It must be emphasised that the above model merely serves as an illustration of how a surface of discontinuity in temperature lapse rate may also, especially if it is sloping, be a surface of discontinuity in horizontal wind-shear. If horizontal shear of wind is an important factor in high altitude bumpiness, and it appears from Section 5 that it may be, then the fact that bumpiness very often occurs just above a sharp change in temperature lapse is, if not fully explained, at least understandable.

No attempt has yet been made to correlate slope of the tropopause surface, as calculated, with bumpiness.

8. Wind-shear in the Vertical

In reference (2), the possible relation between bumpiness and wind shear in the vertical was fully discussed for the R.A.E. Spitfire data, with inconclusive results; accordingly in this report, little attention has been paid to this aspect. The mean vertical wind-shear for all bumpy layers listed in Table II for the M.R.F. flights, was 2.1 kts./1,000' (regardless of sign) and the normal mean vertical wind-shear through the layer 20,000' - 40,000' for all seasons is about 2.0 kts./1,000', (from Table VIII). It would seem, therefore, that there is no strong relation between bumpiness and vertical wind-shear and this is roughly confirmed by a scrutiny of Table II. Table III also supports this view.

9. Turbulence and Wind Speed

Wind speeds were, on the average, above normal on bumpy occasions. The following statistics indicate this.

(a) R.A.E. Spitfire Observations.

Since there is considerable variation of the frequencies of various wind speeds at a given height, from season to season, the wind speeds which occurred on bumpy occasions should be compared with the 'normal' for the appropriate season. Unfortunately the number of observations was not sufficient to make a proper comparison of the frequencies of various wind speeds on bumpy occasions with the 'normal' frequencies. The Table VIII shows the mean wind speeds at which bumpiness was observed, in the four height ranges 20-25,000 ft., 25-30,000 ft., 30-35,000 ft. and 35-40,000 ft., for the four seasons, and also, for comparison, the mean wind speeds in these layers derived from all wind observations made at Larkhill over a period of several years. (Data supplied by M.O.18). The means are also shown for only bumpy occasions on which horizontal wind-shear was estimated.

Table VIII

(i) Mean Wind Speeds on Bumpy Occasions, in Knots.

(ii) " " " " " " for which there were horizontal wind-shear observations.

(iii) Normal mean wind speeds.

/Height

		Height			
Season		20-25,000 ft.	25-30,000 ft.	30-35,000 ft.	35-40,000 ft.
Dec. to Feb.	i	39	101	61	38
	ii	-	101	70	38
	iii	47	53	57	42
March to May	i	54	72	50	62
	ii	54	72	50	45
	iii	37	41	44	26
June to Aug.	i	29	42	54	36
	ii	41	40	54	32
	iii	38	43	46	27
Sept. to Nov.	i	55	87	92	93
	ii	55	76	82	-
	iii	44	51	55	34
All seasons	i	46	62	64	49
	ii	49	61	61	37
	iii	41	47	51	32

The 'normal' mean wind speeds for the top two layers above may be too low, since, as pointed out in Section 5 above, strong winds cannot always be measured at great heights.

(b) M.R.F. Observations

Since the number of observations was small no seasonal comparison was attempted. Table IX shows the mean wind speeds for bumpy occasions in various height ranges and the 'normal' means for Winter, Spring and Summer combined, for comparison. Again it would seem that there is some connection between high winds and the occurrence of bumpiness.

Table IX

(i) Mean Winds on Bumpy Occasions, in Knots.

(ii) " " " " " for which there were horizontal wind-shear observations.

(iii) Normal mean wind speeds. (Period December to August).

		Height			
Season		20-25,000 ft.	25-30,000 ft.	30-35,000 ft.	35-40,000 ft.
Dec. to Aug.	i	56	53	53	50
	ii	77	48	51	39
	iii	41	46	49	32

(c) Other Observations

Since 'normal' wind data are not available for all the areas covered by the flights listed in Table III and since the number of observations is small, no analysis with respect to wind speed has been attempted.

As pointed out in Section 5, large wind shear in the horizontal is more likely to occur with strong winds than with light; this is borne out by the Tables IV and VIII, from which it is seen that large shear is more frequent at 30,000 ft. as is also high wind speed.

/The

The fact that both wind shear in the horizontal and wind speed appear to be related to bumpiness is thus probably not significant, the one resulting from the other. It is possible to attempt a physical explanation resulting from horizontal wind-shear, but it is much less satisfactory to postulate high wind alone as the cause of bumpiness; for this reason the former is preferred.

10. Regions of Large Wind Shear in the Horizontal

Section 5, has indicated that turbulence or bumpiness at high levels is associated with abnormally high horizontal wind-shear. It is of interest, therefore, to know in what regions of the weather map large horizontal wind-shear is usually to be found, above 20,000 ft.

The following notes are the result of a cursory and short study of weather charts for the first seven months of this year and the results found should be treated as tentative and of a preliminary nature. It is hoped to make a more detailed study of the location and development of strong horizontal wind-shear in the near future.

As would be expected, large horizontal wind-shears are found near the main polar front, in temperate latitudes, and it would seem that the largest horizontal wind-shears are usually to be found in the 20-40,000 ft. layer, at any rate in connection with the polar front. Greater than average horizontal wind-shear does occur in other situations but the largest shears noted have been in or near the region of large horizontal temperature gradient.

The 'jet stream' found in a general Zonal (westerly) current over North America has been described by several authors, (4, 5 and 6), and Hovmöller (7) has also pointed to its occurrence over Western Europe. Roughly, in a general westerly current, a north-south vertical cross section of the atmosphere shows that the strongest winds are confined to a small region usually only a few hundred miles across and 10-15,000 ft. in depth, the axis of this jet being at a height of 30-35,000 ft., usually, and above the main polar front surface and a little to the north of the intersection of the polar front with the earth's surface. Figure 3 is a rough diagram showing the type of velocity distribution in this cross section and the usual arrangement of frontal surface and tropopause surfaces. Most of the authors quoted above assume that the tropopause surface is discontinuous in the region of the 'jet stream' and this may well be so though the present writer could discover no real observational evidence for this assumption in their expositions. In several cases observed over the British Isles the tropopause heights observed at the radio-sonde stations were quite consistent with a gradually sloping surface of discontinuity between troposphere and stratosphere; in others, a discontinuous surface seemed likely.

In such a flow there will be quite a large wind shear in the horizontal to north and to south of the jet: in Figure 3, for example, the horizontal wind-shear will be about 0.3 hours^{-1} on the north side. It is often much greater.

This so called 'jet stream' has, so far, only been discussed for approximately westerly steady flow. A study of the wind distribution over the British Isles during the first seven months of 1948 has shown, however, that a 'jet' or region of strong winds aloft, often exists, in the same approximate position relative to a well marked front, whether the front be quasi stationary, (as was the case in most of the American discussions), or moving as a warm or cold front. The height of the axis of the 'jet' varies considerably from case to case, though it does appear to maintain roughly the same height during a particular disturbance; it has been noted over the British Isles as low as 20,000 ft. (7th February, 1948) but usually the height is about 30-35,000 ft.

Figures 4, 5 and 6 show the approximate relation of the axis of the 'jet' to various typical disturbances. It has been noted that the jet is discontinuous across a depression newly beginning to occlude but it is not known if this is always the case. Where the 'jet' appears to over-ride an occlusion (as in Figure 6), the occlusion must be shallow, of course, and without the associated

/warm

warm upper mass of air of the typical 'norwegian' occluded front. It was also noted that in one case of a weak wave disturbance travelling quickly and without significant change, the axis of the 'jet' appeared to run right over the disturbance (which must have had little vertical depth) as if it were not there and the polar front were several hundred miles to the south instead of being bent northwards to the feeble secondary.

It has also been noted that the 'jet' seems to be closer behind the surface cold front (of the order of 300-500 miles) than it is ahead of the warm front, (400-800 miles).

In view of the small number of occasions considered, and the cursory manner of the scrutiny, the above general results should not be taken as hard and fast rules but merely as a rough indication of the regions of strongest winds and therefore often having regions of strong horizontal wind-shear on their boundaries.

11. Conclusions

Bumpiness in clear air, at heights above 20,000 ft., has been shown to be frequently associated with the tropopause or some other discontinuity in the lapse rate of temperature with height (Section 6). It has also been shown to be associated with abnormally large wind shear in the horizontal and also with above average wind speed; the latter is thought to be mainly because of its association with the former, it being easier to see a physical link between turbulence and horizontal wind-shear than between turbulence and wind speed. (Sections 5 and 9).

Sloping surfaces of discontinuity in the lapse rate of temperature may well be sources of horizontal wind-shear so that horizontal wind-shear may be an important factor in the creation of bumpiness often experienced at these surfaces. (Section 7).

Bumpiness has usually been attributed to the aircraft encountering up or down currents with more or less sharp boundaries. It is difficult, however, to understand how vertical turbulent currents can exist in the thermally excessively stable stratifications of an inversion, where bumpiness is often found. It is suggested, therefore, that turbulence experienced at high levels is often associated with variations in the horizontal wind and not with vertical air motion, i.e. the eddies are in approximately horizontal planes. This idea is in keeping with the formation of the eddies by horizontal shear.

No explanation can be put forward, as yet, for the fact that high altitude turbulence is often limited to very shallow layers, and sometimes apparently limited also to a small area in the horizontal.

/References

References

(1) Bumpiness reported above 20,000 ft. on High Altitude Meteorological Flights. A.R.C. No. 10565 Civ.22 , 1947.

(2) An interim report on High Altitude Gust Investigation: by J.F. Holford and D.E. Morris. R.A.E. Report No. Aero. 2235.

(3) The dynamics of long waves in a baroclinic westerly current: by J.G. Charney. Journal of Met. Vol. 4, Oct., 1947.

(4) On the general circulation of the atmosphere in middle latitudes: by members of the University of Chicago. Bulletin of the American Met. Soc., Vol. 28, June, 1947.

(5) Jet Stream in the upper troposphere and cyclone formation: by H. Riehl. Trans. Am. Geoph. Union, Vol. 29, April, 1948.

(6) An analysis of the wind and temperature distribution in the free atmosphere over North America in a case of approximate westerly flow: by E. Palmen and N.M. Nagler. Journal of Met., Vol. 5, April, 1948.

(7) North - south cross section showing distribution of temperature, relative humidity and wind in a well marked zonal current over Western Europe. By E. Hovmöller. Journal of Met., Vol. 5, April, 1948.

HIGH ALTITUDE FLIGHTS - R.E. SPITFIRE

* Tropopause height from radio-sonde.

** " " determined by radio-sonde and aircraft observations.

Date	Time of Flight	Extent of bumpy layer	Height and type of tropopause	Synoptic Situation	Upper Air Situation (500 - 300 mb.)	Upper Cloud	Bump frequency (secs.$^{-1}$)	Height (1,000's of ft.)	$\partial v/\partial z$ (kts./1,000')	$-\partial T/\partial z$ (^{o}F/1,000')	Horizontal Shear (hours^{-1})	Wind Dir.	Wind Speed (kts.)	Notes
17. 7.46.	1010 - 1125	20-24,000'	26-27,000' (I) **	Low over North Sea. Wave Secondary over Belgium moving ENE.	Deep cold pool over England.	AS 20-22,000'	Too small to measure	20 24	) 5.5	4.0 to 4.7	- -	310 310	18 29	
19. 7.46.	1110 - 1255	None	33,900' (II) *	Old low (1004 mb) centred Scotland and quasi-stationary.	Upper low over North England.	None	None	20 30 40	1 2 - 3	2 3.5 0	- .12 - .27 - .11	250 270 270	40 47 40	
23. 7.46.	1440 - 1620	None	43,300' (II) *	High over France and Germany. Low south of Iceland. Shallow secondaries moving NE over NW Ireland and Scotland.	WSW - SW thermal over England.	None	None	20 30 40	3 1 0	3.6 5.4 2.3	- .09 - -	220 240 250	47 54 68	
29. 7.46.	1400 - 1530	Thin layer 23,300' Thin layer 30,000'	39,300' (I) *	Low near Shetlands with trough to E. Holland. Feeble ridge just W. of Ireland followed by warm front. All moving E.	Trough down the North Sea. Strong NW thermal over England.	Trace Ci 28,500'	1/74	23 30	2 2.6	4.1 4.8	0 .18	300 300	70 73	
20. 8.46.	1610 -	None	32,700' (II) **	High Azores with ridge to Ireland. Shallow low over Belgium quasi-stationary. Warm front 200 mi. W. of Ireland moving E.	Cold trough down the west North Sea.	None	None	20 30 40	6 1 - 8.5	4 5.0 0	-	330 330 330	41 70 58	
21. 8.46.	1600 - 1710	None	37,400' (II) **	Shallow low Schleswig. High Azores to France. Weak warm front 8^{o}W moving slowly E.	Upper low over Holland.	None	None	20 30 40	5.7 0 - 4.2	3.7 3.8 - 0.5	.13 .10 .13	360 350 340	46 83 68	
26. 8.46.	1150 - 1300	None	34,500' (I) **	Low over South Baltic and to South of Iceland. High Azores. Weak gradients over England.	Weak upper trough NW England to Belgium.	3/10 Ci 24,000' 1/10 As 32,000'	None	20 30 40	Small " "	4 2.7 - 2.0	Small " "	300 280 -	17 23 -	
27. 8.46.	1100 - 1200	None	36,500' (I) and 32,000'	Deepening and vigorous depression 51N 15W with associated warm front to Corunna moving E.	Weak trough at 30,000', Scotland to Holland.	8/10 26,000' to 35,000'	None	20 30 40		4 5.4 0	Small " "	270 260 -	24 45 -	
28. 8.46.	1500 - 1635	32,000' to 40,000'	36,000' and 32,000'	Deep low SE Ireland moving NNE slowly. Occlusion Firth of Forth to Holland moving NE.	Upper low over SW Ireland.	Cloud at 25,000'	1/188	30 33 40	12.0 12.0 - 15.0	) 1.0) to) - 1.5	.27 .79 -	200 200 200	69 133 71	
5. 9.46.	1530 - 1630	None	34,000' (I)	Old low centred South Scotland filling up.	Upper low over Ireland.	None	None	20 30 40	0 1 - 1	4.3 3.2 0	.06 .21 .12	230 230 220	23 36 35	
23. 9.46.	1540 - 1645	None	38,700' (II) **	Deep old low between Iceland and Norway. Cold front Den Helder - Dover - Ushant moving slowly SE.	Upper (cold) low to North of Shetlands.	None	None	20 30 40	0 0 3	3.1 3.4 0	0 0 -	260 260 260	67 97 68	
24. 9.46.	1100 - 1210	None	41,000' (I) *	Old low to North of British Isles. Shallow wave secondary 48N 10W moving slowly NE.	Old low to north of Shetlands.	5/10 27,000' to 34,000'	None	20 30 40	0 2 0	4.0 4.5 3.0	0.0 0.06 - 0.10	250 260 270	36 54 72	
2.10.46.	1535 - 1625	Thin layer 35,000	36-38,000' * (II), difficult to determine.	High over Scandinavia. Low 43N 20W. Occlusion Prestwick - Spurn Hd - Ostend then a warm front to Bay of Biscay	Upper low to NW of the depression at surface. WSW stream over British Isles.	10/10 from 15,000' to 34,000'	0.7	35	1	3.3	-	260	65	Horizontal wind shear probably large.
3.10.46.	1130 - 1240	Thin layer 23,500' 26,000' to 31,000'	Between 34,700' and 38,000' (II) *	High in Lat. 45N. Depression Iceland. Warm front Isle of Man - Spurn Hd - Den Helder. Cold front 3^{o}W, moving E. Both weak.	Ridge E. Spain to Norway, Trough Scotland to 45N 20W.	Cloud with flat top at 31,000'	No observation 1/7	24 27 30	2 6 6	4.0 3.5 3.5	.06 .36 .44	240 240 240	54 82 92	
24.10.46.	1055 - 1145	None	39,600' (I) *	High centred over Denmark and affecting British Isles.	Weak gradients.	None	None	20 30 40	0 - .5 0	4.3 4.1 - 0.5	0 0 -	Calm 200 220	11 9	

Date	Time of Flight	Extent of bumpy layer	Height and type of tropopause	Synoptic Situation	Upper Air Situation (500 - 300 mb.)	Upper Cloud	Bump frequency (secs.$^{-1}$)	Height (1,000's of ft.)	$\partial v/\partial z$ (kts./1,000')	$-\partial T/\partial z$ (^{o}F/1,000')	Horizontal Shear (hours^{-1})	Wind Dir.	Wind Speed (kts.)	Notes
25.10.46.	1520 - 1620	Thin layer 24,000'	35,300' (I) *	Anticyclone central Europe to Oslo. Old low off SW Ireland quasi-stationary, with an occlusion trough to SW of Scilly and Southwards, moving slowly E.	Trough, north and south, to west of Ireland.	Ci St 24-26,000'	No observation	24	1	4.1	0	200	28	
13.11.46.	1400 - 1510	20,000' to 40,000'	35,200' (I) *	Depression S. Sweden moving E. Another 55N 20W; linking front lay across north France, just south of Brest.	Cold trough Denmark warm ridge just to W. of Ireland.	8/10 Ci Cu 34,000' Some up to 37,000'	1/12	20	3.8	4.1	0	310	59	S.W. side of a "jet".
								25	0	4.7	- .20	330	53	
								30	3	4.9	- .15	310	60	
								35	1	4.0	- .13	300	63	
								40	1	- 1.6	-	310	66	
18.11.46.	1025 - 1145	Thin layer 32,000' Slight 20,000 - 40,000'	37,000' (II) **	Low over North Sea filling up. Another 50N 25W moving E quickly.	Cold trough North Sea. Strong NW stream over Britain.	None	1/11	20	2	3.0	.28	310	108	N.E. side of a "jet".
								25	4.5	3.4	- .02	310	111	
							1/22	30	4.5	4.1	-	310	140	
								35	- 13.3	4.0	-	320	157	
								40	- 4.0	- 1.0	-	310	85	
26.11.46.	1010 - 1145	20-34,000'	36,800' (I) **	Old low NE of Shetlands moving NE. Depression 53N 20W moving E.	Cold trough over Britain.	None	1/9	20	10	1.5	.61	310	76	N.E. side of a "jet".
								25	0	3.6	.41	310	91	
								30	3.4	4.0	.50	300	89	
								35	- 9.6	3.7	.57	300	99	
4.12.46.	1030 - 1130	Thin layer 29,000'	28,800' and probably again about 38,000' *	Shallow low over North Sea filling up. Ridge over Ireland moving E.	Trough down North Sea. Strong NNW thermal.	None	Too small to measure	29	5	0 to - 1.0	.57	340	68	N.E. side of a "jet".
6.12.46.	1415 - 1525	32-34,000'	About 29,000' (II) *	Old occlusion down North Sea. Ridge over Britain followed by active occlusion trough on Atlantic, all moving E.	Slight trough down North Sea. Ridge over Britain.	None	1/11	33	- 4.6	0.3	.42	320	47	
		36-38,000'					no observation	37	- 2.0	- 1.2	.24	290	38	
10.12.46.	1455 - 1610	28-32,000'	38,000' (II) *	Feeble ridge over E. England collapsing. Deep depression West of Ireland, with associated warm and cold fronts, moving E.	Warm ridge over Britain.	Ci St 28-36,000'	1/8	30	5	3.6	.83	330	135	
		Thin layer 36,000'				Bumps heaviest at top of cloud	1/3	36	No obs.	3.8	-	-	-	
		Thin layer 40,000'					1/5	40	"	- 3.0	-	-	-	
3. 1.47.	1030 - 1130	Thin layer 23,000'	37,900' (I) *	Very deep depression near Iceland. High over Spain. Warm front Stornoway to Start Point moving E.	Warm ridge NW Spain to Norway.	3/10 24-26,000'	Too small to measure	23	1	3.3	No obs.	230	39	
		" " 25,000'				1/10 31,000'		25	0	4.2	"	230	39	
		" " 32,000'						32	0	4.4	"	230	34	
		" " 34,000'						34	0	2.9	"	230	33	
4. 1.47.	a.m.	Thin layer 32,000'	41,900' (II) *	Anticyclone West Russia. Quasi-stationary occlusion Edinburgh - Scilly. S to SSW air stream.	Warm ridge France to North Sea.	None	1/86	32	small	3.3	- .07	190	31	
		" " 34,000'					No observation	34	"	3.3	- .07	160	24	
24. 1.47.	1130 - 1240	None	Type II and indeterminate. Aircraft obs. 35,000' radio-sonde 32,000'	Anticyclone over Scotland moving slowly west.	NE thermal up to 20,000'	None	None	20	0	3.7	.04	050	47	
								30	0	3.1	- .12	040	77	
								40	- 1.1	- 0.5	- .07	030	69	
17. 3.47.	1450 - 1600	Very slight bumps 36,000'	37,500' (II) **	Feeble ridge over E. England. Depression S. Sweden moving E; linking front to central France moving north in influence of deep depression SW of Ireland.	Upper low S. Sweden. Warm ridge 6°W.	None	Too small measure	34			(12Z) - .07			Obs. lacking for 1800 Z
								36	3.4	1.0		250	45	
								38			.01			
20. 3.47.	1210 - 1310	20-24,000'	34,000' (I) *	Low eastern North Sea. Very deep low mid-Atlantic. Old weak occlusion NW Ireland - Southampton moving slowly NE.	Cold pool SW North Sea General westerly stream over East Atlantic.	Ci 24-26,000'	1/3	22	6.2	1.7	.12	290	43	
		27-28,500'					1/2	28	2.9	4.7	.21	270	61	
1. 4.47.	1615 - 1730	Thin layer 20,000'	36,000' (II) *	Shallow low over SW Approaches, filling up. Quasi-stationary occlusion Yarmouth to Aberporth.	Trough West Norway across Britain.	None	Too small measure	20	0	3.9	.05	270	25	
		" " 26,000'						26	0	3.6	0	250	19	
		" " 32,000'						32	0	2.6	- .05	270	15	

Date	Time of Flight	Extent of bumpy layer	Height and type of tropopause	Synoptic Situation	Upper Air Situation (500 - 300 mb.)	Upper Cloud	Bump frequency (secs.$^{-1}$)	Height (1,000's of ft.)	$\partial v/\partial z$ (kts./1,000')	$-\partial T/\partial z$ (°F/1,000')	Horizontal Shear (hours^{-1})	Wind Dir.	Wind Speed (kts.)	Notes
9. 4.47.	1515 - 1625	None	34,600' ✕ (II)	High centred over North Biscay; ridge to Shetlands moving E.	Similar to surface.	None	None	20 30 40	1.7 3.0 - 1.8	3.8 3.8 - 0.8	- .09 - -	340 350 340	25 64 44	
11. 4.47.	1645 - 1745	Thin layer 20,000'	44,600' ✕ (I)	High Azores to Central Europe. Depression NE of Iceland. Cold front Oslo - Wash. - Shrewsbury then warm to developing wave 55N 22W moving ENE.	S. Westerly, veering NW above 30,000'.	Thin Ci 33,000'	1/66	20	1.5	3.9	- .12	220	21	
16. 4.47.	1450 - 1600	Thin layer 38,000'	43,100' ✕ (I)	High Southern Europe. Shallow low Biscay. Low E of Iceland. Warm sector wave near Shetlands moving ENE, with cold front to S. Ireland.	SW stream across Britain.	None	Too small to measure	38	2.1	3.0	No obs.	200	79	
17. 4.47.	0950 - 1115	Thin layer 30,000' Thin layer 32,000'	38,000' ✕✕ (I)	High Azores to S. Germany. Shallow inactive low Biscay. Feeble high over Ireland.	SW stream over Britain.	None	1/100 1/82	30) 32)	1.8	3.5	- .11	240	34	
22. 4.47.	1050 - 1210	20-33,000'	32,100' ✕✕ (I)	High Azores to Italy. Low SE of Iceland. Cold front W Denmark to Bordeaux moving SE.	Strong SW thermal.	None	1/106	20 25 30 35	3 0 1 0	2.9 3.3 2.2 - 1.0	.33 .36 .32 .32	220 220 230 230	84 94 96 100	N.W. side of "jet".
24. 4.47.	1340 - 1500	Thin layer 30,000' Thin layer 36,000'	About 39,000' ✕✕ (I)	High South France. Low West Norway; shallow low 47N 20W moving ENE. Linking front E and W just north of Pyrenees.	Cold low west of Bergen.	Ci 36,000'	1/9 1/18	30 36	- -	3 to 4 3.7	- .27 -	280	102	S side of "jet".
25. 4.47.	A.M.	None	41,400' ✕ (I)	High central Europe. Deepening depression off W. Ireland moving NE quickly.	WSW thermal.	8/10 19,500' to 30,000'	None	20 30 40	1.5 0.8 3.0	3.6 4.0 3.8	.09 - .04 - .04	240 250 240	58 61 69	
28. 4.47.	1520 - 1640	20,000' - 30,000'	39,600' ✕ (II)	High Azores to Italy. Low to N and NE of Shetlands. Cold front 55N 10E to just S of Brest, moving slowly SSE; then warm to developing depression mid-Atlantic.	WNW thermal.	None	1/19	20 25 30	2 5 0	5.3 2.6 5.1	- .27 - .50 - .21	290 300 300	88 100 112	S. side of "Jet".
29. 4.47.	1050 - 1220	Thin layer 26,000'	38,700' ✕✕ (I)	Depression just off NW Ireland moving E. Warm front Donaghadee - Holyhead - Portland moving NE.	WNW thermal.	None	too small to measure	26	6	3.9	-.07	280	82	S. side of "jet".
30. 4.47.	1515 - 1630	None	26,000' ✕✕ (I)	Deep depression Schleswig moving ENE. Anticyclone W. of Ireland.	Upper low over North Sea with trough SW'wards.	Ci 24,000'	None	20 30 40	0 1.2 - 0.8	4.1 - 1.5 0	- .02 - .03 -	310 280 260	16 21 21	
7. 5.47.	1450 - 1550	20,000' - 36,000'	37,000' ✕ (II)	High South Baltic. Depression 48N 12W moving NW.	High to east, low to west of Britain.	Ci 24,000' - 34,000'	Too small to measure	20 25 30 35	0 0 3.5 3	3.9 4.2 5.4 4.4	- .10 - .12 - -	190 210 230 240	23 20 38 54	
8. 5.47.	1145 - 1300	None	37,500' ✕✕ (I)	Low to W of Ireland moving NW. Cold front Prestwick - London - S'wards moving slowly ENE.	Ridge France to Shetlands.	Layers of Ci St.	None	20 30 40	0 1.4 0.5	4.1 5.4 0	- .09 - .04 -	180	45	
9. 5.47.	1045 - 1235	Thin layer 28,000' 30-32,000' Thin layer 36,000'	38,000' ✕✕ (I)	Low off Brest moving NNW. Warm front Brest - Larkhill - Wash moving NNW.	General SSE stream over Britain.	Ci St. 5/10, 28,000' 4/10 Ci 30-32,000'	1/13 1/60 1/9	28 30 32 34 36	0 - 1 - 1 - 1 0	4.5 5.4 4.7 3.3 3.0	- .10 - .10 - .05 -	170 170 180 190	46 46 55 54	Hor. wind shear may have been larger than given here.
13. 5.47.	1130 - 1245	Thin layer 34,000'	37,400' ✕✕ (I)	Depression about 52N 20 W moving SE. Weak SE gradient over England.	Slight SSW thermal wind over England	Ci 25,000'	1/153	34	0	4	.07	210	47	
14. 5.47.	1450 - 1610	None	About 38,000' ✕ (II)	Low almost stationary 57N 15W Occlusion Hebrides - London and cold front Prestwick - Lyme Bay moving E.	High to east, low to west of Britain.	Cu 12,000' to 30,000'	None	20 30 40	0.5 0 - 2.0	4.3 4.0 0.5	- .15 - -	170 180 180	47 48 40	

Date	Time of Flight	Extent of bumpy layer	Height and type of tropopause		Synoptic Situation	Upper Air Situation (500 - 300 mb.)	Upper Cloud	Bump frequency (secs.$^{-1}$)	Height (1,000's of ft.)	$\partial v/\partial z$ (kts./1,000')	$-\partial T/\partial z$ (^{o}F/1,000')	Horizontal Shear (hours^{-1})	Wind Dir.	Wind Speed (kts.)	Notes
15. 5.47.	1030 - 1140	20,000' to 40,000'	37-38,000' (I)	**	Depression W. of Scotland. Cold front Orkney to Cologne and occlusion Stornoway - Pembroke - Scilly moving E.	Cold trough Donegal to Isle of Wight.	None	$^{1}/_{13}$	20 25 30 34 39	4.8 5.2 - 4 - 1 - 2	3 3 3 2 - 0.5	.13 .09 .09 .06 -	190 180 170 180 220	24 37 47 30 15	Hor. shear not accurate.
3. 6.47.	1135 - 1235	Thin layer 28,000'	41,400' (I)	*	Shallow low over North Ireland. Quasi-stationary front Carlisle to Bournemouth. Slack S gradient over England.	High Belgium, low near Corunna.	None	$^{1}/_{24}$	28 30	1	5	- .08 .02	160	20	
4. 6.47.	1030 - 1130	21,000 - 30,000'	40,500' (I)	*	Shallow low Liverpool to Brest. Cold front Manchester to the Hague moving slowly NE.	Low over Biscay.	Cloud 17,000' to 29,000'	$^{1}/_{8}$	20 23 27 30 34	0 - 1 - 1 - 1 2.7	3.6 4.1 4.0 4.1 2.9	.07 .12 .27 .39 .18	160 180 190 150 170	26 23 18 13 32	Winds quoted are for 06 Z
6. 6.47.	1023 - 1230	Thin layer 24,000'	25,900' (I)	*	Old low just E of Wick, cold air over Britain.	Upper low over NE. England.	$^{3}/_{10}$ Ci St 24,000'	$^{1}/_{92}$	20 24 27	6	4.7	.11 .20 .14	330	51	
9. 6.47.	1115 - 1230	28,000 - 30,000' Thin layer 38,000'	37,400' (II)	**	Anticyclone Biscay drifting North	As at surface.	Ci St 30,000 - 31,000'	Too small to measure $^{1}/_{23}$	27 29 34 38	- 5 - 9	4.4 0	- .15 - .18 - .04 .15	300 300 300 300	72 55 79 36	Shear evaluated for 06Z Sim. values at 00 & 18Z
10. 6.47.	1200 - 1300	None	41,700' (I)	*	Anticyclone centred Cherbourg.	As at surface.	None	None	20 30 40	0 - 3.8 0	4.1 5.4 0	- .04 - .12 - .07	290 300 300	24 12 29	
11. 6.47.	1130 - 1230	None	41,000'	*	High over Germany and also to north of British Isles. Slack gradients.	Slack gradients.	None	None	20 30 40	1.0 0 -	3.7 4.5 1.0	- .03 .01 -	290 280 -	11 19 -	
13. 6.47.	1130 - 1240	None	37,500' (II)	**	High Iceland. Low NE of Azores moving NE. Occlusion along the channel drifting slowly N.	Ridge over Southern England. Trough Biscay WNW'wards.	Ci 24-26,000'	None	20 30 40	1.6 - -	3.9 5.3 0	- .05 - -	230 210 -	24 21 -	
16. 6.47.	1130 - 1230	26-28,000'	40,400' (I)	*	High East France moving E. Depression 51N 25W drifting N. Warm front Valentia - Lands End moving NE.	NW stream over Britain	Layers to 35,000'	$^{1}/_{30}$	24 27 30 34	Small Small	3.2 4.3	+ .03 - .04 - .15 - .13	270	57	
28. 7.47.	1140 - 1300	20-32,000'	40,400' (I)	*	High over North Sea. Shallow low North Ireland, with cold front to Plymouth, moving slowly E.	SSW stream over England.	None	$^{1}/_{15}$	20 23 27 31	3.7 0 3.6	3.5 3.8 4.1	- .04 - .10 - .14 - .05	200 200 210 230	43 58 58 63	
1. 8.47.	1205 - 1305	Thin layer 28,000' 30-32,000' Thin layer 34,000'	38,100' (I)	**	High over North Sea. Shallow low SW of Ireland.	Ridge Pyrenees to Scotland.	$^{10}/_{10}$ Ci St 29-35,000'	Too small to measure $^{1}/_{33}$ $^{1}/_{8}$	27 31 35	1 1 3	4.4 4.9 5.3	.10 .06 - .02	220 220 240	30 32 44	
5. 8.47.	1520 - 1650	Thin layer 20,000' Thin layer 30,000'	33,500' (I)	**	Anticyclone to SW of British Isles. Cold front N. Scotland - Stranraer - SW Ireland; warm front Stranraer - Scilly. System moving SE.	Upper low over Low Countries and SE England.	None	$^{1}/_{88}$ $^{1}/_{53}$	18 20 24 30	4.4 4.1 4.1	4.1 3.5 3.0	.01 .11 .35 -	330 330	47 68	Wind of doubtful application to S. Farnborough.
7. 8.47.	1110 - 1230	Thin layer 20,000' Thin layer 28,000'	36,300' (I)	**	Ridge Stornoway to South of Ireland moving slowly E.	Light NW winds.	5/10 cloud 20-29,000'	$^{1}/_{56}$ $^{1}/_{57}$	18 20 24 28 31	0 - 3.0	5.2 4.1	.03 .11 .10 - .03 - .19	340 320	28 18	
8. 8.47.	1125 - 1230	Thin layer 36,000'	About 36,000' (I)	**	Feeble high SE England moving NE. Warm front Hebrides to SE Ireland moving very slowly E.	Light gradients	None	$^{1}/_{59}$	36	-	5.4 below - 2.5 above.	-	Light		

Date	Time of Flight	Extent of bumpy layer	Height and type of tropopause	Synoptic Situation	Upper Air Situation (500 - 300 mb.)	Upper Cloud	Bump frequency (secs.$^{-1}$)	Height (1,000's of ft.)	$\partial v/\partial z$ (kts./1,000')	$-\partial T/\partial z$ (^{o}F/1,000')	Horizontal Shear (hours^{-1})	Wind Dir.	Wind Speed (kts.)	Notes
11. 8.47.	1505 - 1555	None	40,200' (II)	High SW of British Isles with ridge to Norway.	Similar to surface.	Slight Ci 33,200'	None	20 30 40	2.4 3.1 - 3.0	3.8 4.7 3.5	- - -			
12. 8.47.	1135 - 1230	Thin layer 34,000'	39,900' (I)	High SW Norway to east of Azores.	High over Ireland and North England.	None	1/34	34	0	4.9	. 01	040	21	
20. 8.47.	1100 - 1200	28,500' - 34,000'	39,100' (I)	High Faeroes. Shallow low pressure Corunna to Central Europe.	Feeble trough NW Germany to North Ireland.	Slight Ci 29-32,000'	1/3	28 29 31 33 35 40	5.6 1.5	4.7 4.7	.06 .33 .40 .13	280 250 250	39 58 64	
20. 8.47.	1500 - 1600	23-38,000'	39,900' (I)	do.	do.	Ci 29-34,000'	1/14	21 24.5 27 31 35 40	0 3.0 2.8 0 - 3.2	3.7 4.1 4.7 3.9 0	Small - .10 - .21 - .13 - .12 - .04	240 260 230 230 250	16 17 36 45 11	Shear measurements not accurate.

$\partial v/\partial z$ = rate of increase of horizontal wind, with height.

$-\partial T/\partial z$ = lapse rate of temperature with height.

All times are G.M.T.

Date/ Time	Area of Flight	Bumpy Layers, etc.	Tropopause and its slope	Weather Situation	Upper Cloud	Height (mb. of pressure)	$\partial u/\partial z$ (kt./1,000')	$-\partial T/\partial z$ (°F/1,000')	Horizontal Shear (hours^{-1})	Wind Dir.	Wind Speed (kt.)	Notes
8.12.47. Mid-day	Farnborough	400 mb. 368 " 322 "	Double: 337 mb. (26,500') 308 mb. (28,500') Impossible to estimate slope.	Low over Thames estuary moving SE. Upper low, with slack gradients, over NW Europe. Strong NW upper current over SW England.	Patches Ci 400 and 368 mb.	400 350 300 250 322	2.3 3.2 2.3 1.6	x 1.3 - 2.9 0 0	-	320 320 310 310	10 24 30 42	x Inversion ($\partial T/\partial z$ = 1°F per 1,000') above, lapse 3.3°F/1,000' below. One bumpy layer connected with tropopause another with an inversion.
19. 1.48. Afternoon	Southampton	329 mb. 315 " 301 " (Slight, all cases)	Double: 376 mb. (22,800') & 300 mb. (27,800') Small slope, perhaps.	Deep low quasi-stationary near Shetland. Moderate W to WNW polar stream over S. England.		329 315 301	- 2.9 Small Small	0 Small Small	less than .10	270 270 270	29 28 28	All connected with the tropopause which had complicated structure usual over deep polar air.
20. 1.48. 1030 – 1230	Southampton	315 mb. (Slight)	Double: 329 mb. (26,200') & 274 mb. (29,700') Upper one the main division. Slope unknown.	Old low E. Scotland filling fast and moving SE as warm sector (about 20°W) moves towards British Isles. Very strong NW upper winds over Ireland.		315	- 1.0	- 1.4	.01	320	34	Between the two tropopauses.
22. 1.48. 1120 – 1240	Cambridge – Wash	325 to 315 mb. and at 206 mb., the limit of climb.	274 mb. (30,400')	Old shallow low Holland, filling up. Old occlusion E & W through the Wash moving slowly S. Strong WNW upper winds over France.		315–325 206	0 - 0.7	- 1.8 - 2.6	.56 .17	300 290	31 28	The lower bumpy layer was confined to the inversion.
6. 2.48. 1330 – 1530	Farnborough	Remarkably bumpy 305–291 mb. Intermittent bumps 291–285. Slight at 319 mb.	Double: 305 mb (28,500') & 277 mb. (31,000') Slope .36X10^{-2}.	Low E. of Iceland. Cold front Hamburg – Ghent – St. Nazaire moving SE. Feeble ridge over Ireland followed by warm front 50N 20W moving NE. Strong WNW "jet" over SW England.		319 300	- 6.2 - 6.2	2.5 - 1.7	Probably large. No obs.	300 300	124 117	The very bumpy layer is in the bottom of the tropopause inversion.
26. 2.48. Afternoon	South Coast, South of Farnborough.	Extremely bumpy (estimated ½ g) for 700 ft. above 250 mb., and again not quite so severe, at 262 mb. Slight at 301, 280 and 216 mb.	Double: 274 mb. (31,700') & 250 mb. (33,700 ft) Slope unknown.	High centred Denmark and affecting British Isles. Upper high same place; very shallow upper low over NE France giving slack upper gradients over British Isles.		301 280 216 250	0 - 2.6 - 2.6 - 2.6	2.0 2.5 - 5 above + 2 below 0	Probably not large. No obs.	330 330 310 330	30 26 11 21	At 250 mb. a fluctuation of 3°F was noticed in 2 minutes. Marked difference (up to 10°F) between ascent and descent. Largest bumps at tropopause: another of layers at temp. lapse check.
27. 2.48. Morning	do.	Slight 274 mb.	Double: 287 mb. (30,700') & 221 mb. (35,900') Slope .2X10^{-2} to .3X10^{-2}	High centred over Poland and affecting West Europe and British Isles. Shallow upper low over England.	Ci 280–274 mb.	300 274 250	- 3.5	- 5 above 3.5 below	.45 - .07	250 260	46 30	Bumpiness associated with the tropopause.
18. 3.48. 1100 – 1235	Midlands	Slight at 329 mb.	238 mb. (34,800') Type I. Slope about .5 x 10^{-2}	High Azores to Biscay. Cold front Hanover to Corunna moving SE. Ridge over Britain moving E before a new frontal system on the Atlantic. Marked upper cold trough (90° veer in wind) over Midlands.		329	Small	3.2	-	280	51	
4. 4.48. Afternoon	South of England	Slight at 485 mb.	360 mb. (24,600') Type I. Slope small.	Secondary depression off Norfolk moving NE. Low over Scotland moving E. Occlusion from secondary to Ostend to St. Nazaire moving E & SE. Upper trough Scotland to Brest.	Frontal cloud tops visible in distance at 400 mb.	485	2.8	3.5	.17	250	63	
6. 4.48. Morning	Kent	Slight 275–260 mb.	Double: 300 mb. (28,900') & 230 mb. (33,600') Slope about 1.0 x 10^{-2}	Stationary low SE of Iceland with occlusion to Shetland – Newcastle – Swansea, then warm to SW'wards and cold WSW'wards, moving ESE across England. Strong NW upper current over Britain.		260 – 275 300	-	0	.55	310 (estimated) 320	100 142	Bumpiness was between the two tropopauses and on the NE side of the "jet".

Date/ Time	Area of Flight	Bumpy Layers, etc.	Tropopause and its slope	Weather situation	Upper Cloud	Height (mb. of pressure)	$\partial v/\partial z$ (kt./1,000')	$-\partial T/\partial z$ (°F/1,000')	Horizontal Shear (hours^{-1})	Wind Dir.	Wind Speed (kt.)	Notes
13. 4.48. 1000 – 1130	South of England	Slight at 206 mb.	216 mb. (39,100') Type I Slope small.	Anticyclone SW of Ireland. Weak wave secondary over Scotland moving ESE. Upper winds WNW over North England light and variable over South.		206	1.0 (08Z) 0 (14Z)	0 *	0 (08Z) .1 (14Z)	Estimated 360	17	* Slight lapse above this level, steep inversion just below. Bumpiness associated with tropopause.
20. 4.48. 0930 – 1100	Farnborough – Salisbury Plain	Slight at 194 mb. (limit of climb)	238 mb. (35,100') Small slope	High Iceland and Scandinavia. Shallow low west of Biscay. Weak gradients. Upper ridge Frankfurt to Hebrides.		194	– 1.0	– 3.5	Small	180	20	
24. 5.48. 1430 – 1600	Farnborough – London	Slight at 360 mb.	238 mb. (33,500') Type I Slope .47 x 10^{-2}	Low Stagerrak moving NE, with cold front to Brussels – St. Nazaire moving slowly SE. Strong SW upper winds over England.		360	3.9	5 below 1 above	.12	230	70	Bumpiness occurred at a check in temp. lapse rate. On the NW side of a "jet", axis 300 mb.
1. 6.48. 0900 – 1000	South Devon	Slight at 300 mb.	329 mb. (27,500') (or might put it 360 mb, 25,500' where lapse rate changes). Large slope, of order .5 x 10^{-2}; or discontinuous	Old low North England with cold front NE Holland – Biarritz moving slowly SE. Upper low over Southern England.	Cirrus below tropopause.	300	Small	– 1.5	–	Estimated NW	60–80	
4. 6.48. 1100 – 1200	South Midlands	Slight at 360 mb. " " 230 " " " 210 "	About 200 mb. (39,000') Type II Slope .36 x 10^{-2}	Low over North Sea; ridge over Britain. Occlusion and warm front system West Ireland. Upper ridge over Irish Sea.	Persistent contrails 230–270 mb.	360 230 210	2.9 – 3.7 – 5.0	– 3.0 4.0 2.5	.14 (09Z) – –	310 310 310 (09Z)	77 96 78	On the NE side of the "jet". Wind observations lacking, but horizontal shear had been large at 03Z.
11. 6.48. 1030 – 1130	Farnborough	Slight at 300 mb.	About 227 mb. (37,100'), Type I Slope .28 x 10^{-2}	High north of Shetland with ridge to Biscay. Cold upper trough Scotland – Hamburg.	Confused Ci patches, base 330 mb, tops above 300 mb but not clear.	300	Small At 09Z At 15Z	4.2	 – .12 – .15	250 (09Z)	14	
16. 6.48. 1000 – 1100	Farnborough	Slight at 250 mb.	262 mb. (33,100') Type II. Slope .27 x 10^{-2}	Stationary low south of Iceland. Occlusion Wick – Ostend moving ENE. Strong WSW upper winds over Southern England.	Short non-persistent contrails 250–238 mb.	300 250	 – 2.2	 0	– .06 (08Z) .18 (14Z) – .07 (08Z)	Estimated 250	80	Bumpiness occurred near centre of WSW "jet" (axis about 300 mb.), and just above tropopause.
22. 6.48. 1530 – 1630	Farnborough	244 mb.	About 274 mb. (32,100') Type II Slope .46 x 10^{-2}	Shallow low over North Sea. Depression mid Atlantic. Cold upper trough Shetlands to Holland.		244	0.4	0	.09	300	64	
1. 7.48. 1500 – 1600	Farnborough	Shallow bumpy layer 210 mb.	216 mb. (37,500') Type I Slope .15 x 10^{-2}	Anticyclone west of Ireland. Low centred Schleswig. Light winds aloft.		210	– 2	– 1	Small	300	10	Bumps connected with tropopause.
8. 7.48. 1000 – 1100	Farnborough and 80 miles to NW.	465 mb. 300–260 mb. especially at 275 mb. 227–200 mb. (limit of climb)	227 mb. (36,700') Type I Slope 1.09 x 10^{-2}	Depression NW Germany moving SE. Anticyclone W. of Ireland. Strong NW upper winds.		465 300 275 250 216	0 0 – 2.3 †	2.5 2.0 – 3.0	.21 * .24 – .04 – .05	320 310 310 (09Z)	85 88 80	* Shear evaluated for 09Z. † Isothermal at the top, inversion at bottom of the layer. Bumpiness apparently associated with horizontal shear and the tropopause. In middle or NE side of a "jet".
8. 7.48. 1400 – 1500	do.	444 mb. and 227 mb.	238 mb. (35,700') Type II	do.	3/10 Ci 400–365 mb. persistent contrails 290–220 mb.	444 227 200	5.0 – 4.3	2.7 0.3	.07 .09	340 330	82 80	The upper bumpy layer associated with the tropopause. Centre of a "jet".
21. 7.48. 1000 – 1120	Farnborough	216 mb.	216 mb. (37,800') Type I Slope .27 x 10^{-2}	Depression Prestwick moving NE. Cold front Heligoland – East of Brussels – to Bordeaux. SW upper winds over England.	Thin Ci patches 274–240 mb.	216	– 2	–2.1 above 0 below	–	220 (09Z)	76	Bumps associated with tropopause.
18. 8.48. 1000 – 1100	Farnborough	400 mb. and 245–238 mb.	232 mb. (35,700') Type I Slope .3 x 10^{-2} or more.	Old stationary low SW Norway. Low over Brest moving ESE. Upper low over SW North Sea.	Patchy Ci 460–238 mb.	400 240 200	2.4 – 2.8	0.2 (5.0 below) 0 *	– .09 – .33 – .02	270 270	39 40	Bumps reported over Lyneham 08–09Z at 20–22,000'. On NE side of "jet", (axis 300 mb). * Lapse above and below this layer.

Date/ Time	Area of Flight	Bumpy Layers, etc.	Tropopause and its slope	Weather Situation	Upper Cloud	Height (1,000's of ft.)	$\partial u/\partial z$ (kt./1,000')	$-\partial T/\partial z$ (°F/1,000 ft.)	Horizontal Shear (hours^{-1})	Wind Dir.	Wind Speed (kts.)	Notes
6. 4.48. 1300 – 1400	Braemar	18,500–20,000' and only within 25 miles of Braemar. Accelerations up to 0.6g.	About 32,000'. Type I. If not discontinuous between Braemar and Lerwick, slope .47 x 10^{-2} or more.	High West of Spain, low South of Iceland. Weak occlusion Wick – just east of Rattray Hd – NE Yorkshire moving E. Strong NW upper winds over West of British Isles.	Thin patches Ci Cu 17,000'. Cu tops 10,000'	18.5 20	7 * 7	– 4 4	.31 .21	290 * 290	64 81	* Interpolated from radio-sonde data for 14Z. The inversion at which bumps occurred may be frontal. Braemar was on the NE side of a strong NW jet, axis over Aldergrove about 34,000'.
21. 4.48. 1210 – 1230	Brittany	20,000 to between 30 and 35,000'. Bumps up to 0.3g.	Probably about 34,000'. Type I. Slope about .13 x 10^{-2}	Elongated low pressure area 57N 25W to near Brest with small centre just SW of Brest moving N, with a cold front from it moving N over Brittany. Upper low over Bay of Biscay.	Ci Cu 20–25,000'							Bumps were over and behind the surface cold front.
22. 4.48. 1245	About 44½N 05½W	30–35,000' A few bumps to 0.3g	Between 30 and 35,000' Slope about .36 x 10^{-2}	Old low NW of Ireland. No proper fronts. Upper cold trough 53N 20W to Bay of Biscay.	None							Radio-sonde ascents indicated that the tropopause probably fell to below 30,000' over the Bay of Biscay, Type II. The aircraft temperatures place it between 30 and 35,000'.
1300	About 47N 04W	30–35,000' Bumps only ± .1g	Between 30 and 35,000' Slope about .27 x 10^{-2}	do.	None							
14. 6.48. 0950 – 1310	Cranfield – Kings Lynn – Irish Sea – Cranfield.	Slight bumps 20–35,000' (limit of climb). Mainly .1g – .2g but 2 at .3g.	39,500' in the last, 37,500' over Irish Sea. Slope about .2 x 10^{-2}	High to east, low to west of Britain. Old occlusion quasi-stationary Leuchars to London. Thundery outbreaks over whole route.	Cu Nimb. with tops to 30,000'. Some Ci St. associated with them.	20 25 30 35	Small " " "	3.8 4.0 4.1 2.6	Small .07 – –	210 210 220 220 (Mean)	25 26 23 21	Flight was between the Cu Nimb. tops. Most bumps were in Ci St. near a Cu Nimb.
6. 7.48. 1510 – 1550	Heligoland and 50 miles to South	32,400–33,900 ft. Bumps of .1g – .2g recorded, of .4g estimated.	32,400' Type I. (At Schleswig, 14Z, was 34,300') Slope .36 x 10^{-2}	High west of Ireland. Old low over South Sweden. Upper cold trough Bergen – Heligoland – Italy.	Large Cu Nimb. about 100 miles away, over land.	33	Small	– 2	Small	350	13	Wind information from Jever 14Z radio-wind determination. (40 miles South of Heligoland). Bumpiness associated with tropopause.
12. 7.48. 1310 – 1620	Cranfield – Inverness – Cranfield.	Slight bumpiness, whole route, 20–35,000' (limit of climb). Nothing above ± .1g	26–28,000' Type II, over route. (Aircraft temps. and radio-sonde). Either discontinuous or steep slope up to SW. (.85 x 10^{-2} in the South, .47 x 10^{-2} in the North).	Old low NW North Sea moving SE. High to SW of British Isles. Occlusion Hull – Lyme Bay moving SE. Upper low off East Scotland. Strong upper winds from NW over Ireland and SW England.	Ci Cu at 25,000' over Firth of Forth.	South 20 30 35 North 20 30 35	Small 3.0 – 4.3 Small 1.0 0 – 0.7	3.3 – 0.5 0 4.1 – 1.6 + 1.3	– – – – .03 .05 0	290 290 300 350 330 340	44 50 42 14 24 24	The flight was along the NE edge of a moderately strong NW "jet" whose axis was probably over Cornwall – S. Ireland. Temperature lapse rates taken from radio-sonde data for 14Z.
15. 7.48. 1005 – 1355	London to Rome	A few slight bumps (± 0.1g); almost all over the Alps or again Genoa – Leghorn region, 35–37,000 ft. (Limit of climb)	About 34,000' at London. Probably about 35,000' over Alps and 41,000' over Rome. * (Slope .33 x 10^{-2} latter part).	Low South Sweden. Feeble ridge west of Ireland. Shallow low over Adriatic. Old occlusion quasi-stationary along Alps. Slack upper low Britain to Baltic. Westerly thermal over Southern Europe and Mediterranean.	10/10 Ci Cu almost to 30,000', over the Alps. No observations made in cloud.							Bumps over the Alps associated with tropopause. There may have been appreciable horizontal shear over North Italy. * Rome tropopause inferred from radio-sonde data. Aircraft temperatures suspect.
16. 7.48. 0918 – 1403	Rome to London (observations to 48°N only)	A few slight bumps (± .1g) mainly in same areas as on 15.7.48 and 30–37,000'.	Probably about 40,000' at Rome falling to between 30,000' and 35,000' by the Alps and remaining that height.	Low over South Baltic. Shallow low N. Italy and Adriatic. Light Northerly flow over Britain. Strong thermal in upper troposphere over Southern Europe and Mediterranean.	Ci Cu tops 31,000' over Alps. (Two bumps in this at 30,000').							Bumps over Alps associated with tropopause, perhaps, which was seen to be Type II 32,000', from radio-sonde observation near there (not contradicted by aircraft observations). There may well have been large horizontal shear over North Italy.

Tropopause normal frequency.

Date/ Time	Area of Flight	Bumpy Layers, etc.	Tropopause and its slope	Weather Situation	Upper Cloud	Height (1,000's of ft.)	$\partial v/\partial z$ (kt./1,000')	$-\partial T/\partial z$ (°F/1,000')	Horizontal Shear (hours^{-1})	Wind Dir.	Wind Speed (kts.)	Notes
20. 7.48. 0855 - 1305	Cranfield - Mid-Scotland and return.	"Rough air" (.1 to .2g) 54½N to 56½N 20-37,000', but most at 35,000'. (Limit of climb 37,000').	About 42,000' at "take off" falling to just above 35,000' Mid Scotland; type I but often type II later.	Low Faeroes. Trailing cold front Dover SW'wards becoming warm front of active depression about 49N 16W moving quickly NE. Strong SW upper winds.	None	South						Most bumps occurred at the tropopause. Horizontal shear large - in the SE boundary of strong SW jet with axis about 34,000' over North Scotland.
						20	0	3.5	- .06	240	52	
						30	.7	3.8	- .01	250	58	
						35	Small	3.1	- .09	250	60	
						38-40	1	3.2	+ .01	260	66 (09Z)	
						North						Tropopause was changing quickly in the north with the advection of the higher tropical tropopause.
						20	0	3.7	0	250	44	
						30	6.2	4.2	- .24	260	113	
						35	- 5.1	2.6	- .14	270	102	
								- 8.7 above				
						38-40	- 7.0	- 1.8	- .18	260	69	
21. 7.48. 0930	Bielefeld (R.A.F. machine)	Slight 23-30,000'	Above 40,000'	Depression N. Ireland moving NE. Cold front Emden - Brussels - Biscay moving SE. NW Germany in warm sector. Upper ridge over Germany.		24	Small	1.9 below 5.1 above	.09	220	25	
						27	"	5.1	.17	220	37	
						30	"	5.1 below 3.4 above	.18	210	37	
21. 7.48. 0940	Hamm (R.A.F. machine)	Slight 25-28,000'	do.	do.		Similar						

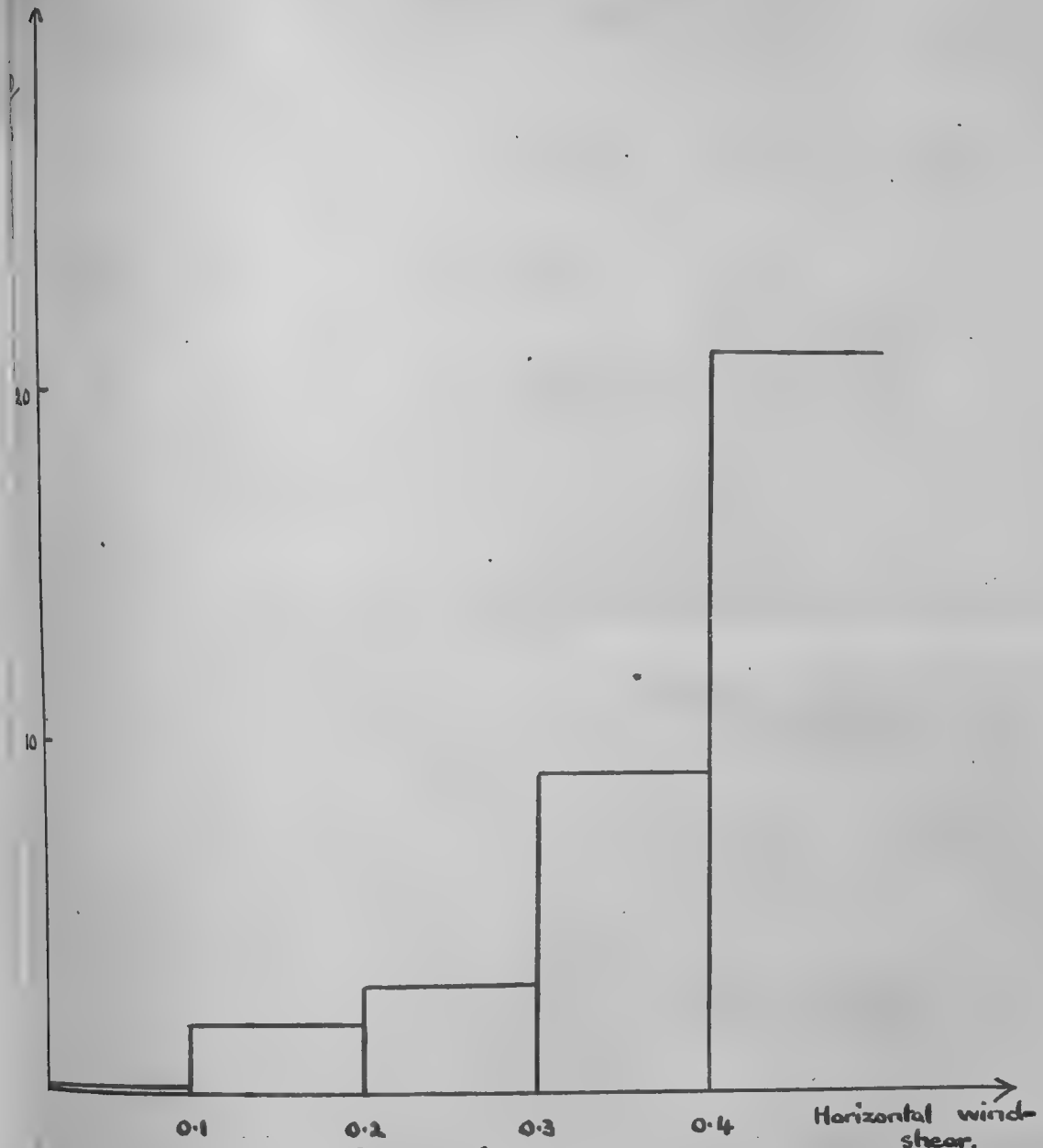


Figure 1.

Ratio of frequency of occurrence of various horizontal wind-shears in deep (>14,000') bumpy layers to the "normal" frequency.

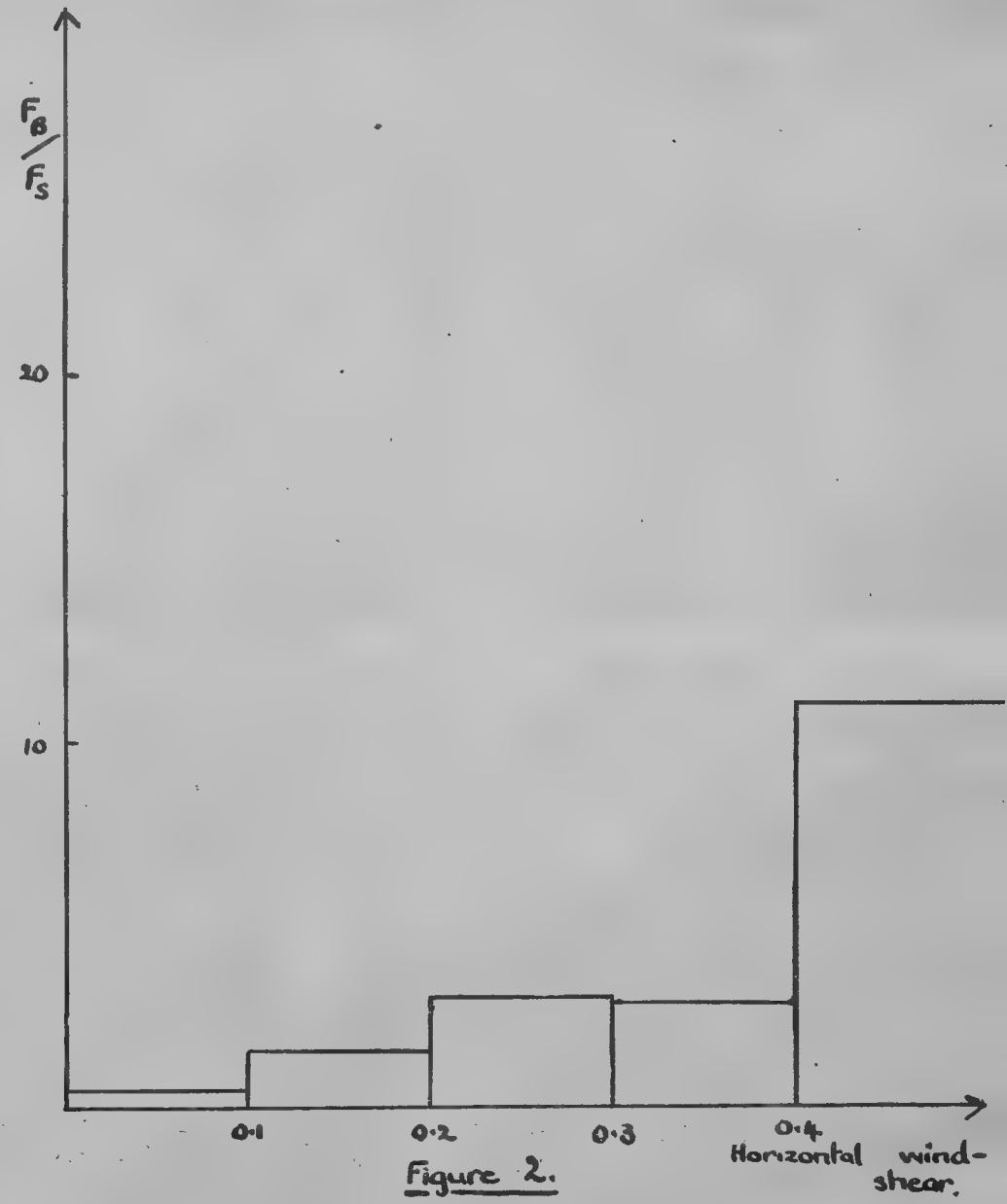


Figure 2.

Ratio of frequency of occurrence of various horizontal wind-shears in bumpy layers (all depths) to the "normal" frequency.

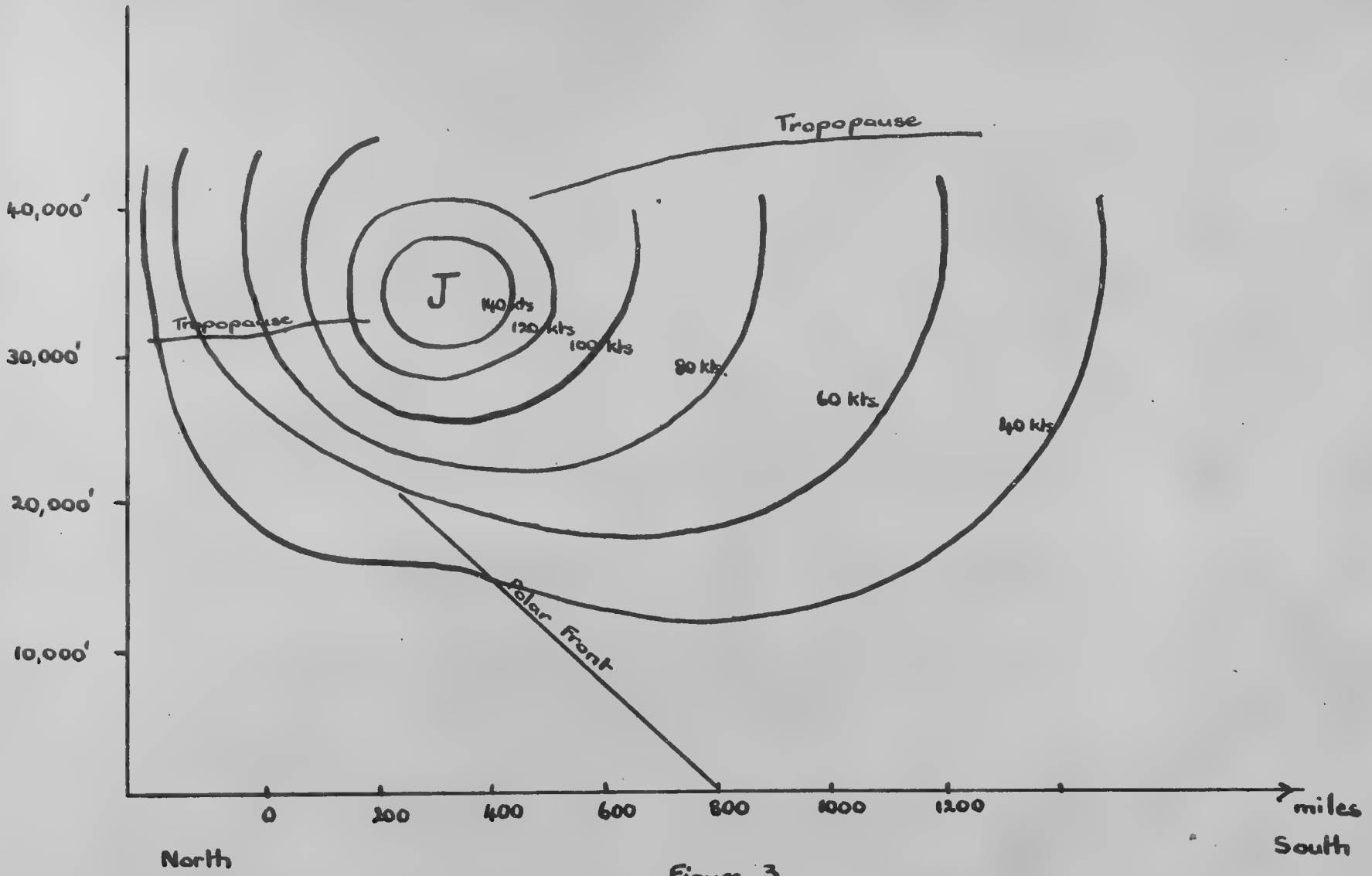


Figure 3

Typical Distribution of Velocity in a West to East Current — "Jet Stream".

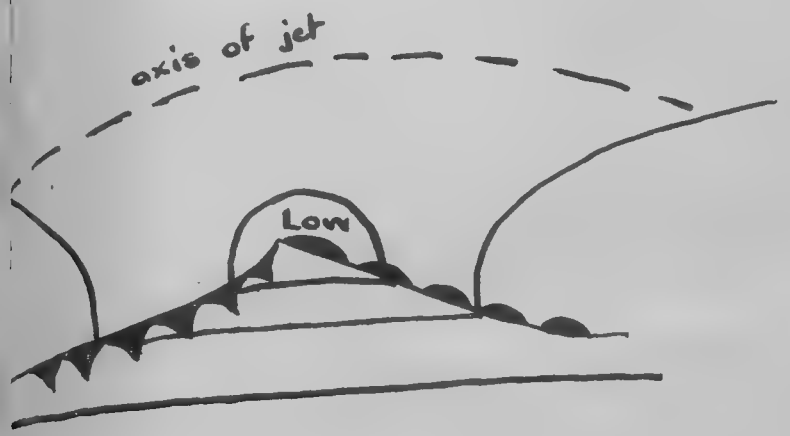


Figure 4.

Axis of "jet" in relation to a developing "wave" disturbance.

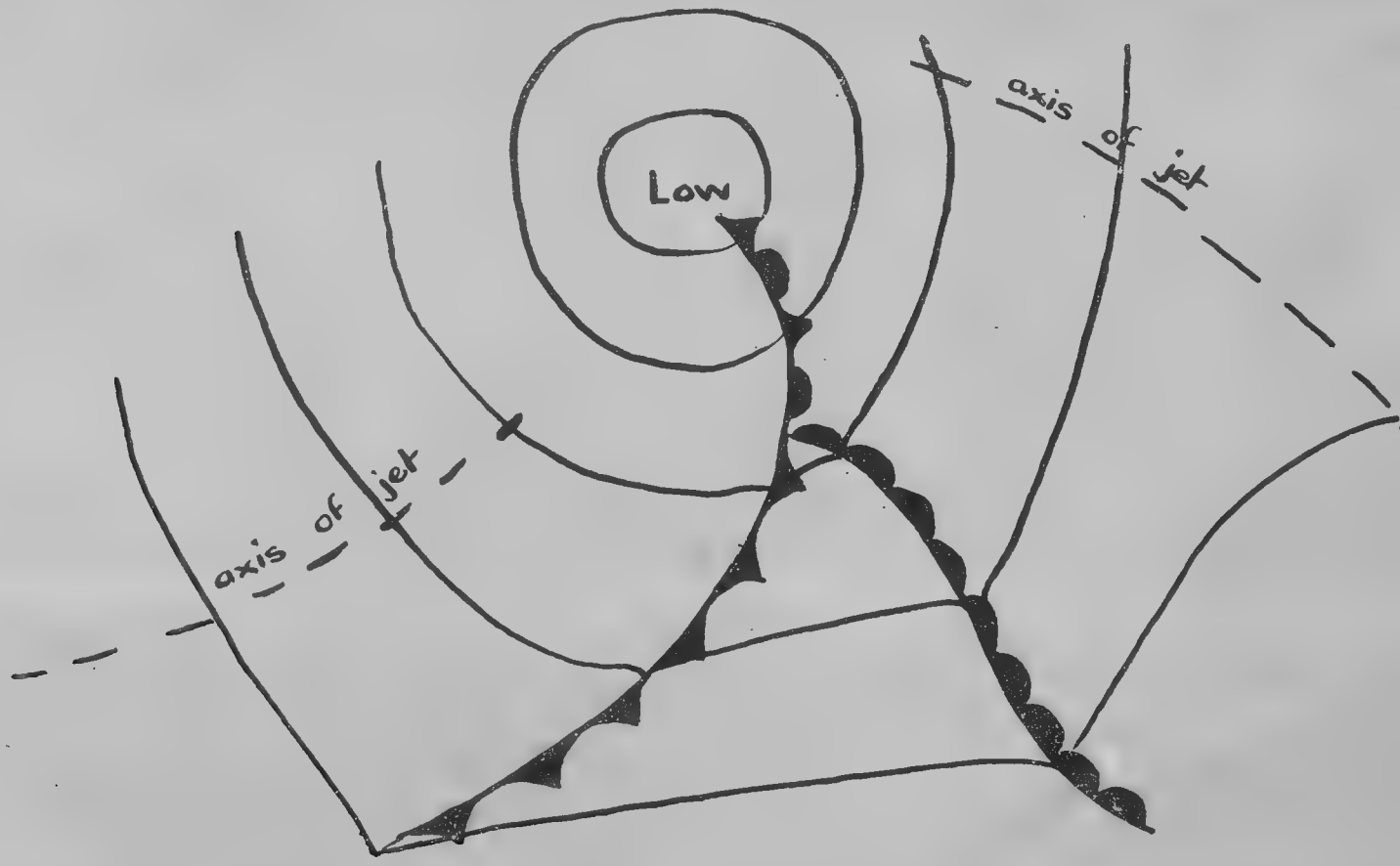


Figure 5.

Axis of "jet" in relation to depression just beginning to occlude. "Jet" discontinuous over the centre of depression.

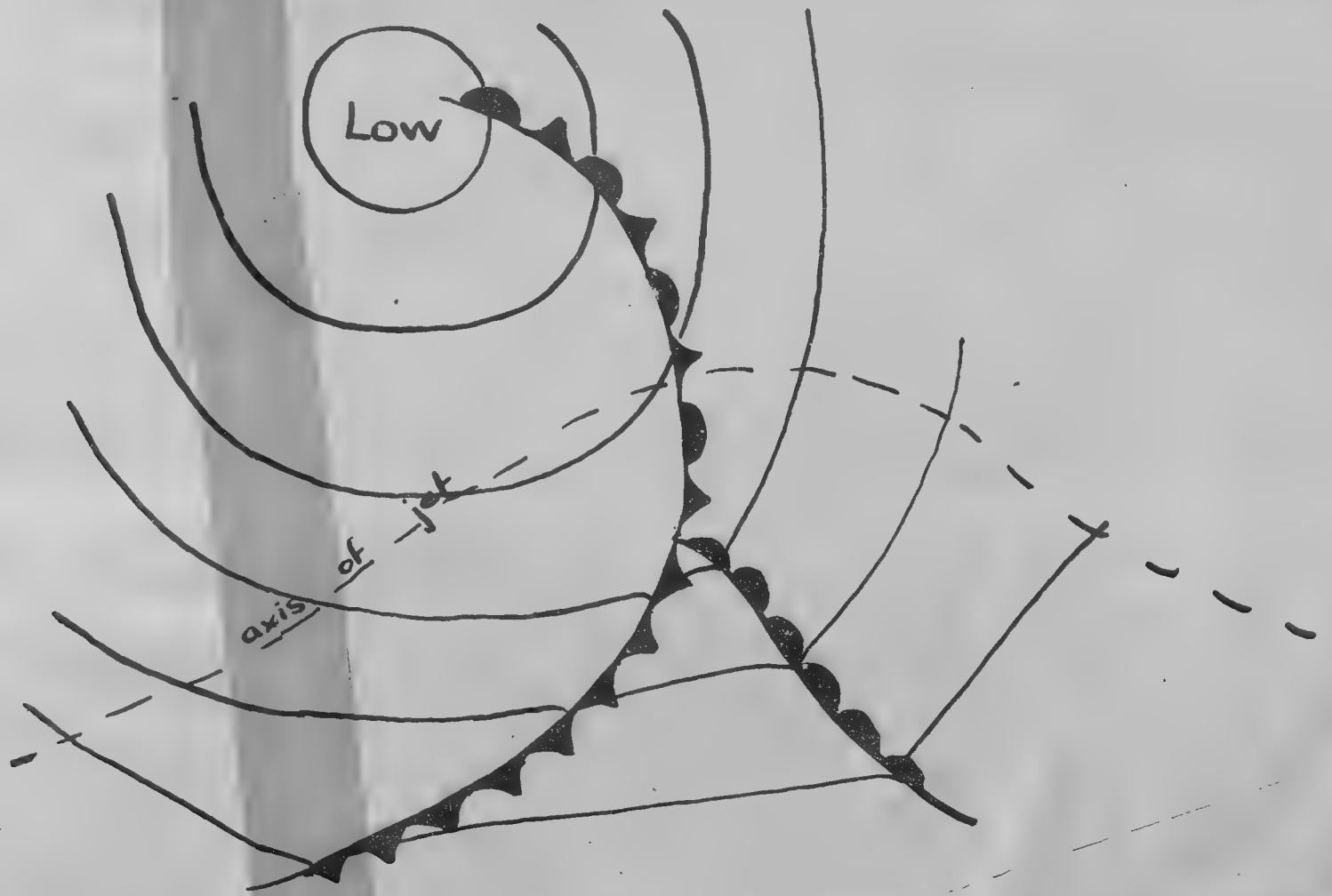


Figure 6.

Axis of "jet" in relation to well occluded depression. "Jet" not discontinuous.

M.R.P. 437
S.C.III/13

AIR MINISTRY

4th October, 1948

METEOROLOGICAL RESEARCH COMMITTEE

A Note on a More General Criterion of Turbulence (after Richardson) and its Application to the Atmosphere

by

J.K.Bannon

LIBRARY
WEATHER BUREAU

Summary

Richardson's criterion for the increase of turbulence in a steady stream is extended to include cases with a velocity shear in the horizontal as well as in the vertical.

From a qualitative discussion of this criterion it is inferred that turbulence may sometimes be most likely to occur just above the tropopause or just above a frontal or other surface of temperature discontinuity in the upper troposphere.

If horizontal shear is as important as vertical shear as a cause of turbulence then it may be deduced that the coefficient of eddy diffusivity of momentum in the horizontal is of the order of 10^9 c.g.s., in fair agreement with independent calculations of this quantity by others.

1. Richardson (1) has given a criterion to determine whether turbulence will increase or decrease ; he considered a turbulent flow in the horizontal in which there was a rate of change of mean velocity with height, i.e. a shear in the vertical. An attempt is here made to extend this criterion to include cases of wind shear in the horizontal. The development follows parallel lines to that of Brunt (2) for the original theory of Richardson.

Let us consider a general (mean) flow in the direction of x, (horizontal). Then the mean velocity U is a function of y and z, in general. (z vertical, y perpendicular to x and z).

At a point (y,z) where pressure, temperature and density are p, T and ρ respectively, let an eddy arrive from a point $(y - a, z - b)$, at which it had the normal temperature at that point, $T - a\,\partial T/\partial y - b\,\partial T/\partial z$.

At its new position its temperature will be

$$T - b\left(\frac{\partial T}{\partial z} + \Gamma\right) - a\frac{\partial T}{\partial y}$$

where Γ is the adiabatic lapse rate of temperature.

Its excess of density over the normal environment at that level will be

$$\frac{b\rho}{T}\left(\frac{\partial T}{\partial z} + \Gamma\right) + \frac{a\rho}{T}\frac{\partial T}{\partial y}$$

and the downward force per unit volume upon it due to this excess density will be

$$\frac{g\rho}{T}\left\{ b\left(\frac{\partial T}{\partial z} + \Gamma\right) + a\frac{\partial T}{\partial y}\right\}$$

If w' is the vertical component of the turbulent velocity, then the amount of work being done against gravity per unit volume per unit time

$$= g\rho\,\frac{\overline{w'b}}{T}\left(\frac{\partial T}{\partial z} + \Gamma\right) + g\rho\,\frac{\overline{w'a}}{T}\frac{\partial T}{\partial y} \qquad \ldots\ldots\ldots\ldots \quad (1)$$

The work done by the eddy stresses per unit volume is

$$\widehat{xz}\,\frac{\partial U}{\partial z} + \widehat{xy}\,\frac{\partial U}{\partial y} \qquad \ldots\ldots\ldots\ldots \quad (2)$$

where (with the usual notation of elasticity) $\widehat{xz}$ is the shearing force in direction x across a plane parallel to $z = 0$, and $\widehat{xy}$ the shearing force in direction x across a plane parallel to $y = 0$.

Expression (2) may be written

$$\rho\left\{ K_{mz}\left(\frac{\partial U}{\partial z}\right)^2 + K_{my}\left(\frac{\partial U}{\partial y}\right)^2 \right\} \qquad \ldots\ldots\ldots\ldots \quad (3)$$

where K_{mz} , K_{my} are the coefficients of eddy diffusivity of momentum upwards and horizontally across the stream, respectively. 7 Accordingly the condition that turbulence should increase may be written, from (1) and (3),

$$K_{mz}\left(\frac{\partial U}{\partial z}\right)^2 + K_{my}\left(\frac{\partial U}{\partial y}\right)^2 > \frac{g}{T}\left\{ \overline{w'b}\left(\frac{\partial T}{\partial z} + \Gamma\right) + \overline{w'a}\,\frac{\partial T}{\partial y}\right\} \ldots\ldots \quad (4)$$

$\overline{w'b}$ is usually denoted by K_{Tz} , the coefficient of eddy diffusivity of heat in the vertical.

The eddies will show a preference for the motion of individual small parcels of air to be along isentropic surfaces. Thus for w' positive, a will usually be in the direction down the isentropic surface through the point in question. (Since we are considering a steady, though turbulent, flow in direction x , the isentropic surfaces will have slope only in the y direction). The sign of $\overline{w'a}$ will thus depend on the sign of $\partial\phi/\partial y$, where ϕ denotes entropy; $\partial\phi/\partial y$ and $\overline{w'a}$ will have the same sign.

2. Qualitative application of the criterion

$$\phi = c_p \log_e T - AR \log_e p + \text{constant} \qquad \ldots\ldots\ldots\ldots \quad (5)$$

where c_p = specific heat of air at constant pressure (= 0.2396 cal/gm/°C)

R = gas constant for air. (= 2.87×10^6).

A = reciprocal of the mechanical equivalent of heat. (= 2.392×10^{-8}).

(See, e.g. Brunt (3)).

Or (5) may be written :-

---

7 The coefficients K_{mz} , K_{my} are not to be confused with the coefficients K_{mx} , K_{my} used by Brunt.

$$\phi = c_p \log T + Ag \int_0^z \frac{dz}{T} - AR \log p_s \quad \ldots\ldots\ldots (6)$$

where p_s = pressure at $z = 0$.

Hence

$$\frac{\partial \phi}{\partial y} \equiv \frac{c_p}{T} \frac{\partial T}{\partial y} - Ag \int_0^z \frac{1}{T^2} \frac{\partial T}{\partial y} dz - AR \frac{1}{p_s} \frac{\partial p_s}{\partial y} \quad \ldots\ldots\ldots (7)$$

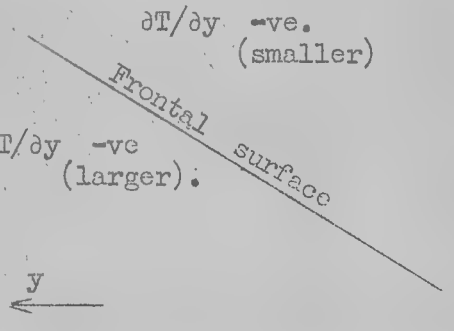
We may discuss equation (7) in relation to the atmosphere, remembering of course that the original assumptions, (steady straight flow in direction x), will not be satisfied in general. It is thought, however, that useful deductions may be made from the consideration of the special case which will be applicable to more general cases.

In the troposphere a large class of cases has $\partial T/\partial y$ of the same sign for all heights, though it is frequently considerably larger in the upper than in the lower troposphere ; in this class, if $\partial T/\partial y$ has values always of the same order of magnitude, then the second term on the right hand side of Eqn.(7) will be considerably smaller than the first, in the troposphere, even up to 40-50,000 ft. The third term is always small, normally about 1/10 the magnitude of the first. Thus it is seen that usually in the troposphere $\partial \phi/\partial y$ takes the sign of $\partial T/\partial y$.

It can occur, however, that $\partial \phi/\partial y$ has opposite sign to $\partial T/\partial y$; if, for example, $\partial T/\partial y$ has had a value L through most of the troposphere and then falls sharply to less than L/3 at 30,000 ft, (but retaining the same sign), then the second term on the right of Eqn.(7) will be greater than the first, just above 30,000 ft, since it depends entirely on the temperature structure in the air below that level while the first term depends only on the temperature gradient at that height.

This is not unusual just above the tropopause, though more often the temperature gradient in the horizontal is entirely reversed in sign at the tropopause. At greater heights, of course, the second term on the right of Eqn. (7) will again become less than the first (regardless of sign) so that normally in the stratosphere $\partial \phi/\partial y$ and $\partial T/\partial y$ will have the same sign.

A frontal surface may also lead to a temporary dissimilarity of the signs of $\partial \phi/\partial y$ and $\partial T/\partial y$; if $|\partial T/\partial y|$ is several times larger in the colder air, beneath the frontal surface, (see Figure - the wind, of course, is blowing into the plane of the paper), than in the warm air above, (not at all unusual), then just above the frontal surface $\partial T/\partial y$ and $\partial \phi/\partial y$ may have opposite signs.



The higher the discontinuity in $\partial T/\partial y$ occurs the more likely it is that $\partial \phi/\partial y$ and $\partial T/\partial y$ will have opposite signs since the absolute value of the second term on the right of Eqm.(7) will have more chance of being larger than the first if the integral is through a great height.

It is seen therefore that under special circumstances the second term on the right of expression (4) may be negative ; in these circumstances, other things being equal, turbulence is more likely to occur. These special circumstances sometimes occur for a limited depth just above the tropopause or just above a frontal or other surface of temperature discontinuity.

It is of interest to note that turbulence has often been noted

just above the tropopause and also just above frontal surfaces or other temperature discontinuities in the upper troposphere. (Bannon, (4)). It has not been determined, yet, whether, in these cases, $\partial\phi/\partial y$ and $\partial T/\partial y$ did in fact have dissimilar signs.

3. Relative magnitudes of K_{mz} and K_{my}

If the two terms on the left hand side of (4) are to be of the same order of magnitude, i.e. if horizontal shear is to be as important as vertical shear in causing turbulence, then we can derive an order of magnitude for K_{my}.

$$\partial U/\partial z = 5 \text{ mph}/1{,}000 \text{ ft}, \text{ i.e. } = 26.4 \text{ hours}^{-1}$$

and $$\partial U/\partial y = 2 \text{ mph}/10 \text{ miles}, \text{ i.e. } = 0.2 \text{ hours}^{-1}$$

are values somewhat larger than average but ones which are not unusual. If, with these values, the two terms on the left of expression (4) are of the same magnitude then

$$\frac{K_{my}}{K_{mz}} = \left(\frac{26.4}{0.2}\right)^2$$

$$= 132^2 = 1.7 \times 10^4$$

K_{mz} in the atmosphere is of the order of 10^5 c.g.s. above the surface friction layer ; (see, for example, various authors quoted by Haurwitz, Dynamic Meteorology, p.210, and Shaw, Manual of Meteorology Vol.IV, p.140).

Thus, in this case,

$$K_{my} = \text{order of } 1.7 \times 10^9 \text{ c.g.s}$$

Rossby (5) and,independently, Grimminger (6) have estimated that K_{my} is of the order 5×10^9 c.g.s and Priestley (7) has shown that a value of that magnitude is plausible in the atmosphere, from other considerations. It may be be inferred, therefore , that horizontal wind-shear of the scale experienced in the atmosphere is important in causing turbulence.

References

1. L.F.Richardson — Proc. Roy. Soc., A. 97, 1920, p.354.
2. D.Brunt — Physical and Dynamical Meteorology, 2nd Ed. p.237.
3. do. — do p.75.
4. J.K.Bannon — Meteorological Factors in the Occurrence of Turbulence at High Altitudes; Interim Report. M.R.P. 436, 1948.
5. C.G.Rossby — Proc. Fifth Int. Congress App. Mech., 1938.
6. G.Grimminger — Trans. Am. Geophys. Union, 1938.
7. C.H.B.Priestley — Quart. J. Roy. Met. Soc., 72 , 1946, p.284.

M.R.P. 437 (Revised)
S.C.III/13

25th January, 1949

AIR WEA... TECHNICAL ...

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

NOV 17 1952

A Note on a More General Criterion of Turbulence (after Richardson) and its Application to the Atmosphere

by

J.K.Bannon

Summary

Richardson's criterion for the increase of turbulence in a steady stream is extended to include cases with a velocity shear in the horizontal as well as in the vertical.

From a qualitative discussion of this criterion it is inferred that turbulence may sometimes be most likely to occur just above the tropopause, in agreement with observation, but the evidence is not conclusive.

If horizontal shear is as important as vertical shear as a cause of turbulence then it may be deduced that the coefficient of eddy diffusivity of momentum in the horizontal is of the order of 10^9 c.g.s., in fair agreement with independent calculations of this quantity by others.

1. Richardson (1) has given a criterion to determine whether turbulence will increase or decrease ; he considered a turbulent flow in the horizontal in which there was a rate of change of mean velocity with height, i.e. a shear in the vertical. An attempt is here made to extend this criterion to include cases of wind shear in the horizontal. The development follows parallel lines to that of Brunt (2) for the original theory of Richardson.

Let us consider a general (mean) flow in the direction of x, (horizontal). Then the mean velocity U is a function of y, z and t, in general. (z vertical , y perpendicular to x and z, t = time .)

At a point (y,z) where pressure, temperature and density are p, T and ρ respectively, let an eddy arrive from a point $(y - a, z - b)$ at which it had the normal temperature at that point, $T - a\,\partial T/Ty - b\,\partial T/\partial z$.

If changes in the eddy during its motion between these two points occurred adiabatically then the potential temperature , θ , of the eddy will have remained constant; i.e. on arrival at (y,z) its potential temperature will be

$$\theta - \Delta\theta = \theta - a\frac{\partial\theta}{\partial y} - b\frac{\partial\theta}{\partial z}$$

$$= \theta\left(1 - a\frac{\partial \log\theta}{\partial y} - b\frac{\partial \log\theta}{\partial z}\right)$$

$$= \theta\left(1 - \frac{a}{c_p}\frac{\partial\phi}{\partial y} - \frac{b}{c_p}\frac{\partial\phi}{\partial z}\right)$$

where

ϕ = entropy = $c_p \log\theta$, with the usual notation.

Since the eddy will have the same pressure as its surroundings, the difference in temperature between eddy and environment will be

$$\frac{T}{\theta}\,\Delta\theta$$

i.e. the temperature of the eddy on arrival at (y,z) will be

$$T - \frac{T}{c_p}\left(a\frac{\partial\phi}{\partial y} + b\frac{\partial\phi}{\partial z}\right)$$

Its excess of density over the normal environment at that level will therefore be

$$\frac{\rho}{c_p}\left(a\frac{\partial\phi}{\partial y} + b\frac{\partial\phi}{\partial z}\right)$$

and the downward force per unit volume upon it, due to this excess density, will be

$$\frac{g\rho}{c_p}\left(a\frac{\partial\phi}{\partial y} + b\frac{\partial\phi}{\partial z}\right)$$

If w' is the vertical component of the turbulent velocity, then the amount of work being done against gravity per unit volume per unit time

$$= \frac{g\rho}{c_p}\left(\overline{aw'}\frac{\partial\phi}{\partial y} + \overline{bw'}\frac{\partial\phi}{\partial z}\right) \qquad \ldots\ldots\ldots(1)$$

The work done by the eddy stresses per unit volume is

$$\widehat{xz}\frac{\partial U}{\partial z} + \widehat{xy}\frac{\partial U}{\partial y} \qquad \ldots\ldots\ldots\ldots(2)$$

where (with the usual notation of elasticity) $\widehat{xz}$ is the shearing force in direction x across a plane parallel to $z = 0$ and $\widehat{xy}$ the shearing force in direction x across a plane parallel to $y = 0$.

Expression (2) may be written

$$\rho\left\{K_{mz}\left(\frac{\partial U}{\partial z}\right)^2 + K_{my}\left(\frac{\partial U}{\partial y}\right)^2\right\} \qquad \ldots\ldots\ldots\ldots(3)$$

where K_{mz}, K_{my} are the coefficients of eddy diffusivity of momentum upwards and horizontally, respectively. [/] Accordingly the condition that turbulence should increase may be written, from (1) and (3),

$$\left.\begin{aligned} K_{my}\left(\frac{\partial U}{\partial y}\right)^2 + K_{mz}\left(\frac{\partial U}{\partial z}\right)^2 &> \frac{g}{c_p}\left(\overline{aw'}\frac{\partial\phi}{\partial y} + \overline{bw'}\frac{\partial\phi}{\partial z}\right) \\ \text{or} \qquad &> \frac{g}{T}\left(\frac{\partial T}{\partial z} + \Gamma\right)\overline{bw'} + \frac{g}{c_p}\overline{aw'}\frac{\partial\phi}{\partial y} \end{aligned}\right\} \ldots (4)$$

where Γ is the adiabatic lapse rate of temperature with height.

[/] The coefficients K_{mz}, K_{my} are not to be confused with the coefficients K_{mx}, K_{my} used by Brunt.

$\overline{bw'}$ is usually denoted by K_{Tz}, the coefficient of eddy diffusivity of heat in the vertical.

The eddies will show a preference for the motion of individual small parcels of air to be along isentropic surfaces. Thus for w' positive, a will usually be in the direction down the isentropic surface through the point in question. (Since we are considering a steady, though turbulent, flow in direction x, the isentropic surfaces will have slope only in the y direction). The sign of $\overline{aw'}$ will thus depend on the sign of $\partial\phi/\partial y$; $\partial\phi/\partial y$ and $\overline{aw'}$ will have the same sign and the second term on the right of (4) will thus always be positive, (as will also the first term, for a statically stable atmosphere).

2. Application of the criterion

The criterion (4) is to determine whether turbulence is increasing or decreasing ; strictly it applies to a straight flow which may be changing with respect to time but is the same all along its length at any one instant. $\overline{aw'}$ and $\overline{bw'}$ will, in general, be functions of y , z and t.

The right hand side of (4) is a measure of the reluctance of the atmosphere to become more turbulent.

From (4) it is seen that, other things being equal, turbulence is more likely to be increasing if $\partial\phi/\partial y$ and $\partial\phi/\partial z$ are small. $\partial\phi/\partial z$ is small, of course, when the stability of the air is small. In the free atmosphere $\partial\phi/\partial y$ is often zero at or near the tropopause [/] and hence from (4), other things being equal, turbulence would be expected to occur there at times when not present at other levels. Turbulence has frequently been observed at or near the tropopause with smooth layers above and below. (Bannon, (3)). This may be considered as observational confirmation of the importance of the horizontal eddy terms in (4) though it must be remembered that just above the tropopause $\partial\phi/\partial z$ is considerably larger than usual tropospheric values, i.e. though one term of the right hand side of (4) may be smaller than previously, on ascending through the tropopause, the other term increases; the evidence is suggestive rather than conclusive.

3. Relative magnitudes of K_{mz} and K_{my}

If the two terms on the left hand side of (4) are to be of the same order of magnitude, i.e. if horizontal shear is to be as important as vertical shear in causing turbulence, then we can derive an order of magnitude for K_{my}.

$$\partial U/\partial z = 5 \text{ mph}/1{,}000 \text{ ft}, \text{ i.e. } = 26.4 \text{ hours}^{-1}$$

$$\text{and } \partial U/\partial y = 2 \text{ mph}/10 \text{ miles}, \text{ i.e. } = 0.2 \text{ hours}^{-1}$$

are values somewhat larger than average but ones which are not unusual. If, with these values, the two terms on the left of expression (4) are of the same magnitude then

---

[/] As an illustration of this, for steady straight flow in direction x ,

$$\frac{\partial U}{\partial z} = -\frac{g}{\lambda}\frac{\partial \phi}{\partial y} \qquad \text{(a form of the thermal wind equation)}$$

where λ is the Coriolis factor. So that if $\partial U/\partial z = 0$ then $\partial\phi/\partial y = 0$. It is well known that U is often a maximum at the tropopause.

$$\frac{K_{my}}{K_{mz}} = \left(\frac{26.4}{0.2}\right)^2$$

$$= 132^2 = 1.7 \times 10^4$$

K_{mz} in the atmosphere is of the order of 10^5 c.g.s above the surface friction layer; (see, for example, various authors quoted by Haurwitz, Dynamic Meteorology, p.210 and Shaw, Manual of Meteorology, Vol.IV, p.140).

Thus in this case,

$$K_{my} = \text{order of } 1.7 \times 10^9 \text{ c.g.s.}$$

Rossby (4) and, independently, Grimminger (5) have estimated that K_{my} is of the order 5×10^9 c.g.s. and Priestley (6) has shown that a value of that magnitude is plausible in the atmosphere, from other considerations. It may be inferred, therefore, that horizontal wind-shear of the scale experienced in the atmosphere is important in causing turbulence.

References

1. L.F. Richardson — Proc. Roy. Soc.,A.97, 1920, p.354.

2. D.Brunt — Physical and Dynamical Meteorology, 2nd Ed.,p.237.

3. J.K.Bannon — Meteorological Factors in the Occurrence of Turbulence at High Altitudes; Interim Report. M.R.P.436, 1948.

4. C.G.Rossby — Proc. Fifth Int. Congress App.Mech., 1938.

5. G.Grimminger — Trans. Am.Geophys. Union, 1938.

6. C.H.B.Priestley — Quart. J. Roy. Met. Soc., 72 , 1946, p.284.